Chapter Twelve

Microwave Amplifier Design

Signal amplification is one of the most basic and prevalent circuit functions in modern RF and microwave systems. Early microwave amplifiers relied on tubes, such as klystrons and traveling-wave tubes, or solid-state reflection amplifiers based on the negative resistance characteristics of tunnel or varactor diodes. However, due to the dramatic improvements and innovations in solid-state technology that have occurred since the 1970s, most RF and microwave amplifiers today use transistor devices such as Si BJTs, GaAs or SiGe HBTs, Si MOSFETs, GaAs MESFETs, or GaAs or GaN HEMTs [1–5]. Microwave transistor amplifiers are rugged, low-cost, and reliable and can be easily integrated in both hybrid and monolithic integrated circuitry. Transistor amplifiers can be used at frequencies in excess of 100 GHz in a wide range of applications requiring small size, low noise figure, broad bandwidth, and medium to high power capacity. Although microwave tubes are still useful for very high power and/or very high frequency applications, continuing improvement in the performance of microwave transistors is steadily reducing the need for microwave tubes.

Our discussion of transistor amplifier design will primarily rely on the terminal characteristics of the transistor, as represented by either scattering parameters or one of the equivalent circuit models introduced in the previous chapter. We will begin with some general definitions of two-port power gains that are useful for amplifier design and then discuss the subject of stability. These results will then be applied to single-stage transistor amplifiers, including designs for maximum gain, specified gain, and low noise figure. Broadband balanced and distributed amplifiers are discussed in Section 12.4. We conclude with a brief treatment of transistor power amplifiers.

12.1 TWO-PORT POWER GAINS

In this section we develop several expressions for the gain and stability of a general two-port amplifier circuit in terms of the scattering parameters of the transistor. These results

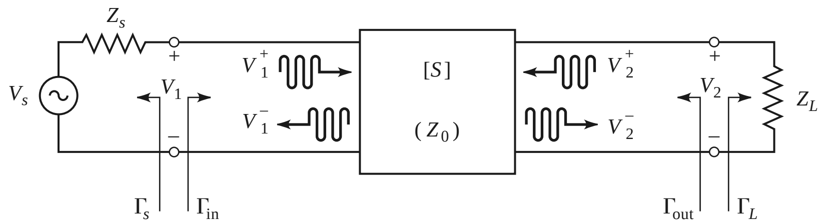


FIGURE 12.1 A two-port network with arbitrary source and load impedances.

will be used in the following sections for amplifier design and in Chapter 13 for oscillator design.

Definition of Two-Port Power Gains

Consider an arbitrary two-port network, characterized by its scattering matrix [S], connected to source and load impedances Z_S and Z_L, respectively, as shown in Figure 12.1. We will derive expressions for three types of power gain in terms of the scattering parameters of the two-port network and the reflection coefficients, Γ_S and Γ_L, of the source and load.

- *Power gain* $= G = P_L/P_{\text{in}}$ is the ratio of power dissipated in the load Z_L to the power delivered to the input of the two-port network. This gain is independent of Z_S, although the characteristics of some active devices may be dependent on Z_S.
- *Available power gain* $= G_A = P_{\text{avn}}/P_{\text{avs}}$ is the ratio of the power available from the two-port network to the power available from the source. This assumes conjugate matching of both the source and the load, and depends on Z_S, but not Z_L.
- *Transducer power gain* $= G_T = P_L/P_{\text{avs}}$ is the ratio of the power delivered to the load to the power available from the source. This depends on both Z_S and Z_L.

These definitions differ primarily in the way the source and load are matched to the two-port device; if the input and output are both conjugately matched to the two-port device, then the gain is maximized and $G = G_A = G_T$.

With reference to Figure 12.1, the reflection coefficient seen looking toward the load is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}, \tag{12.1a}$$

while the reflection coefficient seen looking toward the source is

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}, \tag{12.1b}$$

where Z_0 is the characteristic impedance reference for the scattering parameters of the two-port network.

In general, the input impedance of the terminated two-port network will be mismatched with a reflection coefficient given by Γ_{in}, which can be determined using a signal flow graph (see Example 4.7) or by the following analysis. From the definition of the scattering parameters, and the fact that $V_2^+ = \Gamma_L V_2^-$, we have

$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ = S_{11}V_1^+ + S_{12}\Gamma_L V_2^-, \tag{12.2a}$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ + S_{22}\Gamma_L V_2^-. \tag{12.2b}$$

Eliminating V_2^- from (12.2a) and solving for V_1^-/V_1^+ gives

$$\Gamma_{\text{in}} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0}, \tag{12.3a}$$

where Z_{in} is the impedance seen looking into port 1 of the terminated network. Similarly, the reflection coefficient seen looking into port 2 of the network when port 1 is terminated by Z_S is

$$\Gamma_{\text{out}} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}. \tag{12.3b}$$

By voltage division,

$$V_1 = V_S \frac{Z_{\text{in}}}{Z_S + Z_{\text{in}}} = V_1^+ + V_1^- = V_1^+ (1 + \Gamma_{\text{in}}).$$

Using

$$Z_{\text{in}} = Z_0 \frac{1 + \Gamma_{\text{in}}}{1 - \Gamma_{\text{in}}}$$

from (12.3a) and solving for V_1^+ in terms of V_S gives

$$V_1^+ = \frac{V_S}{2} \frac{(1 - \Gamma_S)}{(1 - \Gamma_S \Gamma_{\text{in}})}. \tag{12.4}$$

If peak values are assumed for all voltages, the average power delivered to the network is

$$P_{\text{in}} = \frac{1}{2Z_0} \left|V_1^+\right|^2 \left(1 - |\Gamma_{\text{in}}|^2\right) = \frac{|V_S|^2}{8Z_0} \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S\Gamma_{\text{in}}|^2} \left(1 - |\Gamma_{\text{in}}|^2\right), \tag{12.5}$$

where (12.4) was used. The power delivered to the load is

$$P_L = \frac{\left|V_2^-\right|^2}{2Z_0} \left(1 - |\Gamma_L|^2\right). \tag{12.6}$$

Solving for V_2^- from (12.2b), substituting into (12.6), and using (12.4) gives

$$P_L = \frac{\left|V_1^+\right|^2}{2Z_0} \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right) |1 - \Gamma_S|^2}{|1 - S_{22}\Gamma_L|^2 \, |1 - \Gamma_S\Gamma_{\text{in}}|^2}. \tag{12.7}$$

The power gain can then be expressed as

$$G = \frac{P_L}{P_{\text{in}}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{\text{in}}|^2\right) |1 - S_{22}\Gamma_L|^2}. \tag{12.8}$$

The power available from the source, P_{avs}, is the maximum power that can be delivered to the network. This occurs when the input impedance of the terminated network is conjugately matched to the source impedance, as discussed in Section 2.6. Thus, from (12.5),

$$P_{\text{avs}} = P_{\text{in}} \Big|_{\Gamma_{\text{in}} = \Gamma_S^*} = \frac{|V_S|^2}{8Z_0} \frac{|1 - \Gamma_S|^2}{\left(1 - |\Gamma_S|^2\right)}. \tag{12.9}$$

Similarly, the power available from the network, P_{avn}, is the maximum power that can be delivered to the load. Thus, from (12.7),

$$P_{avn} = P_L\bigg|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2\left(1-|\Gamma_{out}|^2\right)|1-\Gamma_S|^2}{\left|1-S_{22}\Gamma_{out}^*\right|^2|1-\Gamma_S\Gamma_{in}|^2}\bigg|_{\Gamma_L=\Gamma_{out}^*}. \tag{12.10}$$

In (12.10), Γ_{in} must be evaluated for $\Gamma_L = \Gamma_{out}^*$. From (12.3a), it can be shown that

$$|1-\Gamma_S\Gamma_{in}|^2\bigg|_{\Gamma_L=\Gamma_{out}^*} = \frac{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{out}|^2\right)^2}{\left|1-S_{22}\Gamma_{out}^*\right|^2},$$

which reduces (12.10) to

$$P_{avn} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2|1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{out}|^2\right)}. \tag{12.11}$$

Observe that P_{avs} and P_{avn} have been expressed in terms of the source voltage, V_S, which is independent of the input or load impedances. There would be confusion if these quantities were expressed in terms of V_1^+ since V_1^+ is different for each of the calculations of P_L, P_{avs}, and P_{avn}.

Using (12.11) and (12.9), we obtain the available power gain as

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{out}|^2\right)}. \tag{12.12}$$

From (12.7) and (12.9), the transducer power gain is

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2}. \tag{12.13}$$

A special case of the transducer power gain occurs when both the input and output are matched for zero reflection (in contrast to conjugate matching). Then $\Gamma_L = \Gamma_S = 0$, and (12.13) reduces to

$$G_T = |S_{21}|^2. \tag{12.14}$$

Another special case is the *unilateral transducer power gain*, G_{TU}, where $S_{12} = 0$ (or is negligibly small). This nonreciprocal characteristic is approximately true for many transistors devices. From (12.3a), $\Gamma_{in} = S_{11}$ when $S_{12} = 0$, so (12.13) gives the unilateral transducer power gain as

$$G_{TU} = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-S_{11}\Gamma_S|^2|1-S_{22}\Gamma_L|^2}. \tag{12.15}$$

EXAMPLE 12.1 COMPARISON OF POWER GAIN DEFINITIONS

A silicon bipolar junction transistor has the following scattering parameters at 1.0 GHz, with a 50 Ω reference impedance:

$$S_{11} = 0.38\angle{-158°}$$
$$S_{12} = 0.11\angle 54°$$
$$S_{21} = 3.50\angle 80°$$
$$S_{22} = 0.40\angle{-43°}$$

The source impedance is $Z_S = 25\ \Omega$ and the load impedance is $Z_L = 40\ \Omega$. Compute the power gain, the available power gain, and the transducer power gain.

