Computação Gráfica

Resolução dos Exercícios de Aula

Prof. Alaor Cervati Neto

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Observações

- A resolução dos exercícios será disponibilizada 2 semanas após a respectiva aula.
- Os exercícios podem ser entregues sem descontos na nota até essa data.
- As respostas aqui descritas não são as únicas possíveis para cada exercício. Outras diferentes podem ser consideradas corretas, desde que cheguem a resultados similares.
- Este arquivo será atualizado durante o semestre.
- As questões que dependerem de variáveis individuais para cada estudante serão exibidas em função dessas, mas a resposta apenas será considerada correta se os valores adequados forem utilizados.

Aula 3

Exercício 1

GL_LINE_LOOP(B,C,D,E,F,G)

Exercício 2

GL_TRIANGLES(B,C,G,A,D,F)

Exercício 3

GL_TRIANGLE_STRIP(C,D,G,F)

Aula 4

Exercício 1

$$M = S \cdot T$$

$$M = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} s & 0 & st_x \\ 0 & s & st_y \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 2M \\ 0 & 2 & 2D \\ 0 & 0 & 1 \end{bmatrix}$$

Exercício 2

$$R(M+D) = \begin{bmatrix} \cos(M+D) & -\sin(M+D) & 0 \\ \sin(M+D) & \cos(M+D) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos M \cdot \cos D - \sin M \cdot \sin D & -\cos M \cdot \sin D - \sin M \cdot \cos D & 0 \\ \cos M \cdot \sin D + \sin M \cdot \cos D & \cos M \cdot \cos D - \sin M \cdot \sin D & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(M) \cdot R(D) = \begin{bmatrix} \cos M & -\sin M & 0 \\ \sin M & \cos M & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos D & -\sin D & 0 \\ \sin D & \cos D & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos M \cdot \cos D - \sin M \cdot \sin D & -\cos M \cdot \sin D - \sin M \cdot \cos D & 0 \\ \cos M \cdot \sin D + \sin M \cdot \cos D & \cos M \cdot \cos D - \sin M \cdot \sin D & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore R(M+D) = R(M) \cdot R(D)$$

Exercício 3

$$P = \begin{bmatrix} M-2.5 & M+2.5 & M-2.5 & M+2.5 \\ D-2.5 & D-2.5 & D+2.5 & D+2.5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & 2.5 \\ 0 & 1 & 2.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2.5 \\ 0 & 1 & -2.5 \\ 0 & 0 & 1 \end{bmatrix}$$

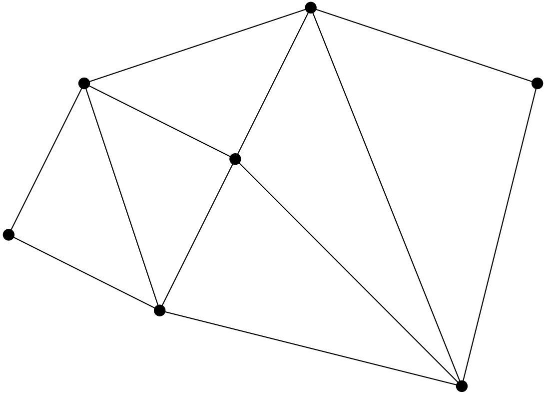
$$= \begin{bmatrix} 1 & 0 & 2.5 \\ 0 & 1 & 2.5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2.5 \\ 0 & 1 & -2.5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2.5 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{5-5\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$P' = M \cdot P = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2.5 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{5-5\sqrt{2}}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M-2.5 & M+2.5 & M-2.5 & M+2.5 \\ D-2.5 & D-2.5 & D+2.5 & D+2.5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

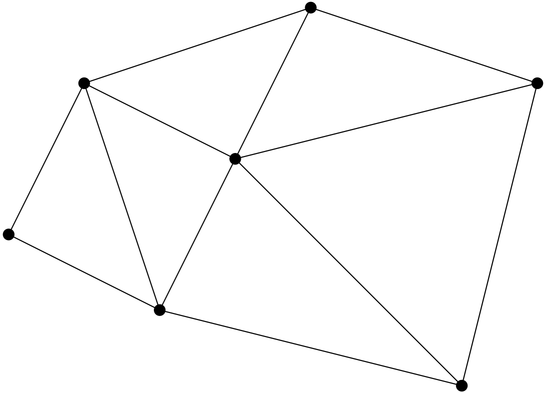
$$= \begin{bmatrix} \frac{\sqrt{2}(M-D)+5}{2} & \frac{\sqrt{2}(M-D+5)+5}{2} & \frac{\sqrt{2}(M-D-5)+5}{2} & \frac{\sqrt{2}(M-D)+5}{2} \\ \frac{\sqrt{2}(M+D-10)+5}{2} & \frac{\sqrt{2}(M+D-5)+5}{2} & \frac{\sqrt{2}(M+D-5)+5}{2} & \frac{\sqrt{2}(M+D)+5}{2} \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Aula 5

