AULA MECÂNICA 07/12 ①

QUERIA RESOLVER. $U: \mathbb{R}^{3N} \to \mathbb{R}$

$$m_i \ddot{X}_i = -\frac{\partial U}{\partial X_i} \quad (*)$$

ENERGIA POTENCIAL.

MAS SUJEITO A VÍNCULOS GEOMETRICOS.

DO TIPO

$$G(X(t)) \equiv 0.$$

$G: \mathbb{R}^{3N} \to \mathbb{R}^{k}$ E ASSUMIMOS

QUE $\forall\, y \in \mathbb{R}^{3N} \,/\, G(y) = 0$ sé

$DG(y)$ TEM POSTO k.

(a)

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a) É NECESSÁRIO QUE:

$\forall t \in I$ $\dot{X}(t)$ PRECISA ESTAR EM

$N(DG(X(t)))$.

b) IMPOSSÍVEL RESOLVER $(*)$ MESMO COM CONDIÇÕES INICIAIS $X(0) \in G^{-1}(\vec{0})$.

$\dot{X}(0) \in N(DG(X(0)))$!

– PRINCÍPIO DE D'ALEMBERT.

ENCONTRAR UM CAMPO DE FORÇAS

$R = (R_1, \dots, R_N)$

$m_i \ddot{X}_i = -\frac{\partial U}{\partial X_i} + R_i$

$R_i : \mathbb{R}^{3N} \times \mathbb{R}^{3N} \to \mathbb{R}^3$ $\quad R_i(X, \dot{X})$

G A PROPRIEDADE EXTRA QUE:

$\langle R, V\rangle = 0, \ \forall V$

$\langle R(X,X), V\rangle = 0$ P/ TODO V EM $N(DG(X))$.

EXERCÍCIO: R ESTÁ COMPLETAMENTE DETERMINADA NESTAS CONDIÇÕES.

$g: R^{3N-K} \to R^{K}$.

$g(Y) = Z$. $\qquad G^{-1}(0)$

$G(Y,Z) = Z - g(Y)$.

$Dg(a) = 0$.
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$X(t)$ UMA CURVA EM $g^{-1}(0)$,

COM $X(0) = (0,0)$.

$\dot{X}(0) = (Y_0, 0)$.

$$X(t) = (Y(t), Z(t)) = (Y(t), g(Y(t))).$$

$$\dot{X}(t) = (\dot{Y}(t), Dg(Y(t))\dot{Y}(t)).$$

$$\underset{0}{\parallel}$$

$$\ddot{X}(t) = (\ddot{Y}(t), \langle D^2 g\, \dot{Y}, \dot{Y}\rangle + \overset{0}{\overbrace{Dg(Y(0))}}\ddot{Y})$$

$$\ddot{X}(0) = (\ddot{Y}(t), \langle D H g\, \dot{Y}, \dot{Y}\rangle).$$

SE R SATISFAZ O PRINCÍPIO DE D'ALEMBERT, (5)