Solution
From (12.1a) and (12.1b) the reflection coefficients at the source and load are

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{25 - 50}{25 + 50} = -0.333,$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{40 - 50}{40 + 50} = -0.111.$$

From (12.3a) and (12.3b) the reflection coefficients seen looking at the input and output of the terminated network are

$$\Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.365\angle - 152^\circ,$$

$$\Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.545\angle - 43^\circ.$$

Then from (12.8) the power gain is

$$G = \frac{|S_{21}|^2\left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{\text{in}}|^2\right)|1 - S_{22}\Gamma_L|^2} = 13.1.$$

From (12.12) the available power gain is

$$G_A = \frac{|S_{21}|^2\left(1 - |\Gamma_S|^2\right)}{|1 - S_{11}\Gamma_S|^2\left(1 - |\Gamma_{\text{out}}|^2\right)} = 19.8.$$

From (12.13) the transducer power gain is

$$G_T = \frac{|S_{21}|^2\left(1 - |\Gamma_S|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_S\Gamma_{\text{in}}|^2\,|1 - S_{22}\Gamma_L|^2} = 12.6.$$

■

Further Discussion of Two-Port Power Gains

A single-stage microwave transistor amplifier can be modeled by the circuit of Figure 12.2, where matching networks are used on both sides of the transistor to transform the input and output impedance Z_0 to the source and load impedances Z_S and Z_L. The most useful gain definition for amplifier design is the transducer power gain of (12.13), which accounts for both source and load mismatch. From (12.13) we can define separate effective gain factors for the input (source) matching network, the transistor itself, and the output (load)

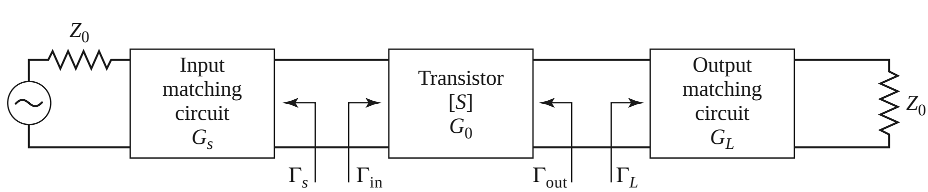


FIGURE 12.2 The general transistor amplifier circuit.

matching network as follows:

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - \Gamma_{\text{in}}\Gamma_S|^2}, \tag{12.16a}$$

$$G_0 = |S_{21}|^2, \tag{12.16b}$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}. \tag{12.16c}$$

The overall transducer gain is then $G_T = G_S G_0 G_L$. The effective gains G_S and G_L of the matching networks may be greater than unity. This is because the unmatched transistor would incur power loss due to reflections at the input and output of the transistor, and the matching sections can reduce these losses.

If the transistor is unilateral, so that $S_{12} = 0$ (or is small enough to be ignored), then (12.3) reduces to $\Gamma_{\text{in}} = S_{11}$, $\Gamma_{\text{out}} = S_{22}$, and the unilateral transducer gain reduces to $G_{TU} = G_S G_0 G_L$, where

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2}, \tag{12.17a}$$

$$G_0 = |S_{21}|^2, \tag{12.17b}$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}. \tag{12.17c}$$

The above results have been derived using the scattering parameters of the transistor, but it is possible to obtain alternative expressions for gain in terms of the equivalent circuit parameters of the transistor. As an example, consider the evaluation of the unilateral transducer gain for a conjugately matched FET using the equivalent circuit of Figure 11.21 (with $C_{gd} = 0$). To conjugately match the transistor we choose source and load impedances as shown in Figure 12.3. Setting the series source inductive reactance $X = 1/\omega C_{gs}$ will make $Z_{\text{in}} = Z_S^*$, and setting the shunt load inductive susceptance $B = -\omega C_{ds}$ will make $Z_{\text{out}} = Z_L^*$; this effectively eliminates the reactive elements from the transistor equivalent circuit. Then by voltage division $V_c = V_S/2j\omega R_i C_{gs}$, and the gain can be easily evaluated as

$$G_{TU} = \frac{P_L}{P_{\text{avs}}} = \frac{\frac{1}{8}\,|g_m V_c|^2\, R_{ds}}{\frac{1}{8}\,|V_S|^2\,/R_i} = \frac{g_m^2 R_{ds}}{4\omega^2 R_i C_{gs}^2} = \frac{R_{ds}}{4R_i}\left(\frac{f_T}{f}\right)^2. \tag{12.18}$$

where the last step has been written in terms of the cutoff frequency, f_T, from (11.24). This shows the interesting result that the gain of a conjugately matched FET amplifier drops off as $1/f^2$, or 6 dB per octave. A photograph of a low-noise MMIC amplifier is shown in Figure 12.4.

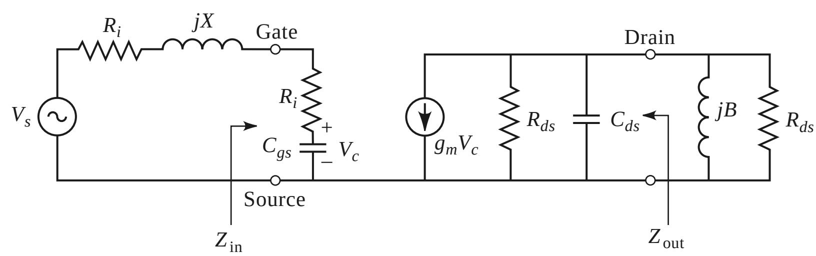


FIGURE 12.3 Unilateral FET equivalent circuit and source and load terminations for the calculation of unilateral transducer power gain.

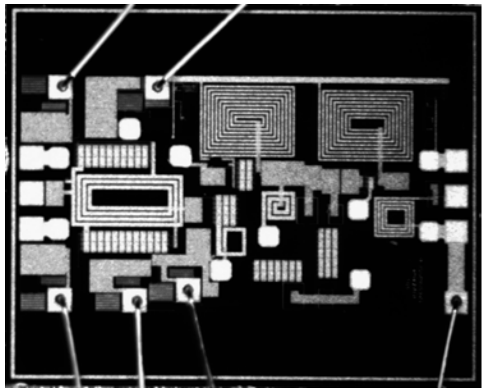


FIGURE 12.4 Photograph of a low-noise MMIC amplifier that is switchable between 2.4, 3.6, and 5.8 GHz. The amplifier uses pHEMTs in a cascode configuration with source inductance, followed by a common source stage with feedback. Gain is approximately 13 dB in each band. Chip dimensions are 1.85 mm by 1 mm.

Courtesy of J. Shatzman and R. W. Jackson of the University of Massachusetts at Amherst and H. Yu of TriQuint, Lowell, Mass.

12.2 STABILITY

We now discuss the necessary conditions for a transistor amplifier to be stable. In the circuit of Figure 12.2, oscillation is possible if either the input or output port impedance has a negative real part; this would then imply that $|\Gamma_{in}| > 1$ or $|\Gamma_{out}| > 1$. Because Γ_{in} and Γ_{out} depend on the source and load matching networks, the stability of the amplifier depends on Γ_S and Γ_L as presented by the matching networks. Thus, we define two types of stability:

- *Unconditional stability*: The network is unconditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ for all passive source and load impedances (i.e., $|\Gamma_S| < 1$ and $|\Gamma_L| < 1$).
- *Conditional stability*: The network is conditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ only for a certain range of passive source and load impedances. This case is also referred to as *potentially unstable*.

Note that the stability condition of an amplifier circuit is usually frequency dependent since the input and output matching networks generally depend on frequency. It is therefore possible for an amplifier to be stable at its design frequency but unstable at other frequencies. Careful amplifier design should consider this possibility. We must also point out that the following discussion of stability is limited to two-port amplifier circuits of the type shown in Figure 12.2, and where the scattering parameters of the active device can be measured without oscillations over the frequency band of interest. The rigorous general treatment of stability requires that the network scattering parameters (or other network parameters) have no poles in the right-half complex frequency plane, in addition to the conditions that $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ [6]. This can be a difficult assessment in practice, but for the special case considered here, where the scattering parameters are known to be pole free (as confirmed by measurability), the following stability conditions are adequate.

Stability Circles

Applying the above requirements for unconditional stability to (12.3) gives the following conditions that must be satisfied by Γ_S and Γ_L if the amplifier is to be unconditionally stable:

$$|\Gamma_{\text{in}}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1, \tag{12.19a}$$

$$|\Gamma_{\text{out}}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1. \tag{12.19b}$$

If the device is unilateral ($S_{12} = 0$), these conditions reduce to the simple results that $|S_{11}| < 1$ and $|S_{22}| < 1$ are sufficient for unconditional stability. Otherwise, the inequalities of (12.19) define a range of values for Γ_S and Γ_L where the amplifier will be stable. Finding this range for Γ_S and Γ_L can be facilitated by using the Smith chart and plotting the input and output *stability circles*. The stability circles are defined as the loci in the Γ_L (or Γ_S) plane for which $|\Gamma_{\text{in}}| = 1$ (or $|\Gamma_{\text{out}}| = 1$). The stability circles then define the boundaries between stable and potentially unstable regions of Γ_S and Γ_L. Γ_S and Γ_L must lie on the Smith chart ($|\Gamma_S| < 1$, $|\Gamma_L| < 1$ for passive matching networks).

We can derive the equation for the output stability circle as follows. First use (12.19a) to express the condition that $|\Gamma_{\text{in}}| = 1$ as

$$\left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1, \tag{12.20}$$

or

$$|S_{11}(1 - S_{22}\Gamma_L) + S_{12}S_{21}\Gamma_L| = |1 - S_{22}\Gamma_L|.$$

Now define Δ as the determinant of the scattering matrix:

$$\Delta = S_{11}S_{22} - S_{12}S_{21}. \tag{12.21}$$

Then we can write the above result as

$$|S_{11} - \Delta\Gamma_L| = |1 - S_{22}\Gamma_L|. \tag{12.22}$$

Now square both sides and simplify to obtain

$$|S_{11}|^2 + |\Delta|^2|\Gamma_L|^2 - \left(\Delta\Gamma_L S_{11}^* + \Delta^*\Gamma_L^* S_{11}\right) = 1 + |S_{22}|^2|\Gamma_L|^2 - \left(S_{22}^*\Gamma_L^* + S_{22}\Gamma_L\right)$$

$$\left(|S_{22}|^2 - |\Delta|^2\right)\Gamma_L\Gamma_L^* - \left(S_{22} - \Delta S_{11}^*\right)\Gamma_L - \left(S_{22}^* - \Delta^* S_{11}\right)\Gamma_L^* = |S_{11}|^2 - 1$$

$$\Gamma_L\Gamma_L^* - \frac{\left(S_{22} - \Delta S_{11}^*\right)\Gamma_L + \left(S_{22}^* - \Delta^* S_{11}\right)\Gamma_L^*}{|S_{22}|^2 - |\Delta|^2} = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2}. \tag{12.23}$$

