Aula_11 Integral Indefinida

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$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

EXERCÍCIOS

1) $\int 3x^3\,dx = 3\int x^3\,dx = \frac{3.x^4}{4} + C$

VERIFICANDO:

$F(x) = \frac{3x^4}{4} \Rightarrow F'(x) = f(x)$

$= \frac{3.\cancel{4}x^{4-1}}{\cancel{4}} = 3x^3$

2) $\int \frac{x^3}{3}\,dx = \frac{1}{3}\int x^3\,dx = \frac{1}{3}.\frac{x^4}{4} + C = \frac{x^4}{12} + C$

VERIFICANDO:

$F(x) = \frac{x^4}{12} \Rightarrow F'(x) = \frac{\cancel{4}x^3}{\cancel{12}} = \frac{x^3}{3} \Rightarrow f(x)$

3) $\int \underbrace{4x^4}_{f(x)} + \underbrace{\frac{x^2}{2}}_{g(x)}\,dx = \int 4x^4\,dx + \int \frac{x^2}{2}\,dx$

$= 4\int x^4\,dx + \frac{1}{2}\int x^2\,dx$

$= 4\frac{x^5}{5} + \frac{1}{2}.\frac{x^3}{3} + C = \frac{4x^5}{5} + \frac{x^3}{6} + C$

VERIFICANDO:

$F(x) = \frac{4x^5}{5} + \frac{x^3}{6}$

$F'(x) = \frac{4.\cancel{5}x^{5-1}}{\cancel{5}} + \frac{\cancel{3}.x^{3-1}}{\cancel{6}}$

$= 4x^4 + \frac{x^2}{2}$

4) $\int 9x^3 - 2x^5\,dx = \int 9x^3\,dx - \int 2x^5\,dx$

$= 9\int x^3\,dx - 2\int x^5\,dx$

$= 9\frac{x^4}{4} - 2\frac{x^6}{6} + C = \frac{9x^4}{4} - \frac{x^6}{3} + C$

VERIFICANDO:

$F(x) = \frac{9x^4}{4} - \frac{x^6}{3}$

$F'(x) = \frac{9\cancel{4}x^3}{\cancel{4}} - \frac{6x^5}{3}$

$= 9x^3 - 2x^5$

5) $\int \frac{3x^2}{4} - \frac{2}{x^3}\,dx = \int \frac{3x^2}{4}\,dx - \int \frac{2}{x^3}\,dx$

$= \frac{3}{4}\int x^2\,dx - 2\int \frac{1}{x^3}\,dx = \frac{3}{4}\int x^2\,dx - 2\int x^{-3}\,dx =$

$= \frac{\cancel{3}}{4}.\frac{x^3}{\cancel{3}} - \cancel{2}.\frac{x^{-2}}{-\cancel{2}} + C = \frac{x^3}{4} + \frac{1}{x^2} + C$

VERIFICANDO: $F(x) = \frac{x^3}{4} + \frac{1}{x^2} \Rightarrow F'(x) = \frac{3x^2}{4} + (-2.x^{-3})$

$\frac{1}{x^2} \rightarrow x^{-2}$

$= \frac{3x^2}{4} - \frac{2}{x^3}$

6) $\int \frac{2x^3}{3} + 3^x\,dx = \int \frac{2x^3}{3}\,dx + \int 3^x\,dx = \frac{2}{3}\int x^3\,dx + \int 3^x\,dx =$

$\left\{ \int a^x\,dx = \frac{a^x}{\ln(a)} + C \right.$

6) $\int \frac{2x^3}{3} + 3^x \, dx = \int \frac{2x^3}{3} dx + \int 3^x dx = \frac{2}{3}\int x^3 dx + \int 3^x dx =$

$\underset{\text{CUIDADO!!!}}{\hookrightarrow a^x}$

$\int a^x dx = \frac{a^x}{\ln(a)} + C$

$= \frac{\cancel{2}}{3}\cdot\frac{x^4}{\cancel{4}} + \frac{3^x}{\ln(3)} + C = \frac{x^4}{6} + \frac{3^x}{\ln(3)} + C$

VERIFICANDO: $F(x) = \frac{x^4}{6} + \frac{3^x}{\ln(3)} \Rightarrow F'(x) = \frac{\cancel{4}x^3}{\cancel{6}} + \frac{1}{\cancel{\ln(3)}}\cdot 3^x \cdot \cancel{\ln(3)}$

$= \frac{2x^3}{3} + 3^x$

7) $\int 4^x - e^x \, dx = \int \overset{\hookrightarrow a^x}{4^x} dx - \int \overset{\hookrightarrow e^x}{e^x} dx = \frac{4^x}{\ln(4)} - e^x + C$