Exercício 1



Exercício 2



Exercício 3

```
<svg width="700" height="500">
        <path d="M 0 2 L 2 1 3 3 4 5 7 4 6 0 3 3 1 4 2 1 6 0
           M 3 3 L 7 4 M 4 5 L 1 4 0 2" stroke="black"
           stroke-width="0.01" fill="none"
           transform="translate(0,500),scale(100,-100)"/>
</svg>
```

Aula 6

Exercício 1[1]

```
modelo = load_model_from_file('objetos/cubo.obj')
```

[1]A função para leitura do arquivo .obj deve ser definida no programa em que é utilizada, mas sua chamada é a forma como este é carregado.

Exercício 2[2]

```
mat_model = model(angle=90, r_x=0, r_y=0, r_z=1.0, t_x=0.0,
    t_y=0.0, t_z=-25.0, s_x=0.5, s_y=0.5, s_z=0.5)
vertices_list = []
for v in np.array(modelo['vertices']):
        vertices_list.append([v[0], v[1], v[2], 1])
vert = np.array(vertices_list, np.float32)
vert_t = mat_model@vert.transpose()
```

[2] Para esta operação, é preciso criar uma variável auxiliar que armazene o vetor considerando a coordenada homogênea, e fazer a operação da matriz *Model* aparte da execução do OpenGL.

Exercício 3[3]

```
print(vertices_list)
print(vert_t.transpose())
```

[3] O resultado da operação sobre os vértices está na forma matricial. Para exibi-lo com cada vértice em uma linha, é necessário transpor a matriz resultante.

Aula 7

Exercício 1

$$M_{\text{Model}} = \begin{bmatrix} M & 0 & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} M & 0 & 0 & \frac{M}{2} \\ 0 & D & 0 & 0 \\ 0 & 0 & M & \frac{M}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P' = M_{\text{Model}} \cdot P = \begin{bmatrix} M & 0 & 0 & \frac{M}{2} \\ 0 & D & 0 & 0 \\ 0 & 0 & M & \frac{M}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.5 & 0.5 & -0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -0.5 & -0.5 & 0.5 & 0.5 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & M & 0 & M & \frac{M}{2} \\ 0 & 0 & 0 & 0 & D \\ 0 & 0 & M & M & \frac{M}{2} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Exercício 2

$$n = \frac{N}{|N|} = \frac{(0,0,M)}{M} = (0,0,1)\,, u = \frac{V \times n}{|V \times n|} = \frac{(1,0,0)}{1} = (1,0,0)\,, v = n \times u = (0,1,0)$$

$$M_{\text{View}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -M \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P'' = M_{\text{View}} \cdot P' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -M \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & M & 0 & M & \frac{M}{2} \\ 0 & 0 & 0 & 0 & D \\ 0 & 0 & M & M & \frac{M}{2} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & M & 0 & M & \frac{M}{2} \\ 0 & 0 & 0 & 0 & D \\ -M & -M & 0 & 0 & -\frac{M}{2} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Exercício 3

$$M_{\text{normpers}} = \begin{bmatrix} \frac{\cot\left(\frac{\theta}{2}\right)}{aspect} & 0 & 0 & 0 \\ 0 & \cot\left(\frac{\theta}{2}\right) & 0 & 0 \\ 0 & 0 & -\frac{z_{\text{far}}+z_{\text{near}}}{z_{\text{far}}-z_{\text{near}}} & -\frac{2\cdot z_{\text{far}}\cdot z_{\text{near}}}{z_{\text{far}}-z_{\text{near}}} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{\cot\left(\frac{\theta}{2}\right)}{aspect} & 0 & 0 & 0 \\ 0 & \cot\left(\frac{\theta}{2}\right) & 0 & 0 \\ 0 & 0 & -\frac{1001}{999} & -\frac{20D}{999} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$P''' = M_{\text{normpers}} \cdot P'' = \begin{bmatrix} \frac{\cot\left(\frac{\theta}{2}\right)}{aspect} & 0 & 0 & 0 \\ 0 & \cot\left(\frac{\theta}{2}\right) & 0 & 0 \\ 0 & 0 & -\frac{1001}{999} & -\frac{20D}{999} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & M & 0 & M & \frac{M}{2} \\ 0 & 0 & 0 & 0 & D \\ -M & -M & 0 & 0 & -\frac{M}{2} \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{M\cdot\cot\left(\frac{\theta}{2}\right)}{aspect} & 0 & \frac{M\cdot\cot\left(\frac{\theta}{2}\right)}{aspect} & \frac{M\cdot\cot\left(\frac{\theta}{2}\right)}{2\cdot aspect} \\ 0 & 0 & 0 & 0 & D \cdot \cot\left(\frac{\theta}{2}\right) \\ \frac{1001M-20D}{999} & \frac{1001M-20D}{999} & -\frac{20D}{999} & -\frac{20D}{999} & \frac{1001M-40D}{1998} \\ M & M & 0 & 0 & \frac{M}{2} \end{bmatrix}$$