Next, complete the square by adding $\left|S_{22} - \Delta S_{11}^*\right|^2 / \left(|S_{22}|^2 - |\Delta|^2\right)^2$ to both sides:

$$\left| \Gamma_L - \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} \right|^2 = \frac{\left|S_{11}^2\right| - 1}{|S_{22}|^2 - |\Delta|^2} + \frac{\left|S_{22} - \Delta S_{11}^*\right|^2}{\left(|S_{22}|^2 - |\Delta|^2\right)^2},$$

or

$$\left| \Gamma_L - \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} \right| = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|. \tag{12.24}$$

In the complex Γ plane, an equation of the form $|\Gamma - C| = R$ represents a circle having a center at C (a complex number) and a radius R (a real number). Thus, (12.24) defines the output stability circle with a center C_L and radius R_L, where

$$C_L = \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} \qquad \text{(center)}, \tag{12.25a}$$

$$R_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right| \qquad \text{(radius)}. \tag{12.25b}$$

Similar results can be obtained for the input stability circle by interchanging S_{11} and S_{22}:

$$C_S = \frac{\left(S_{11} - \Delta S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} \qquad \text{(center)}, \tag{12.26a}$$

$$R_S = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right| \qquad \text{(radius)}. \tag{12.26b}$$

Given the scattering parameters of the transistor, we can plot the input and output stability circles to define where $|\Gamma_{\text{in}}| = 1$ and $|\Gamma_{\text{out}}| = 1$. On one side of the input stability circle we will have $|\Gamma_{\text{out}}| < 1$, while on the other side we will have $|\Gamma_{\text{out}}| > 1$. Similarly, we will have $|\Gamma_{\text{in}}| < 1$ on one side of the output stability circle, and $|\Gamma_{\text{in}}| > 1$ on the other side. We need to determine which areas on the Smith chart represent the stable region, for which $|\Gamma_{\text{in}}| < 1$ and $|\Gamma_{\text{out}}| < 1$.

Consider the output stability circles plotted in the Γ_L plane for $|S_{11}| < 1$ and $|S_{11}| > 1$, as shown in Figure 12.5. If we set $Z_L = Z_0$, then $\Gamma_L = 0$, and (12.19a) shows that $|\Gamma_{\text{in}}| = |S_{11}|$. Now if $|S_{11}| < 1$, then $|\Gamma_{\text{in}}| < 1$, so $\Gamma_L = 0$ must be in a stable region. This means that the center of the Smith chart ($\Gamma_L = 0$) is in the stable region, so all of the Smith chart ($|\Gamma_L| < 1$) that is exterior to the stability circle defines the stable range for Γ_L. This region is shaded in Figure 12.5a. Alternatively, if we set $Z_L = Z_0$ but have $|S_{11}| > 1$, then $|\Gamma_{\text{in}}| > 1$ for $\Gamma_L = 0$, and the center of the Smith chart must be in an unstable region. In this case the stable region is the inside region of the stability circle that intersects the Smith chart, as illustrated in Figure 12.5b. Similar results apply to the input stability circle.

If the device is unconditionally stable, the stability circles must be completely outside (or totally enclose) the Smith chart. We can state this result mathematically as

$$\big||C_L| - R_L\big| > 1 \quad \text{for} \quad |S_{11}| < 1, \tag{12.27a}$$

$$\big||C_S| - R_S\big| > 1 \quad \text{for} \quad |S_{22}| < 1. \tag{12.27b}$$

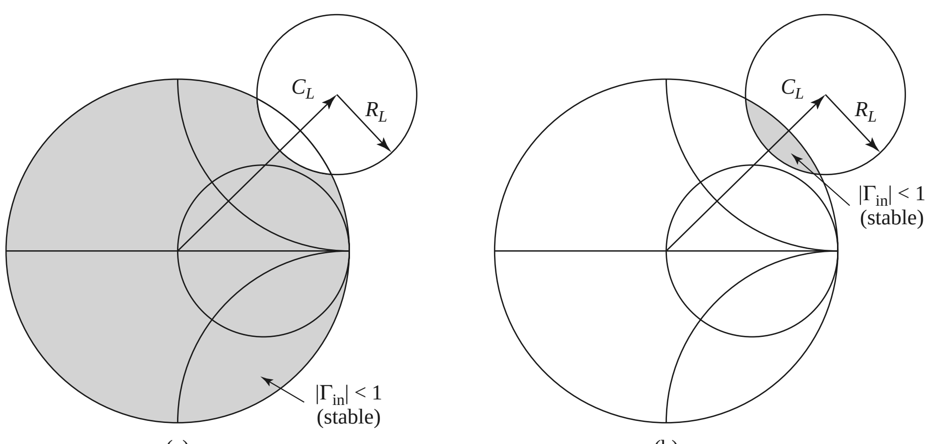


FIGURE 12.5 Output stability circles for a conditionally stable device. (a) $|S_{11}| < 1$. (b) $|S_{11}| > 1$.

If $|S_{11}| > 1$ or $|S_{22}| > 1$, the amplifier cannot be unconditionally stable because we can always have a source or load impedance of Z_0 leading to $\Gamma_S = 0$ or $\Gamma_L = 0$, thus causing $|\Gamma_{in}| > 1$ or $|\Gamma_{out}| > 1$. If the device is only conditionally stable, operating points for Γ_S and Γ_L must be chosen in stable regions, and it is good practice to check stability at several frequencies over the range where the device operates. Also note that the scattering parameters of a transistor depend on the bias conditions, and so stability will also depend on bias conditions. If it is possible to accept a design with less than maximum gain, a transistor can usually be made to be unconditionally stable by using resistive loading.

Tests for Unconditional Stability

The stability circles discussed above can be used to determine regions for Γ_S and Γ_L where the amplifier circuit will be conditionally stable, but simpler tests can be used to determine unconditional stability. One of these is the $K - \Delta$ *test*, where it can be shown that a device will be unconditionally stable if *Rollet's condition*, defined as

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1, \tag{12.28}$$

along with the auxiliary condition that

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1, \tag{12.29}$$

are simultaneously satisfied. These two conditions are necessary and sufficient for unconditional stability, and are easily evaluated. If the device scattering parameters do not satisfy the $K - \Delta$ test, the device is not unconditionally stable, and stability circles must be used to determine if there are values of Γ_S and Γ_L for which the device will be conditionally stable. Also recall that we must have $|S_{11}| < 1$ and $|S_{22}| < 1$ if the device is to be unconditionally stable.

While the $K - \Delta$ test of (12.28)–(12.29) is a mathematically rigorous condition for unconditional stability, it cannot be used to compare the relative stability of two or more devices because it involves constraints on two separate parameters. Recently, however, a new criterion has been proposed [7] that combines the scattering parameters in a test involving only a single parameter, μ, defined as

$$\mu = \frac{1 - |S_{11}|^2}{\left|S_{22} - \Delta S_{11}^*\right| + |S_{12}S_{21}|} > 1. \tag{12.30}$$

Thus, if $\mu > 1$, the device is unconditionally stable. In addition, it can be said that larger values of μ imply greater stability.

We can derive the μ-test of (12.30) by starting with the expression from (12.3b) for Γ_{out}:

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = \frac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S}, \tag{12.31}$$

where Δ is the determinant of the scattering matrix defined in (12.21). Unconditional stability implies that $|\Gamma_{out}| < 1$ for any passive source termination, Γ_S. The reflection coefficient for a passive source impedance must lie within the unit circle on a Smith chart, and the outer boundary of this circle can be written as $\Gamma_S = e^{j\phi}$. The expression given in (12.31) maps this circle into another circle in the Γ_{out} plane. We can show this by substituting $\Gamma_S = e^{j\phi}$ into (12.31) and solving for $e^{j\phi}$:

$$e^{j\phi} = \frac{S_{22} - \Gamma_{out}}{\Delta - S_{11}\Gamma_{out}}.$$

Taking the magnitude of both sides gives

$$\left|\frac{S_{22} - \Gamma_{out}}{\Delta - S_{11}\Gamma_{out}}\right| = 1.$$

Squaring both sides and expanding gives

$$|\Gamma_{out}|^2\left(1 - |S_{11}|^2\right) + \Gamma_{out}\left(\Delta^* S_{11} - S_{22}^*\right) + \Gamma_{out}^*\left(\Delta S_{11}^* - S_{22}\right) = |\Delta|^2 - |S_{22}|^2.$$

Now divide by $1 - |S_{11}|^2$ to obtain

$$|\Gamma_{out}|^2 + \frac{\left(\Delta^* S_{11} - S_{22}^*\right)\Gamma_{out} + \left(\Delta S_{11}^* - S_{22}\right)\Gamma_{out}^*}{1 - |S_{11}|^2} = \frac{|\Delta|^2 - |S_{22}|^2}{1 - |S_{11}|^2}.$$

Complete the square by adding $\dfrac{\left|\Delta^* S_{11} - S_{22}^*\right|^2}{\left(1 - |S_{11}|^2\right)^2}$ to both sides:

$$\left|\Gamma_{out} + \frac{\Delta S_{11}^* - S_{22}}{1 - |S_{11}|^2}\right|^2 = \frac{|\Delta|^2 - |S_{22}|^2}{1 - |S_{11}|^2} + \frac{\left|\Delta^* S_{11} - S_{22}^*\right|^2}{\left(1 - |S_{11}|^2\right)^2} = \frac{|S_{12}S_{21}|^2}{\left(1 - |S_{11}|^2\right)^2}. \tag{12.32}$$

This equation is of the form $|\Gamma_{out} - C| = R$, representing a circle with center C and radius R in the Γ_{out} plane. Thus the center and radius of the mapped $|\Gamma_S| = 1$ circle are given by

$$C = \frac{S_{22} - \Delta S_{11}^*}{1 - |S_{11}|^2}, \tag{12.33a}$$

$$R = \frac{|S_{12}S_{21}|}{1 - |S_{11}|^2}. \tag{12.33b}$$

If points within this circular region are to satisfy $|\Gamma_{out}| < 1$, then we must have that

$$|C| + R < 1. \tag{12.34}$$

Substituting (12.33) into (12.34) gives

$$\left|S_{22} - \Delta S_{11}^*\right| + |S_{12}S_{21}| < 1 - |S_{11}|^2,$$

which after rearranging yields the μ-test of (12.30):

$$\frac{1 - |S_{11}|^2}{\left|S_{22} - \Delta S_{11}^*\right| + |S_{12}S_{21}|} > 1.$$

The $K - \Delta$ test of (12.28)–(12.29) can be derived from a similar starting point, or more simply from the μ-test of (12.30). Rearranging (12.30) and squaring gives

$$\left|S_{22} - \Delta S_{11}^*\right|^2 < \left(1 - |S_{11}|^2 - |S_{12}S_{21}|\right)^2. \tag{12.35}$$