VERIFICANDO: $F(x) = \frac{4^x}{\ln(4)} - e^x \Rightarrow F'(x) = \frac{1}{\cancel{\ln(4)}}\cdot 4^x \cdot \cancel{\ln(4)} - e^x$

$= 4^x - e^x$

8) $\int \frac{x^3}{3} + \sqrt{x} \, dx = \int \frac{x^3}{3} dx + \int \overset{\hookrightarrow x^{1/2}}{\sqrt{x}} dx = \frac{1}{3}\int x^3 dx + \int x^{1/2} dx$

$= \frac{1}{3}\cdot\frac{x^4}{4} + \frac{x^{3/2}}{3/2} + C = \frac{x^4}{12} + \frac{2}{3}\cdot x^{3/2} + C = \frac{x^4}{12} + \frac{2\sqrt{x^3}}{3} + C$

VERIFICANDO: $F(x) = \frac{x^4}{12} + \frac{2\sqrt{x^3}}{3} \Rightarrow F'(x) = \frac{\cancel{4}x^3}{\cancel{12}} + \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}\cdot x^{1/2}$

$= \frac{x^3}{3} + \sqrt{x}$

EXERCÍCIOS (SUBSTITUIÇÃO)

$(x+10)^3 \neq x^3 + 10^3$

$\hookrightarrow x^3 + 3.x^2.10 + 3x.10^2 + 10^3$

1) $\int (x+10)^3 dx = \int u^3 \underset{\hookrightarrow du}{dx} = \int u^3 du = \frac{u^4}{4} + C = \frac{(x+10)^4}{4} + C$

$f(u) = u^3$

$u(x) = 1\cdot x + 10 \Rightarrow \frac{du}{dx} = u'(x) \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$

2) $\int (x - \frac{1}{2})^4 dx = \int u^4 \underset{\hookrightarrow du}{dx} = \int u^4 du = \frac{u^5}{5} + C = \frac{(x - 1/2)^5}{5} + C$

$f(u) = u^4$

$u(x) = x - \frac{1}{2} \Rightarrow \frac{du}{dx} = u'(x) \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$

3) $\int \frac{2x}{\sqrt{\boxed{x^2+1}}} dx = \int \frac{2x}{\sqrt{u}} \underset{\hookrightarrow du}{dx} = \int \frac{\cancel{2x}}{\sqrt{u}}\cdot\frac{1}{\cancel{2x}} du = \int \frac{1}{\sqrt{u}} du = \int u^{-1/2} du = \frac{u^{-1/2+1}}{-1/2+1} + C$

$= \frac{(x^2+1)^{1/2}}{\cdot} + C$

3) $\int \frac{2x}{\sqrt{\boxed{x^2+1}}}\,dx = \int \frac{2x}{\sqrt{u}}\,\underset{\hookrightarrow du}{dx} = \int \frac{\cancel{2x}}{\sqrt{u}}\,\frac{1}{\cancel{2x}}\,du = \int \frac{1}{\sqrt{u}}\,du = \int u^{\ldots}\,du = \frac{\ldots}{-1/2+1}$

$= \frac{(x^2+1)^{1/2}}{1/2} + C$

$= 2\sqrt{x^2+1} + C$

$u(x) = \begin{cases} 2x \\ \sqrt{x^2+1} \\ x^2+1 \leftarrow 2x \end{cases}$

$u(x) = x^2+1 \Rightarrow \frac{du}{dx} = u'(x) \Rightarrow \frac{du}{\boxed{dx}} = 2x \Rightarrow dx = \frac{1}{2x}\,du$

$f(u) = \frac{1}{\sqrt{u}}$

4) $\int y(y^2+5)^8\,dy = \int y\cdot u^8\,\underset{\hookrightarrow du}{dy} = \int \cancel{y}\cdot u^8 \frac{1}{2\cancel{y}}\,du = \frac{1}{2}\int u^8\,du = \frac{1}{2}\frac{u^9}{9} + C = \frac{(y^2+5)^9}{18} + C$

$u(y) = \begin{cases} y \\ (y^2+5)^8 \\ y^2+5 \leftarrow 2y \end{cases}$

$u(y) = y^2+5$

$f(u) = u^8$

$\frac{du}{dy} = u'(y) \Rightarrow \frac{du}{\boxed{dy}} = 2y \Rightarrow dy = \frac{1}{2y}\,du$