Aula 8

Exercício 1[4]

$$\vec{AB} = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{aspect}, 0, \frac{1001M - 20D}{999}\right) - \left(0, 0, \frac{1001M - 20D}{999}\right) = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{aspect}, 0, 0\right)$$

$$\vec{AC} = \left(0, 0, -\frac{20D}{999}\right) - \left(0, 0, \frac{1001M - 20D}{999}\right) = \left(0, 0, -\frac{1001M}{999}\right)$$

$$\vec{AE} = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{2 \cdot aspect}, D \cdot \cot\left(\frac{\theta}{2}\right), \frac{1001M - 40D}{1998}\right) - \left(0, 0, \frac{1001M - 20D}{999}\right) = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{2 \cdot aspect}, D \cdot \cot\left(\frac{\theta}{2}\right), -\frac{1001M}{999}\right)$$

$$\vec{BD} = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{aspect}, 0, \frac{20D}{999}\right) - \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{aspect}, 0, \frac{1001M - 20D}{999}\right) = \left(0, 0, -\frac{1001M}{999}\right)$$

$$\vec{BE} = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{2 \cdot aspect}, D \cdot \cot\left(\frac{\theta}{2}\right), \frac{1001M - 40D}{1998}\right) - \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{aspect}, 0, \frac{1001M - 20D}{999}\right) = \left(-\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{2 \cdot aspect}, D \cdot \cot\left(\frac{\theta}{2}\right), -\frac{1001M}{999}\right)$$

[4] $P''' = \begin{bmatrix} A & B & C & D & E \end{bmatrix}$

Exercício 1

$$\vec{CD} = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{aspect}, 0, -\frac{20D}{999}\right) - \left(0, 0, -\frac{20D}{999}\right) = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{aspect}, 0, 0\right)$$

$$\vec{CE} = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{2 \cdot aspect}, D \cdot \cot\left(\frac{\theta}{2}\right), \frac{1001M - 40D}{1998}\right) - \left(0, 0, -\frac{20D}{999}\right) = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{2 \cdot aspect}, D \cdot \cot\left(\frac{\theta}{2}\right), \frac{1001M}{1998}\right)$$

$$\vec{DB} = -\vec{BD} = \left(0, 0, \frac{1001M}{999}\right)$$

$$\vec{DE} = \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{2 \cdot aspect}, D \cdot \cot\left(\frac{\theta}{2}\right), \frac{1001M - 40D}{1998}\right) - \left(\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{aspect}, 0, -\frac{20D}{999}\right) = \left(-\frac{M \cdot \cot\left(\frac{\theta}{2}\right)}{2 \cdot aspect}, D \cdot \cot\left(\frac{\theta}{2}\right), \frac{1001M}{999}\right)$$

Exercício 1

$$\vec{N_1} = \vec{AB} \times \vec{AC} = \left(0, \frac{1001M^2 \cdot \cot\left(\frac{\theta}{2}\right)}{999 \cdot aspect}, 0\right)$$

$$\vec{N_2} = \vec{AE} \times \vec{AB} = \left(0, -\frac{1001M^2 \cdot \cot\left(\frac{\theta}{2}\right)}{999 \cdot aspect}, -\frac{DM \cdot \cot^2\left(\frac{\theta}{2}\right)}{2 \cdot aspect}\right)$$

$$\vec{N_3} = \vec{BE} \times \vec{BD} = \left(-\frac{1001DM \cdot \cot\left(\frac{\theta}{2}\right)}{999}, -\frac{1001M^2 \cdot \cot\left(\frac{\theta}{2}\right)}{1998 \cdot aspect}, 0\right)$$

$$\vec{N_4} = \vec{CD} \times \vec{CE} = \left(0, -\frac{DM \cdot \cot^2\left(\frac{\theta}{2}\right)}{aspect}, \frac{DM \cdot \cot^2\left(\frac{\theta}{2}\right)}{2 \cdot aspect}\right)$$

$$\vec{N_5} = \vec{DB} \times \vec{DE} = \left(-\frac{1001DM \cdot \cot\left(\frac{\theta}{2}\right)}{999}, -\frac{1001M^2 \cdot \cot\left(\frac{\theta}{2}\right)}{1998 \cdot aspect}, 0\right)$$

Exercício 2

$$\vec{N_1}_z = 0$$

$$\vec{N_2}_z = -\frac{DM \cdot \cot^2\left(\frac{\theta}{2}\right)}{2 \cdot aspect}$$