It can be verified by direct expansion that

$$\left|S_{22} - \Delta S_{11}^*\right|^2 = |S_{12}S_{21}|^2 + \left(1 - |S_{11}|^2\right)\left(|S_{22}|^2 - |\Delta|^2\right),$$

so (12.35) expands to

$$|S_{12}S_{21}|^2 + \left(1 - |S_{11}|^2\right)\left(|S_{22}|^2 - |\Delta|^2\right) < \left(1 - |S_{11}|^2\right)\left(1 - |S_{11}|^2 - 2|S_{12}S_{21}|\right) + |S_{12}S_{21}|^2.$$

Simplifying gives

$$|S_{22}|^2 - |\Delta|^2 < 1 - |S_{11}|^2 - 2|S_{12}S_{21}|,$$

which yields Rollet's condition of (12.28) after rearranging:

$$\frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = K > 1.$$

In addition to (12.28), the $K - \Delta$ test also requires the auxiliary condition of (12.29) to guarantee unconditional stability. Although we derived Rollet's condition from the necessary and sufficient result of the μ-test, the squaring step used in (12.35) introduces an ambiguity in the sign of the right-hand side, thus requiring the additional condition. This can be derived by requiring that the right-hand side of (12.35) be positive before squaring. Thus,

$$|S_{12}S_{21}| < 1 - |S_{11}|^2.$$

Because similar conditions can be derived for the input side of the circuit, we can interchange S_{11} and S_{22} to obtain the analogous condition that

$$|S_{12}S_{21}| < 1 - |S_{22}|^2.$$

Adding these two inequalities gives

$$2|S_{12}S_{21}| < 2 - |S_{11}|^2 - |S_{22}|^2.$$

From the triangle inequality we know that

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| \leq |S_{11}S_{22}| + |S_{12}S_{21}|,$$

so we have that

$$|\Delta| < |S_{11}||S_{22}| + 1 - \frac{1}{2}|S_{11}|^2 - \frac{1}{2}|S_{22}|^2 < 1 - \frac{1}{2}\left(|S_{11}|^2 - |S_{22}|^2\right) < 1,$$

which is identical to (12.29).

EXAMPLE 12.2 TRANSISTOR STABILITY

The Triquint T1G6000528 GaN HEMT has the following scattering parameters at 1.9 GHz ($Z_0 = 50\ \Omega$):

$$\begin{aligned} S_{11} &= 0.869\angle{-159^\circ}, \\ S_{12} &= 0.031\angle{-9^\circ}, \\ S_{21} &= 4.250\angle{61^\circ}, \\ S_{22} &= 0.507\angle{-117^\circ}. \end{aligned}$$

Determine the stability of this transistor by using the $K - \Delta$ test and the μ-test, and plot the stability circles on a Smith chart.

Solution
From (12.28) and (12.29) we compute K and $|\Delta|$ as

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| = 0.336,$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 0.383.$$

Thus we have $|\Delta| < 1$ but not $K > 1$, so the unconditional stability criteria of (12.28)–(12.29) are not satisfied, and the device is potentially unstable. The stability of this device can also be evaluated using the μ-test, for which (12.30) gives $\mu = 0.678$, again indicating potential instability.

The centers and radii of the stability circles are given by (12.25) and (12.26):

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} = 1.59\angle 132°,$$

$$R_L = \frac{|S_{12}S_{21}|}{|S_{22}|^2 - |\Delta|^2} = 0.915,$$

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} = 1.09\angle 162°,$$

$$R_S = \frac{|S_{12}S_{21}|}{|S_{11}|^2 - |\Delta|^2} = 0.205.$$

These data can be used to plot the input and output stability circles, as shown in Figure 12.6. Since $|S_{11}| < 1$ and $|S_{22}| < 1$, the central part of the Smith chart represents the stable operating region for Γ_S and Γ_L. The unstable regions are shaded. ■

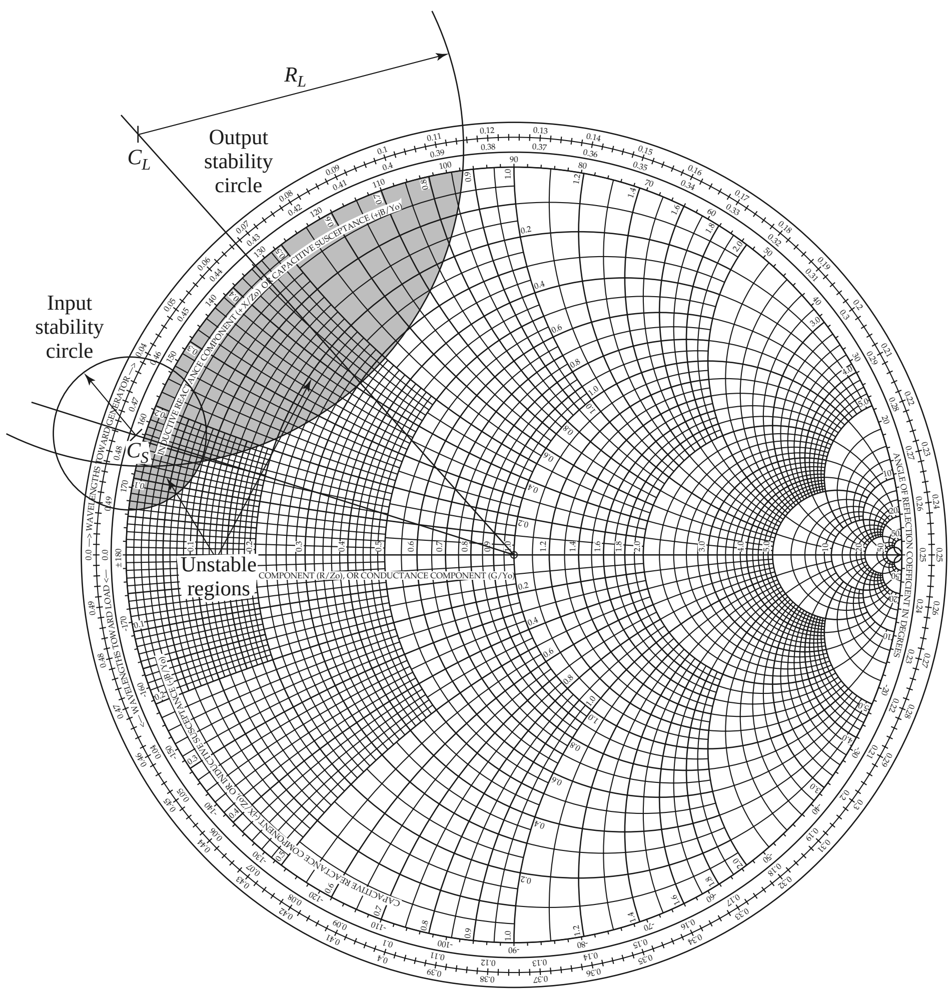


FIGURE 12.6 Stability circles for Example 12.2.

12.3 SINGLE-STAGE TRANSISTOR AMPLIFIER DESIGN

Design for Maximum Gain (Conjugate Matching)

After the stability of the transistor has been determined and the stable regions for Γ_S and Γ_L have been located on the Smith chart, the input and output matching sections can be designed. Since G_0 of (12.16b) is fixed for a given transistor, the overall transducer gain of the amplifier will be controlled by the gains, G_S and G_L, of the matching sections. Maximum gain will be realized when these sections provide a conjugate match between the amplifier source or load impedance and the transistor. Because most transistors exhibit a significant impedance mismatch (large $|S_{11}|$ and $|S_{22}|$), the resulting frequency response may be narrowband. In the following section we will discuss how to design for less than maximum gain, with a corresponding improvement in bandwidth. Broadband amplifier design will be discussed in Section 12.4.

With reference to Figure 12.2 and our discussion in Section 2.6 on conjugate impedance matching, we know that maximum power transfer from the input matching network to the transistor will occur when

$$\Gamma_{\text{in}} = \Gamma_S^*, \tag{12.36a}$$

and that maximum power transfer from the transistor to the output matching network will occur when

$$\Gamma_{\text{out}} = \Gamma_L^*. \tag{12.36b}$$

With the assumption of lossless matching sections, these conditions will maximize the overall transducer gain. From (12.13), this maximum gain will be given by

$$G_{T_{\max}} = \frac{1}{1 - |\Gamma_S|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}. \tag{12.37}$$

In addition, with conjugate matching and lossless matching sections, the input and output ports of the amplifier will be matched to Z_0.

In the general case with a *bilateral* ($S_{12} \neq 0$) transistor, Γ_{in} is affected by Γ_{out} and vice versa, so the input and output sections must be matched simultaneously. Using (12.36) in (12.3) gives the necessary equations:

$$\Gamma_S^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}, \tag{12.38a}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}. \tag{12.38b}$$

We can solve for Γ_S by first rewriting these equations as follows:

$$\Gamma_S = S_{11}^* + \frac{S_{12}^* S_{21}^*}{1/\Gamma_L^* - S_{22}^*},$$

$$\Gamma_L^* = \frac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S},$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$. Substituting the expression for Γ_L^* into the expression for Γ_S and expanding gives

$$\Gamma_S\left(1 - |S_{22}|^2\right) + \Gamma_S^2\left(\Delta S_{22}^* - S_{11}\right) = \Gamma_S\left(\Delta S_{11}^* S_{22}^* - |S_{11}|^2 - \Delta S_{12}^* S_{21}^*\right) + S_{11}^*\left(1 - |S_{22}|^2\right) + S_{12}^* S_{21}^* S_{22}.$$

Using the result that $\Delta\left(S_{11}^* S_{22}^* - S_{12}^* S_{21}^*\right) = |\Delta|^2$ allows this to be rewritten as a quadratic equation for Γ_S:

$$\left(S_{11} - \Delta S_{22}^*\right)\Gamma_S^2 + \left(|\Delta|^2 - |S_{11}|^2 + |S_{22}|^2 - 1\right)\Gamma_S + \left(S_{11}^* - \Delta^* S_{22}\right) = 0. \qquad (12.39)$$