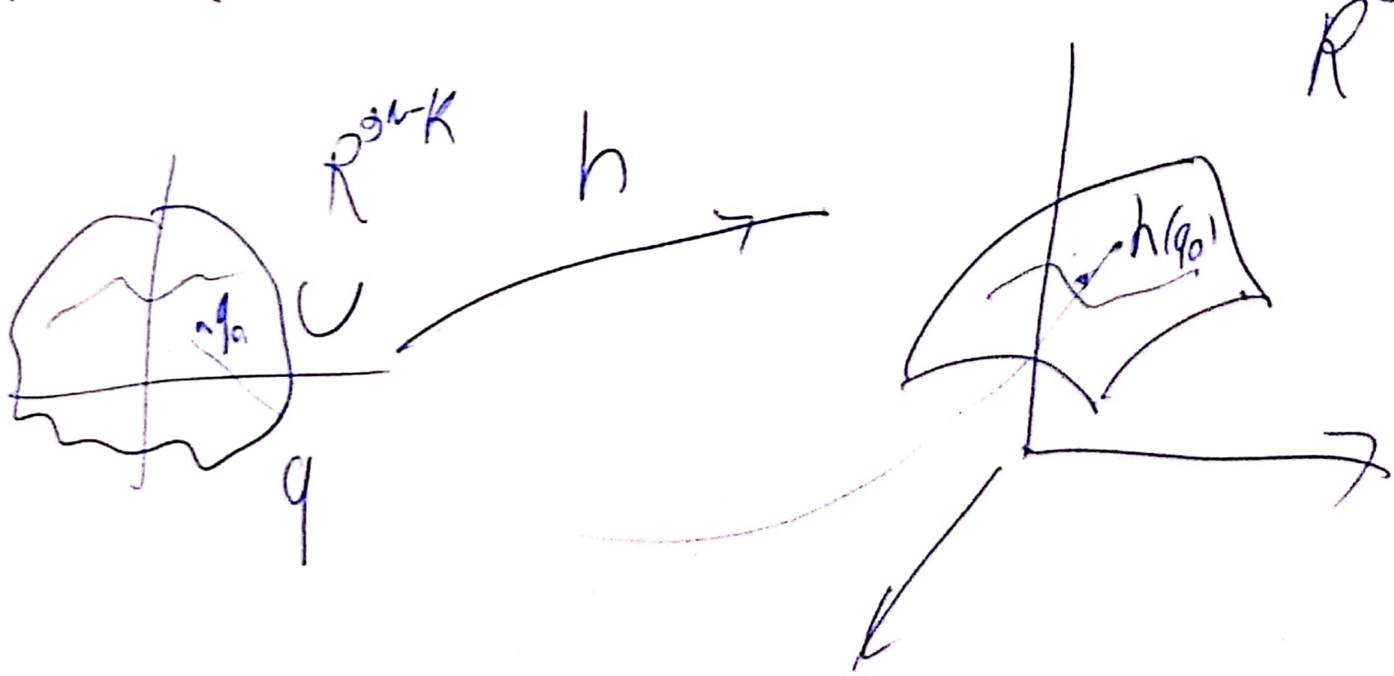
ENTÃO
$$H(X) = T(X,\dot{X}) + U(X) \text{ É CONSTANTE}$$

$$T(\dot{X}) = \frac{\sum m_i \|\dot{X}_i\|^2}{2}.$$

USAR LAGRANGIANO EM COORDENADAS LOCAIS.

$G^{-1}(0)$ = IMAGEM DE UMA FUNÇÃO C^2

DE R^{3N-K} EM R^{3N}.



DEFINIR LAGRANGIANO EM $R^{3N-k} \times R^{3N-k}$. 6

COORDENADAS GENERALIZADAS

$(q, \dot{q})$

$$L(q,\dot{q}) = \overline{T}(q,\dot{q}) - \overline{U}(q).$$

$$T(\dot{X}) = \frac{\sum_{i=1}^{N} m_i \|\dot{X}_i\|^2}{2}$$

$(q, \dot{q})$

$\gamma(0) = q_0$

$\dot{\gamma}(0) = \dot{q}_0$

$\gamma : I \to R^{3N-k}$. CURVA.

$X = h(\gamma) : I \to R^{3N}$

$\dot{X}(0) = Dh(q_0) \cdot \dot{q}_0$

$$M = \begin{bmatrix} m_1 & & & & & & & & & \\ & m_1 & & & & & & 0 & & \\ & & m_1 & & & & & & & \\ & & & m_2 & & & & & & \\ & & & & m_2 & & & & & \\ & & & & & m_2 & & & & \\ & & & & & & \ddots & & & \\ & & 0 & & & & & m_N & & \\ & & & & & & & & m_N & \\ & & & & & & & & & m_N \end{bmatrix}$$

$$T(\dot{x}) = \frac{\langle M\dot{x}, \dot{x}\rangle}{2}.$$

$$\bar{T}(q,\dot{q}) = \frac{\langle M\,Dh(q)\dot{q},\, Dh(q)\dot{q}\rangle}{2} =$$

$$= \frac{\langle Dh^T(q)\,M\,Dh(q)\,\dot{q}, \dot{q}\rangle}{2}.$$

$$B(q) = Dh^T(q)\,M\,Dh(q).$$

$$\bar{T}(q,\dot{q}) = \frac{1}{2}\langle B(q)\dot{q}, \dot{q}\rangle.$$

E $B(q)$ SIMÉTRICA.