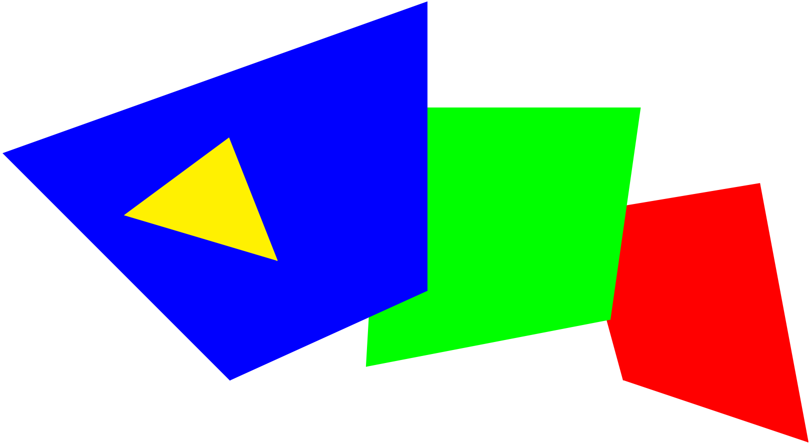
$$\vec{N_3}_z = 0$$

$$\vec{N_4}_z = \frac{DM \cdot \cot^2\left(\frac{\theta}{2}\right)}{2 \cdot aspect}$$

$$\vec{N_5}_z = 0$$

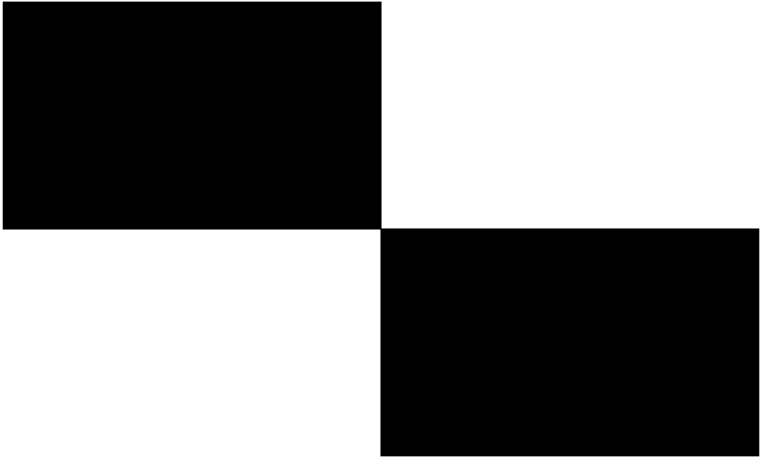
A única face com $\vec{N}_z > 0$ e, portanto, visível, é *CDE*.

Exercício 3

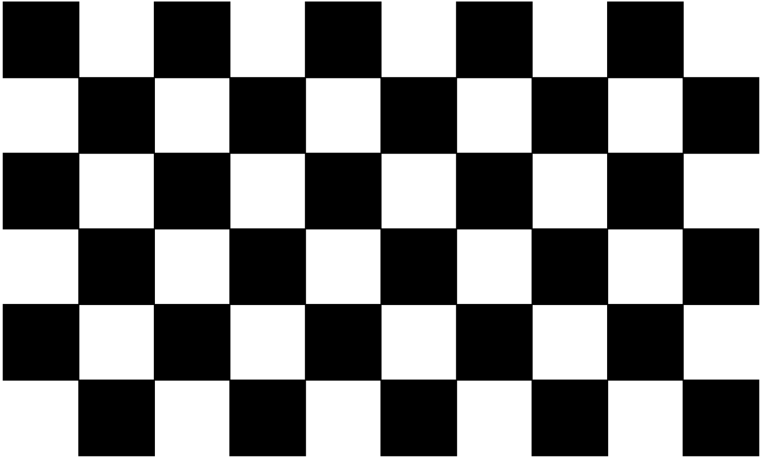


Aula 9

Exercício 1



Exercício 2



Exercício 3[5]

```
load_texture_from_file(0, 'caixa/textura2.jpg')
```

[5] Alterando a textura do objeto "caixa", descrito no exemplo de aula.

Aula 10

Exercício 1

$$I_{\text{amb}} = k_a I_a = \frac{D}{40} \cdot \frac{4}{5} = \frac{D}{50}$$

Exercício 2

$$I_{\text{diff}} = k_a I_a + k_d I_l \cos\theta = \frac{D}{50} + \frac{M}{20} \cos\theta$$

Exercício 3

$$I_{\text{spec}} = k_a I_a + k_d I_l \cos\theta + k_s I_l \cos^{n_s}\alpha = \frac{D}{50} + \frac{M}{20}\cos\theta + \frac{3M}{4D+80}\cos^{M\cdot D}\alpha$$