The solution is

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}. \qquad (12.40a)$$

Similarly, the solution for Γ_L can be written as

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}. \qquad (12.40b)$$

The variables B_1, C_1, B_2, C_2 are defined as

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2, \qquad (12.41a)$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2, \qquad (12.41b)$$

$$C_1 = S_{11} - \Delta S_{22}^*, \qquad (12.41c)$$

$$C_2 = S_{22} - \Delta S_{11}^*. \qquad (12.41d)$$

Solutions to (12.40) are only possible if the quantity within the square root is positive, and it can be shown that this is equivalent to requiring $K > 1$. Thus, unconditionally stable devices can always be conjugately matched for maximum gain, and potentially unstable devices can be conjugately matched if $K > 1$ and $|\Delta| < 1$. The results are much simpler for the unilateral case. When $S_{12} = 0$, (12.38) shows that $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$, and then maximum transducer gain of (12.37) reduces to

$$G_{TU_{\max}} = \frac{1}{1 - |S_{11}|^2}|S_{21}|^2 \frac{1}{1 - |S_{22}|^2}. \qquad (12.42)$$

The maximum transducer power gain given by (12.37) occurs when the source and load are conjugately matched to the transistor, as given by the conditions of (12.36). If the transistor is unconditionally stable, so that $K > 1$, the maximum transducer power gain of (12.37) can be simply rewritten as follows:

$$G_{T_{\max}} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right). \qquad (12.43)$$

This result can be obtained by substituting (12.40) and (12.41) for Γ_S and Γ_L into (12.37) and simplifying. The maximum transducer power gain is also sometimes referred to as the *matched gain*. The maximum gain does not provide a meaningful result if the device is only conditionally stable since simultaneous conjugate matching of the source and load is not possible if $K < 1$ (see Problem 12.8). In this case a useful figure of merit is the *maximum stable gain*, defined as the maximum transducer power gain of (12.43) with $K = 1$. Thus,

$$G_{\text{msg}} = \frac{|S_{21}|}{|S_{12}|}. \qquad (12.44)$$

The maximum stable gain is easy to compute and offers a convenient way to compare the gain of various devices under stable operating conditions.

EXAMPLE 12.3 CONJUGATELY MATCHED AMPLIFIER DESIGN

Design an amplifier for maximum gain at 4 GHz using single-stub matching sections. Calculate and plot the input return loss and the gain from 3 to 5 GHz. The transistor is a GaAs MESFET with the following scattering parameters ($Z_0 = 50\ \Omega$):

f(GHz)	S_{11}	S_{12}	S_{21}	S_{22}
3.0	$0.80\angle{-89°}$	$0.03\angle 56°$	$2.86\angle 99°$	$0.76\angle{-41°}$
4.0	$0.72\angle{-116°}$	$0.03\angle 57°$	$2.60\angle 76°$	$0.73\angle{-54°}$
5.0	$0.66\angle{-142°}$	$0.03\angle 62°$	$2.39\angle 54°$	$0.72\angle{-68°}$

Solution

In practice, scattering parameters are usually provided by the manufacturer over a wide frequency range, and it is prudent to check stability over the entire range. Here we have limited the data to three frequencies to illustrate the point without undue computational burden. Using (12.28) and (12.29) to calculate K and Δ from the scattering parameters at each frequency in the above table gives the following results:

f(GHz)	K	Δ
3.0	0.77	0.592
4.0	1.19	0.487
5.0	1.53	0.418

We see that $K > 1$ and $|\Delta| < 1$ at 4 and 5 GHz, so the transistor is unconditionally stable at these frequencies, but it is only conditionally stable at 3 GHz. We can proceed with the design at 4 GHz, but should check stability at 3 GHz after we find the matching networks (which determine Γ_S and Γ_L).

For maximum gain, we should design the matching sections for a conjugate match to the transistor. Thus, $\Gamma_S = \Gamma_{in}^*$ and $\Gamma_L = \Gamma_{out}^*$, and Γ_S, Γ_L can be determined from (12.40):

$$\Gamma_S = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} = 0.872\angle 123°,$$

$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} = 0.876\angle 61°.$$

The effective gain factors of (12.16) can be calculated as

$$G_S = \frac{1}{1 - |\Gamma_S|^2} = 4.17 = 6.20 \text{ dB},$$

$$G_0 = |S_{21}|^2 = 6.76 = 8.30 \text{ dB},$$

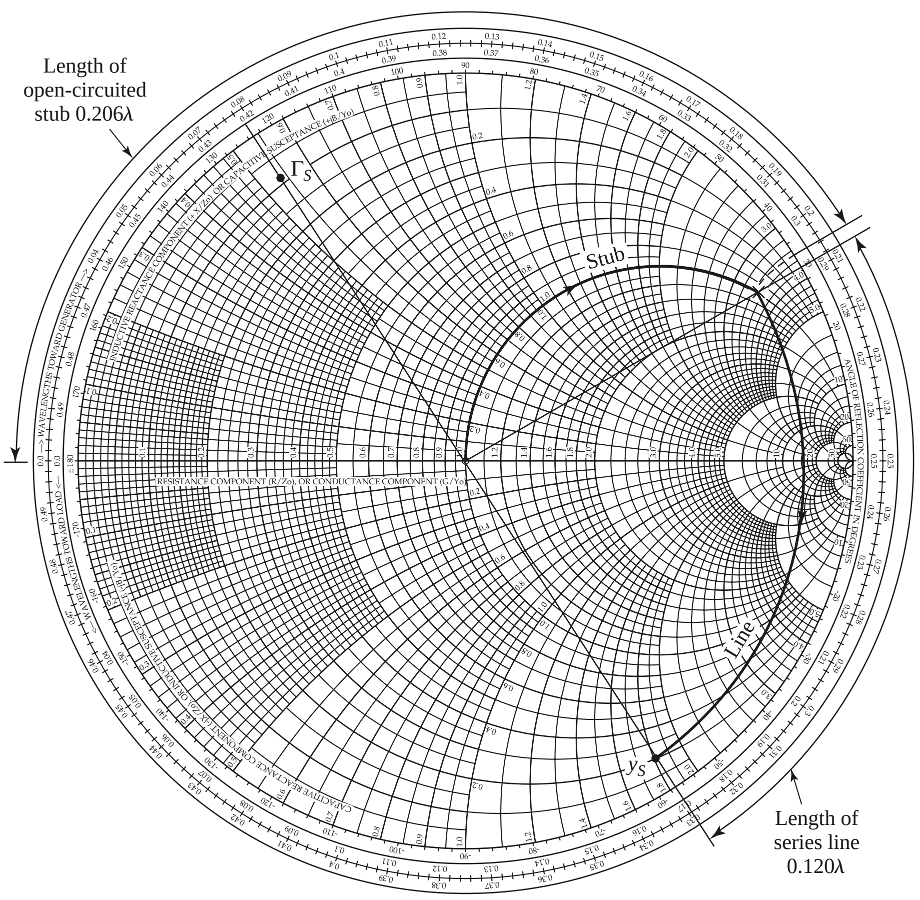
$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} = 1.67 = 2.22 \text{ dB}.$$

Then the overall transducer gain is

$$G_{T_{\max}} = 6.20 + 8.30 + 2.22 = 16.7 \text{ dB}.$$

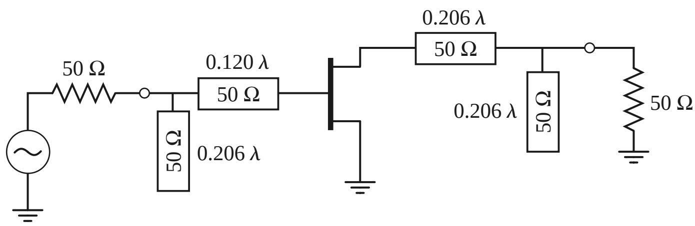
The matching networks can easily be determined using the Smith chart. For the input matching section, first plot Γ_S, as shown in Figure 12.7a. The impedance, Z_S, represented by this reflection coefficient is the impedance seen looking into the matching section toward the source impedance, Z_0. Thus, the matching section must transform Z_0 to the impedance Z_S. There are several ways of doing this, but we will use an open-circuited shunt stub followed by a length of line. We convert to the normalized admittance y_s, and work backward (toward the load on the Smith chart) to find that a line of length 0.120λ will bring us to the $1 + jb$ circle. Then we see that the required stub admittance is $+j3.5$, for an open-circuited stub length of 0.206λ. A similar procedure gives a line length of 0.206λ and a stub length of 0.206λ for the output matching circuit.

The final amplifier circuit is shown in Figure 12.7b. This circuit only shows the RF components; the amplifier will also require bias circuitry. The return loss and gain were calculated using a CAD package, interpolating the necessary

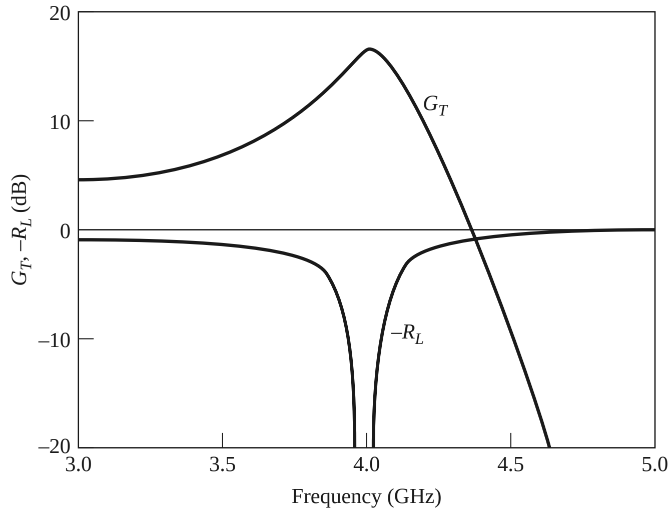


(a)

FIGURE 12.7 Circuit design and frequency response for the transistor amplifier of Example 12.3. (a) Smith chart for the design of the input matching network.



(b)



(c)

FIGURE 12.7 Continued. (b) RF circuit. (c) Frequency response.

scattering parameters from the data given above. The results are plotted in Figure 12.7c, and show the expected gain of 16.7 dB at 4 GHz, with a very good return loss. The bandwidth where the gain drops by 1 dB is about 2.5%.

With regard to the potential instability at 3 GHz, we leave it to the reader to show that the designed matching sections present source and load impedances that lie within the stable regions of the appropriate stability circles. Note that the matching sections are frequency dependent, so the impedances and reflection coefficients are different at 3 GHz than their design values at 4 GHz. The fact that CAD simulation did not show any indication of instability over the frequency range of 3–5 GHz is evidence that the circuit is stable over this frequency range. ■

Constant-Gain Circles and Design for Specifie Gain

In many cases it is preferable to design for less than the maximum obtainable gain, to improve bandwidth or to obtain a specific value of amplifier gain. This can be done by designing the input and output matching sections to have less than maximum gains; in other words, mismatches are purposely introduced to reduce the overall gain. The design procedure is facilitated by plotting *constant-gain circles* on the Smith chart to represent loci of Γ_S and Γ_L that give fixed values of gain (G_S and G_L). To simplify our discussion, we will only treat the case of a unilateral device; the more general case of a bilateral device must sometimes be considered in practice, and is discussed in detail in references [1–2].