$\bar{T}(q,\dot{q}) > 0$ SE $\dot{q} \neq \vec{0}$.

$$\overrightarrow{U(q)} = U(h(q)).$$

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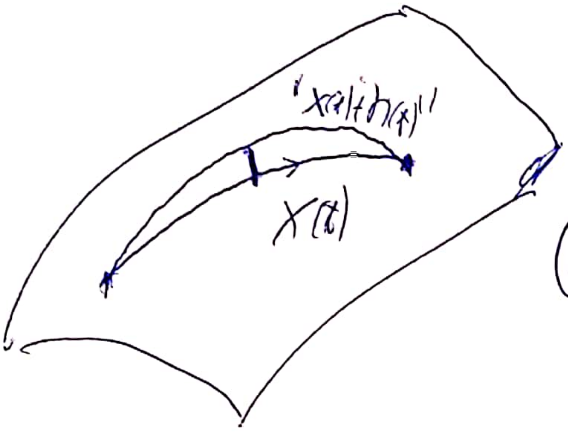
→ DEFINIMOS UM LAGRANGIANO NAS COORDENADAS GENERALIZADAS.

$\gamma(t)$ UMA CURVA EXTREMAL P/ O LAGRANGIANO NAS COORDENADAS GENERALIZADAS

$X(t)$ A CURVA CORRESPONDENTE EM $\mathbb{R}^{3N}$.

TEOREMA: γ É UM EXTREMAL P/ O LAGRANGIANO GENERALIZADO SE, E SÓ SE,

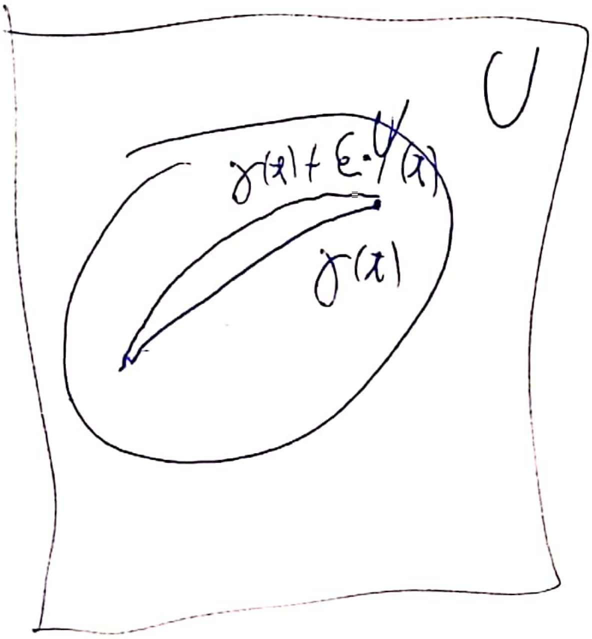
$X(t)$ SATISFAZ AS EQUAÇÕES DE NEWTON - LAGRANGE (SATISFAZ A RELAÇÃO VINCULAR DE D'ALEMBERT



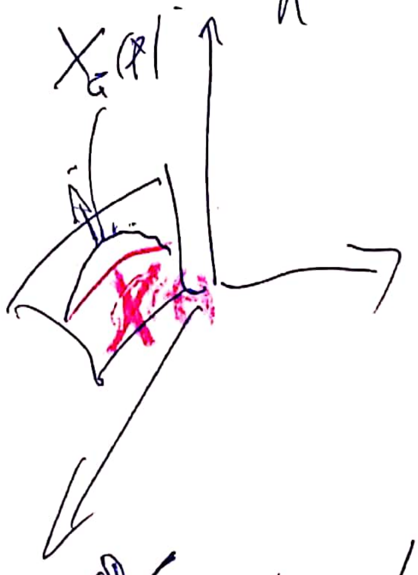
$\sigma^{-1}(a)$

$X(t) \in \sigma^{-1}(a)$,

$X(t) + \varepsilon h(t) \in \sigma^{-1}(a)$.



R^{3N-k}



$X_\varepsilon(t) = h(\gamma(t) + \varepsilon \psi(t))$.

$$X_\varepsilon(t) = X(t) + \varepsilon\, Dh(\gamma(t))\cdot\psi(t) + \text{T.O.S.}(\varepsilon).$$

$$\boxed{X_\varepsilon(t)} = h(\gamma(t) + \varepsilon\,\psi(t)) =$$

$$= \boxed{h(\gamma(t)) + Dh(\gamma(t))\cdot\varepsilon\,\psi(t) + \text{T.O.S.}(\varepsilon).}$$