5) $\int \frac{1}{\sqrt{4-y}}\,dy = \int \frac{1}{\sqrt{u}}\,\underset{\hookrightarrow du}{dy} = \int u^{-1/2}(-du) = -1\int u^{-1/2}\,du = -\frac{u^{-1/2+1}}{-1/2+1} + C = -\frac{u^{1/2}}{1/2} + C$

$= -2\sqrt{u} + C$

$= -2\sqrt{4-y} + C$

$u(y) = 4-y \Rightarrow \frac{du}{dy} = u'(y) \Rightarrow \frac{du}{dy} = -1 \Rightarrow dy = -du$

6) $\int x e^{(-x^2)}\,dx = \int x e^{u}\,\underset{\hookrightarrow du}{dx} = \int \cancel{x}\cdot e^{u}\cdot\left(-\frac{1}{2\cancel{x}}\,du\right) = -\frac{1}{2}\int e^{u}\,du = -\frac{e^{u}}{2} + C$

$= -\frac{e^{(-x^2)}}{2} + C$

$u(x) = -x^2 \Rightarrow \frac{du}{dx} = u'(x) \Rightarrow \frac{du}{\boxed{dx}} = -2x \Rightarrow dx = -\frac{1}{2x}\,du$

7) $\int t^2 e^{(t^3+1)}\,dt = \int t^2 e^{u}\,\underset{\hookrightarrow du}{dt} = \int \cancel{t^2} e^{u}\left(\frac{1}{3\cancel{t^2}}\,du\right) = \frac{1}{3}\int e^{u}\,du = \frac{1}{3}e^{u} + C$

$= \frac{e^{(t^3+1)}}{3} + C$

$u(t) = t^3+1 \Rightarrow \frac{du}{dt} = u'(t) \Rightarrow \frac{du}{\boxed{dt}} = 3t^2 \Rightarrow dt = \frac{1}{3t^2}\,du$

8) $\int r\sqrt{r^2+1}\,dr = \int r\sqrt{u}\,\underset{\hookrightarrow du}{dr} = \int \cancel{r}\,u^{1/2}\left(\frac{1}{2\cancel{r}}\,du\right) = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\,\frac{u^{1/2+1}}{1/2+1} + C$

$= \frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\cdot u^{3/2} + C$

$= \frac{\sqrt{(r^2+1)^3}}{3} + C$

$u(r) = r^2+1 \Rightarrow \frac{du}{dr} = u'(r) \Rightarrow \frac{du}{\boxed{dr}} = 2r \Rightarrow dr = \frac{1}{2r}\,du$

9) $\int \frac{[\ln(z)]^2}{z}\,dz = \int \frac{u^2}{z}\,\underset{\hookrightarrow du}{dz} = \int \frac{u^2}{\cancel{z}}\,\cancel{z}\cdot du = \int u^2\,du = \frac{u^3}{3} + C$

$= \frac{[\ln(z)]^3}{3} + C$

$u(z) = \ln(z) \Rightarrow \frac{du}{dz} = u'(z) \Rightarrow \frac{du}{\boxed{dz}} = \frac{1}{z} \Rightarrow dz = z\cdot du$

$\ldots \int \frac{\cancel{x+1}}{\ldots}\left(\frac{\ldots}{\ldots}\,du\right) = \ldots\int \frac{1}{u}\,du = \ldots\int u^{-1}\,du$

10) $\displaystyle\int \frac{x+1}{x^2+2x+19}\,dx = \int \frac{x+1}{u}\,\underset{\hookrightarrow du}{dx} = \int \frac{\cancel{x+1}}{u}\left(\frac{1}{2\cancel{(x+1)}}\,du\right) = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\int u^{-1}\,du$

$= \frac{1}{2}\ln|u| + C$

$u(x) = x^2+2x+19 \qquad \frac{du}{dx} = u'(x) \Rightarrow \frac{du}{\boxed{dx}} = 2x+2 = 2(x+1) \Rightarrow dx = \frac{1}{2(x+1)}\,du$

$= \dfrac{\ln|x^2+2x+19|}{2} + C$

11) $\displaystyle\int \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy = \int \frac{e^{u}}{\sqrt{y}}\,\underset{\hookrightarrow du}{dy} = \int \frac{e^{u}}{\cancel{\sqrt{y}}}\left(2\cancel{\sqrt{y}}\,du\right) = 2\int e^{u}\,du = 2e^{u} + C$

$= 2e^{\sqrt{y}} + C$

$u(y) = y^{1/2} \Rightarrow \frac{du}{dy} = u'(y) \Rightarrow \frac{du}{\boxed{dy}} = \frac{1}{2}y^{-1/2} = \frac{1}{2\sqrt{y}} \Rightarrow dy = 2\sqrt{y}\,du$