For many transistors $|S_{12}|$ is small enough to be ignored, and the device can be assumed to be unilateral. This greatly simplifies the design procedure. The error in the transducer gain caused by approximating $|S_{12}|$ as zero is given by the ratio G_T/G_{TU}. It can be shown that this ratio is bounded by

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}, \tag{12.45}$$

where U is defined as the *unilateral figure of merit*,

$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{\left(1-|S_{11}|^2\right)\left(1-|S_{22}|^2\right)}. \tag{12.46}$$

Usually an error of a few tenths of a dB or less justifies the unilateral assumption.

The expression for G_S and G_L for the unilateral case are given by (12.17a) and (12.17c):

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2},$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}.$$

These gains are maximized when $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$, resulting in the maximum values given by

$$G_{S_{\max}} = \frac{1}{1-|S_{11}|^2}, \tag{12.47a}$$

$$G_{L_{\max}} = \frac{1}{1-|S_{22}|^2}. \tag{12.47b}$$

Define normalized gain factors g_S and g_L as

$$g_S = \frac{G_S}{G_{S_{\max}}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}\left(1-|S_{11}|^2\right), \tag{12.48a}$$

$$g_L = \frac{G_L}{G_{L_{\max}}} = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}\left(1-|S_{22}|^2\right). \tag{12.48b}$$

Then we have that $0 \le g_S \le 1$ and $0 \le g_L \le 1$.

For fixed values of g_S and g_L, (12.48) represents circles in the Γ_S or Γ_L plane. To show this, consider (12.48a), which can be expanded to give

$$g_S|1-S_{11}\Gamma_S|^2 = \left(1-|\Gamma_S|^2\right)\left(1-|S_{11}|^2\right),$$

$$\left(g_S|S_{11}|^2 + 1 - |S_{11}|^2\right)|\Gamma_S|^2 - g_S\left(S_{11}\Gamma_S + S_{11}^*\Gamma_S^*\right) = 1 - |S_{11}|^2 - g_S,$$

$$\Gamma_S\Gamma_S^* - \frac{g_S\left(S_{11}\Gamma_S + S_{11}^*\Gamma_S^*\right)}{1-(1-g_S)|S_{11}|^2} = \frac{1-|S_{11}|^2-g_S}{1-(1-g_S)|S_{11}|^2}. \tag{12.49}$$

Now add $\left(g_S^2|S_{11}|^2\right)/\left[1-(1-g_S)|S_{11}|^2\right]^2$ to both sides to complete the square:

$$\left|\Gamma_S - \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}\right|^2 = \frac{\left(1-|S_{11}|^2-g_S\right)\left[1-(1-g_S)|S_{11}|^2\right]+g_S^2|S_{11}|^2}{\left[1-(1-g_S)|S_{11}|^2\right]^2}.$$

Simplifying gives

$$\left|\Gamma_S - \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}\right| = \frac{\sqrt{1-g_S}\left(1-|S_{11}|^2\right)}{1-(1-g_S)|S_{11}|^2}, \tag{12.50}$$

which is the equation of a circle with its center and radius given by

$$C_S = \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}, \tag{12.51a}$$

$$R_S = \frac{\sqrt{1-g_S}\left(1-|S_{11}|^2\right)}{1-(1-g_S)|S_{11}|^2}. \tag{12.51b}$$

The results for the constant gain circles of the output section can be shown to be

$$C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2}, \tag{12.52a}$$

$$R_L = \frac{\sqrt{1-g_L}\left(1-|S_{22}|^2\right)}{1-(1-g_L)|S_{22}|^2}. \tag{12.52b}$$

The centers of each family of circles lie along straight lines given by the angle of S_{11}^* or S_{22}^*. Note that when g_S (or g_L) $= 1$ (maximum gain), the radius R_S (or R_L) $= 0$, and the center reduces to S_{11}^* (or S_{22}^*), as expected. In addition, it can be shown that the 0 dB gain circles ($G_S = 1$ or $G_L = 1$) will always pass through the center of the Smith chart. These results can be used to plot a family of circles of constant gain for the input and output sections. Then Γ_S and Γ_L can be chosen along these circles to provide the desired gains. The choices for Γ_S and Γ_L are not unique, but it makes sense to choose points close to the center of the Smith chart to minimize mismatch, and thus maximize bandwidth. Alternatively, as we will see in the next section, the input network mismatch can be chosen to provide a low-noise design.

EXAMPLE 12.4 AMPLIFIER DESIGN FOR SPECIFIED GAIN

Design an amplifier to have a gain of 11 dB at 4.0 GHz. Plot constant-gain circles for $G_S = 2$ and 3 dB, and $G_L = 0$ and 1 dB. Calculate and plot the input return loss and overall amplifier gain from 3 to 5 GHz. The transistor has the following scattering parameters ($Z_0 = 50\ \Omega$):

f (GHz)	S_{11}	S_{12}	S_{21}	S_{22}
3	$0.80\angle{-90°}$	0	$2.8\angle 100°$	$0.66\angle{-50°}$
4	$0.75\angle{-120°}$	0	$2.5\angle 80°$	$0.60\angle{-70°}$
5	$0.71\angle{-140°}$	0	$2.3\angle 60°$	$0.58\angle{-85°}$

Solution

Since $S_{12} = 0$ and $|S_{11}| < 1$ and $|S_{22}| < 1$, the transistor is unilateral and unconditionally stable at each frequency in the above table. From (12.47) we calculate the maximum matching section gains as

$$G_{S_{\max}} = \frac{1}{1 - |S_{11}|^2} = 2.29 = 3.6 \text{ dB},$$

$$G_{L_{\max}} = \frac{1}{1 - |S_{22}|^2} = 1.56 = 1.9 \text{ dB}.$$

The gain of the mismatched transistor is

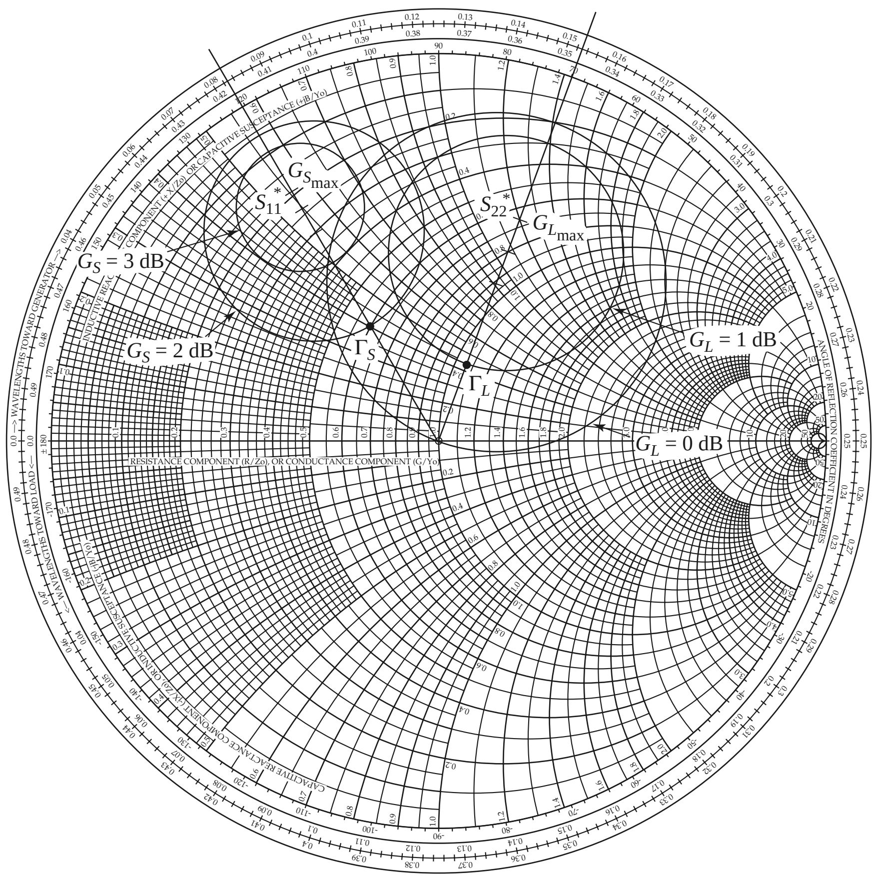
$$G_0 = |S_{21}|^2 = 6.25 = 8.0 \text{ dB},$$

so the maximum unilateral transducer gain is

$$G_{TU_{\max}} = 3.6 + 1.9 + 8.0 = 13.5 \text{ dB}.$$

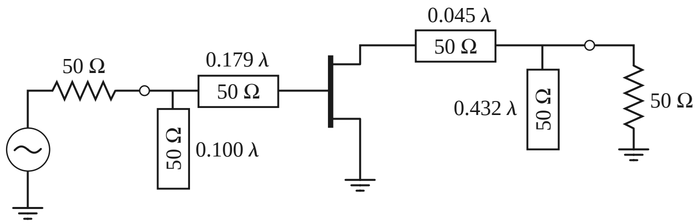
We therefore have 2.5 dB more available gain than is required by the specifications.

Next, use (12.48), (12.51), and (12.52) to calculate the following data for the

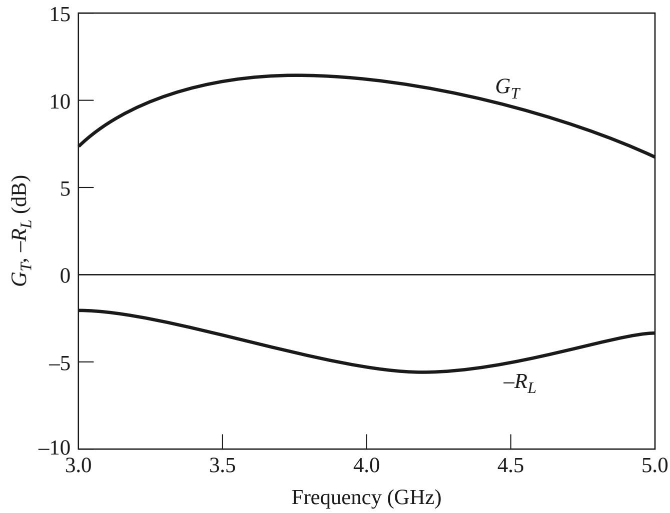


(a)

FIGURE 12.8 Circuit design and frequency response for the transistor amplifier of Example 12.4. (a) Constant-gain circles.