$\bar{L}$ LAGRANGIANO NAS COORDENADAS GENERALIZADAS

$$\frac{\partial \bar{L}}{\partial q}(\gamma(t),\dot\gamma(t)) - \frac{d}{d}\,\frac{\partial \bar{L}}{\partial \dot q}(\gamma(t),\dot\gamma(t)) = 0.$$

$$\Leftrightarrow \quad \cancel{\frac{\partial h}{\partial q}}\ \left\langle \frac{\partial L}{\partial X} - \frac{d}{dt}\frac{\partial L}{\partial \dot X}, \xi \right\rangle = 0 \quad \forall \xi \in T_{x_0}G.$$

SE γ É CURVA EXTREMAL P/ $\bar{L}$,

$\Rightarrow$ ~~QUALQUER PERTUBAÇÃO DE γ TE~~

$\gamma : [a,b] \to R^{3N-K}$

$I = [a,b]$

$\forall \psi : I \to R^{3N-K}$, $\psi(a) = \psi(b) = 0$.

~~$\lim A \, (\gamma + \varepsilon\psi)$~~

$$A_{\psi}(\varepsilon) = \int_a^b \bar{L}(\gamma + \varepsilon\psi, \dot{\gamma} + \varepsilon\dot{\psi})\, dt$$

TEMOS QUE $\left.\dfrac{dA_{\psi}(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} = 0.$

$$\Rightarrow \frac{\partial \bar{L}}{\partial q} - \frac{d}{dt}\frac{\partial \bar{L}}{\partial \dot{q}} = 0.$$

como $A_\varphi(\varepsilon) =$

$$\int_a^b L(X_\varepsilon(t), \dot{X}_\varepsilon(t))\,dt.$$

onde $X_\varepsilon(t) = h(\gamma(t) + \varepsilon\,\varphi(t))$

$$\cancel{\varepsilon \quad A_\varphi(\varepsilon) = \int_a^b L(X(t) + \varepsilon\, Dh(t)\cdot\varphi(t) + \overline{\Psi}_\varepsilon(t)}$$

$$\cancel{A_\varphi(\varepsilon) = \int_a^b L(X(t) + \varepsilon\, Dh(\gamma(t))\varphi(t) + \Psi_\varepsilon(t), \dot{X}(t) + \varepsilon\, Dh\,\dot{\varphi}(t)}$$

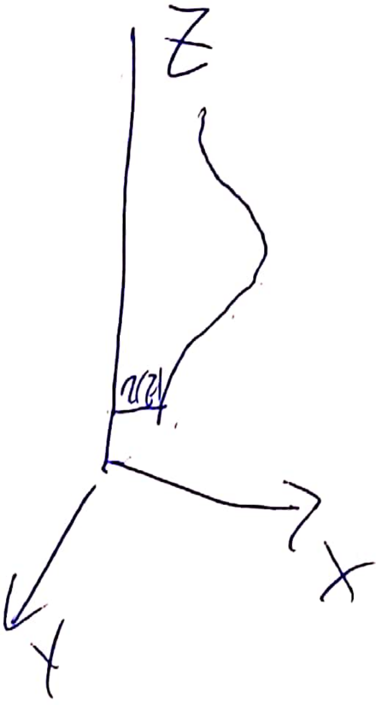
$$L(X, \dot{X}) = \sum_{i=1}^{n} \frac{m_i \|\dot{X}_i\|^2}{2} - U(X).$$

$$L(\alpha_\epsilon(t)) = \sum_{i=1}^{N} \frac{m_i \|\dot{\alpha}_{\epsilon,i}\|^2}{2} - U(\alpha_\epsilon(t)).$$

$$X_\epsilon(t) = X(t) + \epsilon \cdot Dh(\gamma(t)) \cdot \psi(t) + T.O.S.$$

$$\dot{X}_\epsilon(t) = \dot{X}(t) + \epsilon \cdot Hh(\gamma(t)) \cdot \dot{\gamma}(t)\psi(t) +$$

EXEMPLO: SUPERFÍCIE DE REVOLUÇÃO:



EXISTE UMA FUNÇÃO

$r(z)$

VÍNCULA.

$$G(x,y,z) = x^2 + y^2 - r^2(z) = 0.$$

LAGRANGIANO ONDE $U \equiv 0$. (14)

$(z, \cancel{\theta}) \to \mathbb{R}^3$.