(b)



(c)

FIGURE 12.8 Continued. (b) RF circuit. (c) Transducer gain and return loss.

constant-gain circles:

$$
\begin{array}{llll}
G_S = 3 \text{ dB} & g_S = 0.875 & C_S = 0.706\angle 120^\circ & R_S = 0.166 \\
G_S = 2 \text{ dB} & g_S = 0.691 & C_S = 0.627\angle 120^\circ & R_S = 0.294 \\
G_L = 1 \text{ dB} & g_L = 0.806 & C_L = 0.520\angle 70^\circ & R_L = 0.303 \\
G_L = 0 \text{ dB} & g_L = 0.640 & C_L = 0.440\angle 70^\circ & R_L = 0.440
\end{array}
$$

The constant-gain circles are shown in Figure 12.8a. We choose $G_S = 2$ dB and $G_L = 1$ dB, for an overall amplifier gain of 11 dB. Then we select Γ_S and Γ_L along these circles as shown, to minimize the distance from the center of the chart (this places Γ_S and Γ_L along the radial lines at 120° and 70°, respectively). Thus, $\Gamma_S = 0.33\angle 120^\circ$ and $\Gamma_L = 0.22\angle 70^\circ$, and the matching networks can be designed using shunt stubs as in Example 12.3.

The final amplifier circuit is shown in Figure 12.8b. The response was calculated using CAD software, with interpolation of the given scattering parameter data. The results are shown in Figure 12.8c, where it is seen the desired gain of 11 dB is achieved at 4.0 GHz. The bandwidth over which the gain varies by ± 1 dB or less is about 25%, which is considerably better than the bandwidth of the maximum gain design in Example 12.3. The return loss, however, is not very good, being only about 5 dB at the design frequency. This is due to the deliberate mismatch introduced into the matching sections to achieve the specified gain. ■

Low-Noise Amplifie Design

Besides stability and gain, another important design consideration for a microwave amplifier is its noise figure. In receiver applications especially it is often required to have a preamplifier with as low a noise figure as possible since, as we saw in Chapter 10, the first stage of a receiver front end has the dominant effect on the noise performance of the overall system. Generally it is not possible to obtain both minimum noise figure and maximum gain for an amplifier, so some sort of compromise must be made. This can be done by using constant-gain circles and *circles of constant noise figure* to select a usable trade-off between noise figure and gain. Here we will derive the equations for constant–noise figure circles and show how they are used in transistor amplifier design.

As shown in references [1] and [2], the noise figure of a two-port amplifier can be expressed as

$$F = F_{\min} + \frac{R_N}{G_S}|Y_S - Y_{\text{opt}}|^2, \tag{12.53}$$

where the following definitions apply:

$Y_S = G_S + jB_S =$ source admittance presented to transistor.
$Y_{\text{opt}} =$ optimum source admittance that results in minimum noise figure.
$F_{\min} =$ minimum noise figure of transistor, attained when $Y_S = Y_{\text{opt}}$.
$R_N =$ equivalent noise resistance of transistor.
$G_S =$ real part of source admittance.

Instead of the admittance Y_S and Y_{opt}, we can use the reflection coefficients Γ_S and Γ_{opt}, where

$$Y_S = \frac{1}{Z_0}\frac{1-\Gamma_S}{1+\Gamma_S}, \tag{12.54a}$$

$$Y_{\text{opt}} = \frac{1}{Z_0}\frac{1-\Gamma_{\text{opt}}}{1+\Gamma_{\text{opt}}}. \tag{12.54b}$$

Γ_S is the source reflection coefficient defined in Figure 12.1. The quantities $F_{\min}, \Gamma_{\text{opt}}$, and R_N are characteristics of the particular transistor being used, and are called the *noise parameters* of the device; they may be given by the manufacturer or measured.

Using (12.54), we can express the quantity $|Y_S - Y_{\text{opt}}|^2$ in terms of Γ_S and Γ_{opt}:

$$|Y_S - Y_{\text{opt}}|^2 = \frac{4}{Z_0^2}\frac{|\Gamma_S - \Gamma_{\text{opt}}|^2}{|1+\Gamma_S|^2|1+\Gamma_{\text{opt}}|^2}. \tag{12.55}$$

In addition,

$$G_S = \text{Re}\{Y_S\} = \frac{1}{2Z_0}\left(\frac{1-\Gamma_S}{1+\Gamma_S} + \frac{1-\Gamma_S^*}{1+\Gamma_S^*}\right) = \frac{1}{Z_0}\frac{1-|\Gamma_S|^2}{|1+\Gamma_S|^2}. \tag{12.56}$$

Using these results in (12.53) gives the noise figure as

$$F = F_{\min} + \frac{4R_N}{Z_0}\frac{|\Gamma_S - \Gamma_{\text{opt}}|^2}{\left(1-|\Gamma_S|^2\right)|1+\Gamma_{\text{opt}}|^2}. \tag{12.57}$$

For a fixed noise figure F we can show that this result defines a circle in the Γ_S plane. First define the *noise figure parameter, N*, as

$$N = \frac{|\Gamma_S - \Gamma_{\text{opt}}|^2}{1-|\Gamma_S|^2} = \frac{F - F_{\min}}{4R_N/Z_0}|1+\Gamma_{\text{opt}}|^2, \tag{12.58}$$

which is a constant for a given noise figure and set of noise parameters. Then rewrite (12.58) as

$$(\Gamma_S - \Gamma_{\text{opt}})\left(\Gamma_S^* - \Gamma_{\text{opt}}^*\right) = N\left(1 - |\Gamma_S|^2\right),$$

$$\Gamma_S\Gamma_S^* - \left(\Gamma_S\Gamma_{\text{opt}}^* + \Gamma_S^*\Gamma_{\text{opt}}\right) + \Gamma_{\text{opt}}\Gamma_{\text{opt}}^* = N - N|\Gamma_S|^2,$$

$$\Gamma_S\Gamma_S^* - \frac{\left(\Gamma_S\Gamma_{\text{opt}}^* + \Gamma_S^*\Gamma_{\text{opt}}\right)}{N+1} = \frac{N - |\Gamma_{\text{opt}}|^2}{N+1}.$$

Add $|\Gamma_{\text{opt}}|^2/(N+1)^2$ to both sides to complete the square to obtain

$$\left|\Gamma_S - \frac{\Gamma_{\text{opt}}}{N+1}\right| = \frac{\sqrt{N\left(N+1-|\Gamma_{\text{opt}}|^2\right)}}{(N+1)}. \tag{12.59}$$

This result defines circles of constant noise figure with centers at

$$C_F = \frac{\Gamma_{\text{opt}}}{N+1}, \tag{12.60a}$$

and radii of

$$R_F = \frac{\sqrt{N\left(N+1-|\Gamma_{\text{opt}}|^2\right)}}{N+1}. \tag{12.60b}$$

EXAMPLE 12.5 LOW-NOISE AMPLIFIER DESIGN

A GaAs MESFET is biased for minimum noise figure, with the following scattering parameters and noise parameters at 4 GHz ($Z_0 = 50\ \Omega$): $S_{11} = 0.6\angle{-60°}$, $S_{12} = 0.05\angle 26°$, $S_{21} = 1.9\angle 81°$, $S_{22} = 0.5\angle{-60°}$, $F_{\min} = 1.6$ dB, $\Gamma_{\text{opt}} = 0.62\angle 100°$, and $R_N = 20\ \Omega$. For design purposes, assume the device is unilateral, and calculate the maximum error in G_T resulting from this assumption. Then design an amplifier having a 2.0 dB noise figure with the maximum gain that is compatible with this noise figure.

Solution

We first calculate that $K = 2.78$ and $\Delta = 0.37$, so the device is unconditionally stable even without the approximation of a unilateral device. Next, compute the unilateral figure of merit from (12.46):

$$U = \frac{|S_{12}S_{21}S_{11}S_{22}|}{\left(1 - |S_{11}|^2\right)\left(1 - |S_{22}|^2\right)} = 0.059.$$

From (12.45) the ratio G_T/G_{TU} is bounded as

$$\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2},$$

or

$$0.891 < \frac{G_T}{G_{TU}} < 1.130.$$

In dB, this is

$$-0.50 < G_T - G_{TU} < 0.53\ \text{dB},$$

where G_T and G_{TU} are now in dB. Thus, we should expect less than about ± 0.5 dB error in gain.

Now use (12.58) and (12.60) to compute the center and radius of the 2 dB noise figure circle:

$$N = \frac{F - F_{\min}}{4R_N/Z_0}|1 + \Gamma_{\text{opt}}|^2 = \frac{1.58 - 1.445}{4(20/50)}|1 + 0.62\angle 100°|^2$$
$$= 0.0986,$$
$$C_F = \frac{\Gamma_{\text{opt}}}{N+1} = 0.56\angle 100°,$$
$$R_F = \frac{\sqrt{N\left(N + 1 - |\Gamma_{\text{opt}}|^2\right)}}{N+1} = 0.24.$$

This noise figure circle is plotted in Figure 12.9a. Minimum noise figure ($F_{\min} =$ 1.6 dB) occurs for $\Gamma_S = \Gamma_{\text{opt}} = 0.62\angle 100°$.

Next we calculate data for several input section constant-gain circles. From (12.51), we have the following results:

G_S (dB)	g_S	C_S	R_S
1.0	0.805	$0.52\angle 60°$	0.300
1.5	0.904	$0.56\angle 60°$	0.205
1.7	0.946	$0.58\angle 60°$	0.150

These circles are plotted in Figure 12.9a. We see that the $G_S = 1.7$ dB gain circle just intersects the $F = 2$ dB noise figure circle, and that any higher gain will result in a worse noise figure. From the Smith chart the optimum solution is $\Gamma_S = 0.53\angle 75°$, yielding $G_S = 1.7$ dB and $F = 2.0$ dB.

For the output section we choose $\Gamma_L = S_{22}^* = 0.5\angle 60°$ for a maximum G_L of

$$G_L = \frac{1}{1 - |S_{22}|^2} = 1.33 = 1.25 \text{ dB}.$$

The transistor gain is

$$G_0 = |S_{21}|^2 = 3.61 = 5.58 \text{ dB},$$

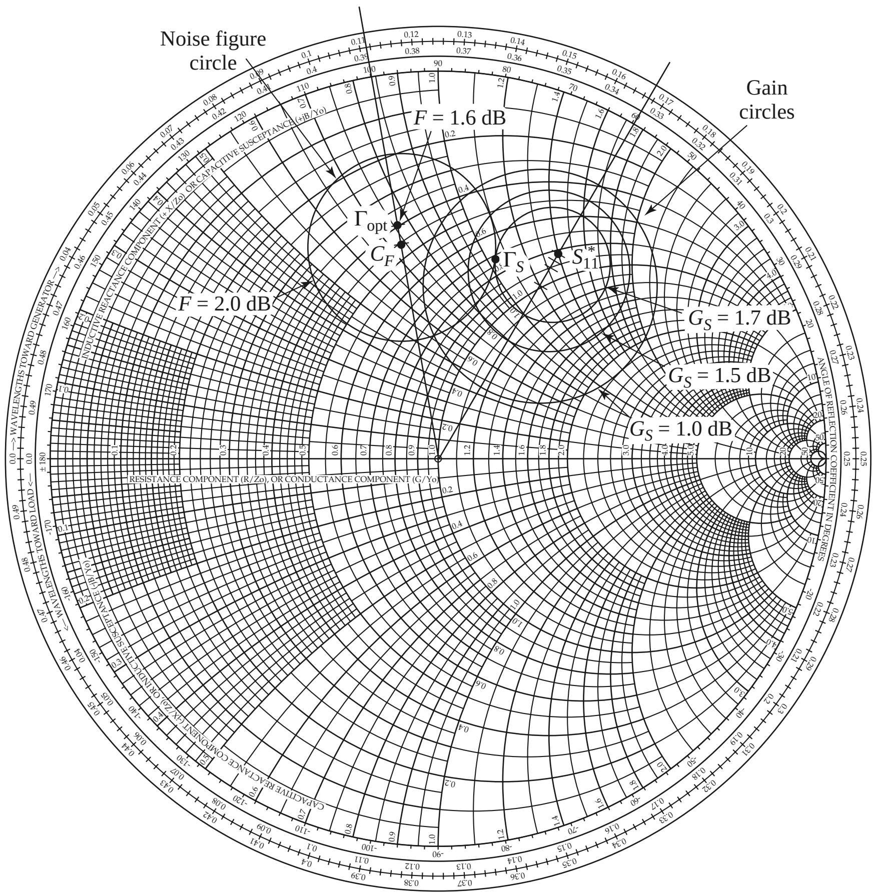
so the overall transducer gain will be

$$G_{TU} = G_S + G_0 + G_L = 8.53 \text{ dB}.$$

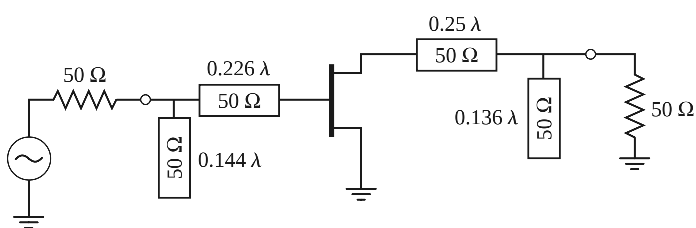
A complete AC circuit for the amplifier, using open-circuited shunt stubs in the matching sections, is shown in Figure 12.9b. A computer analysis of the circuit gives a gain of 8.36 dB. ■

Low-Noise MOSFET Amplifie

MOSFETs have a relatively low AC input resistance, making them difficult to impedance match. An external series resistance can be added to the gate, but this approach increases noise power and degrades efficiency. By using a series inductor at the source of a MOSFET,



(a)



(b)

FIGURE 12.9 Circuit design for the transistor amplifier of Example 12.5. (a) Constant-gain and constant–noise figure circles. (b) RF circuit.

however, it is possible to create a resistive input impedance without adding noisy resistors. This technique is called *inductive source degeneration*; similar methods can be used with MESFETs and other transistors. The conceptual circuit is shown in Figure 12.10a, where the inductor L_s is placed in series with the source of the device.

The equivalent circuit of the amplifier is shown in Figure 12.10b, where we have simplified the model by assuming the transistor is unilateral, and that R_i, R_{ds}, and C_{ds} can be ignored. For an input current I at the gate of the transistor, the capacitor voltage is

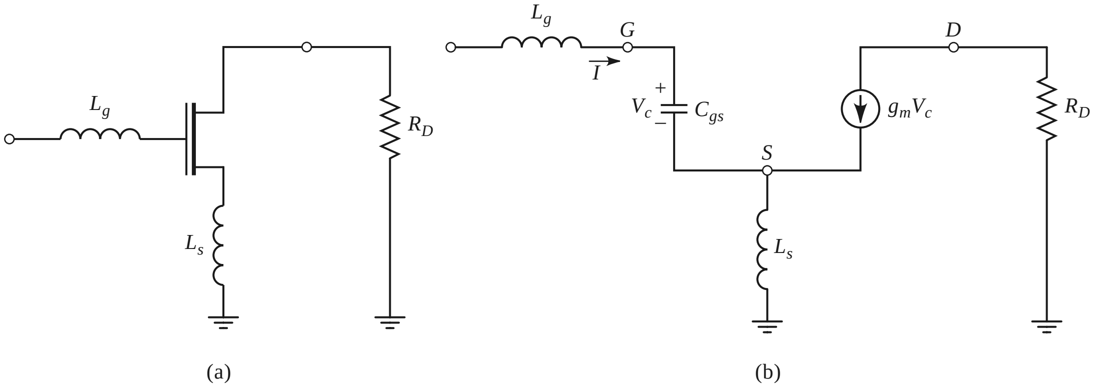


FIGURE 12.10 Low-noise MOSFET amplifier. (a) Basic AC circuit. (b) Equivalent circuit using a simplified unilateral FET model.

$V_c = I/j\omega C_{gs}$. The gate voltage, relative to ground, is then

$$\begin{aligned} V &= \frac{I}{j\omega C_{gs}} + j\omega L_s\,(I + g_m V_c) \\ &= I\left(\frac{1}{j\omega C_{gs}} + j\omega L_s + \frac{g_m L_s}{C_{gs}}\right). \end{aligned} \tag{12.61}$$

The input impedance at the gate is

$$Z = \frac{V}{I} = \frac{g_m L_s}{C_{gs}} + j\left(\omega L_s - \frac{1}{\omega C_{gs}}\right), \tag{12.62}$$

showing that the circuit has produced an input resistance of $g_m L_s/C_{gs}$. The series inductor, L_s, can be chosen to match the input resistance of the amplifier to a source impedance, Z_0. The inductor at the gate, L_g, can then be chosen to cancel the residual input reactance, which is usually capacitive. The combination of the series matching inductor, the gate capacitance, and the effective input resistance forms a series RLC resonator. The Q of this resonator is

$$Q = \frac{\omega L_g C_{gs}}{g_m L_s}. \tag{12.63}$$

The bandwidth of this circuit may be relatively narrow if this Q is high.

EXAMPLE 12.6 LOW-NOISE MOSFET AMPLIFIER DESIGN

An Infineon BF1005 n-channel MOSFET transistor having $C_{gs} = 2.1$ pF and $g_m = 24$ mS is used in a 900 MHz low-noise amplifier with inductive source degeneration, as shown in Figure 12.10. Determine the source and gate inductors, and estimate the bandwidth of the amplifier. Assume a source impedance of $Z_0 = 50\ \Omega$.

Solution
From (12.62), matching the input resistance to Z_0 determines the source inductor as

$$L_s = \frac{Z_0 C_{gs}}{g_m} = \frac{(50)(2.1 \times 10^{-12})}{0.024} = 4.37 \text{ nH}.$$

The net reactance at the input is $jX = j\left(\omega L_s - \dfrac{1}{\omega C_{gs}}\right) = -j59.5\ \Omega$, so the required series inductance for matching is

$$L_g = \frac{-X}{\omega} = \frac{59.5}{2\pi\left(900 \times 10^6\right)} = 10.5\ \text{nH}.$$

From (12.63) we can estimate the Q as

$$Q = \frac{\omega L_g C_{gs}}{g_m L_s} = 1.2,$$

so the bandwidth of the amplifier could be as high as 80%. This value is probably higher than what would be obtained in practice, due to the approximations that have been made in our analysis. ■

12.4 BROADBAND TRANSISTOR AMPLIFIER DESIGN

The ideal amplifier would have constant gain and good input matching over the desired frequency bandwidth. As the examples of the last section have shown, conjugate matching will give maximum gain only over a relatively narrow bandwidth, while designing for less than maximum gain will improve the gain bandwidth, but the input and output ports of the amplifier will be poorly matched. These problems are primarily a result of the fact that microwave transistors typically are not well matched to 50 Ω, and large impedance mismatches are governed by the Bode–Fano gain–bandwidth criterion discussed in Chapter 5. Another consideration, as shown earlier in this chapter, is that $|S_{21}|$ decreases with frequency at the rate of 6 dB/octave. For these reasons, special consideration must be given to the problem of designing broadband amplifiers. Some of the common approaches to this problem are listed below; note in each case that an improvement in bandwidth is achieved only at the expense of gain, complexity, or similar factors.

- *Compensated matching networks*: Input and output matching sections can be designed to compensate for the gain rolloff in $|S_{21}|$, but generally at the expense of the input and output matching.
- *Resistive matching networks*: Good input and output matching can be obtained by using resistive matching networks, with a corresponding loss in gain and increase in noise figure.
- *Negative feedback*: Negative feedback can be used to flatten the gain response of the transistor, improve the input and output match, and improve the stability of the device. Amplifier bandwidths in excess of a decade are possible with this method, at the expense of gain and noise figure.
- *Balanced amplifiers*: Two amplifiers having 90° couplers at their input and output can provide good matching over an octave bandwidth, or more. The gain is equal to that of a single amplifier, however, and the design requires two transistors and twice the DC power.
- *Distributed amplifiers*: Several transistors are cascaded together along a transmission line, giving good gain, matching, and noise figure over a wide bandwidth. The circuit is large, and does not give as much gain as a cascade amplifier with the same number of stages.
- *Differential amplifiers*: Driving two devices in a differential mode, with input signals of opposite polarity, results in an effective series connection of device capacitance, thus roughly doubling f_T. Differential amplifiers can also provide a larger output voltage swing than a single device, and common mode noise rejection.