$(z, \theta) \to \cancel{R\cos\theta, R}\,(r(z)\cos\theta, r(z)\sin\theta, z)$

$T(x, y, z, \dot{x}, \dot{y}, \dot{z}) = \dfrac{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}{2}$

$\dot{x} = r'\dot{z}\cos\theta + -r(z)\sin\theta\,\dot{\theta}$

$\overline{T}(\dot{\theta}, \dot{z}) = \frac{1}{2}\left[(1 + r'(z)^2)\dot{z}^2 + r(z)^2\dot{\theta}^2\right] = \overline{L}(\theta, z, \dot{\theta}, \dot{z})$.

$\dfrac{\partial \overline{L}}{\partial q} - \dfrac{d}{dt}\dfrac{\partial \overline{L}}{\partial \dot{q}} = 0$.

$\dfrac{\partial \overline{L}}{\partial \theta} = 0$.

$\Downarrow$

$\dfrac{\partial \overline{L}}{\partial \dot{\theta}} \equiv cte$.

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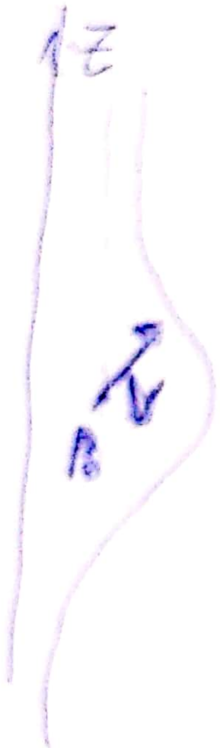
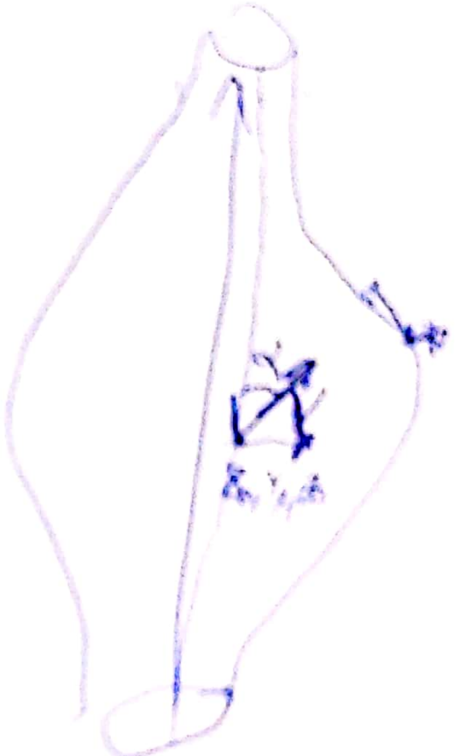
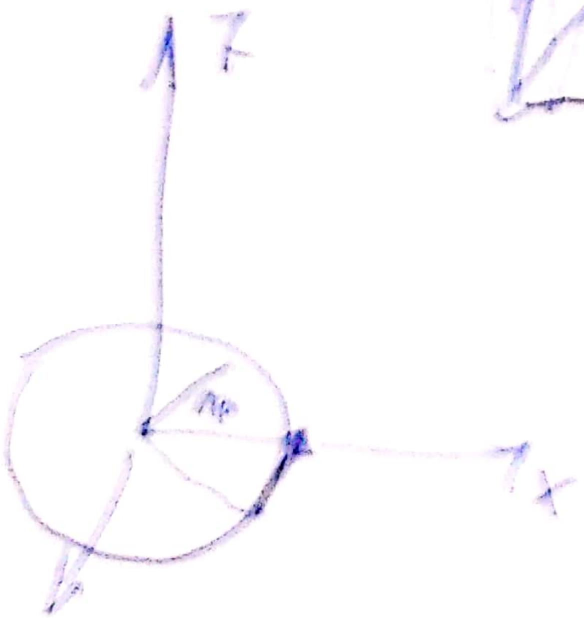
$$\Rightarrow \boxed{r^2(z)\cdot\dot{\theta} = CTE}$$

ALÉM DISSO TEMOS QUE A ENERGIA TOTAL É CONSTANTE







$r\dot{\theta} = \|V\| \cdot \sin\alpha$

$r^2\dot{\theta} = r \cdot \|V\| \cdot \sin\alpha$

c_2

$c_1 = r^2\dot{\theta} = r\,\|V\| \cdot \sin\alpha =$

$r \cdot \sin\alpha$ é constante