ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO

Observações de

Euler e Lagrange

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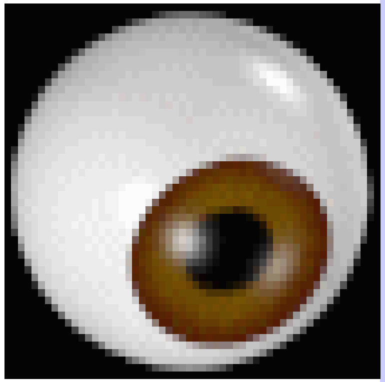
título

objetivo

relacionar as <u>variações temporais</u> de uma observação do espaço **fixa** com uma observação que se **movimenta com o fluido**



$$\frac{D\varphi}{Dt} = \frac{\partial \varphi}{\partial t} + \vec{v}\,.\,\mathrm{gr}\overleftarrow{\mathrm{a}}\mathrm{d}\;\varphi$$

sistema M =

= porção de matéria analisada

[observação à Lagrange]

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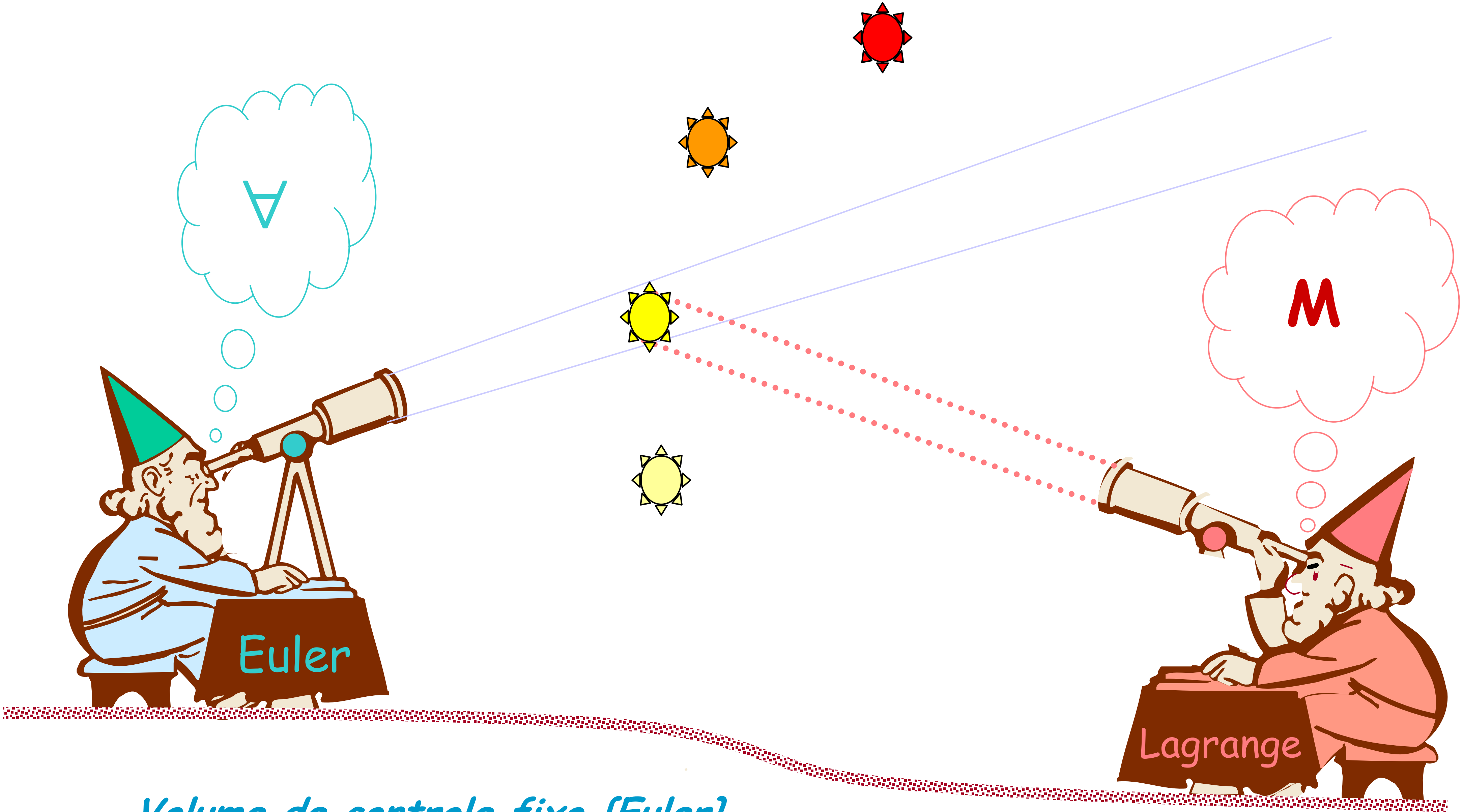
volume de controle $\forall$ =

= região do espaço considerada para estudo

[observação à Euler]

ambas

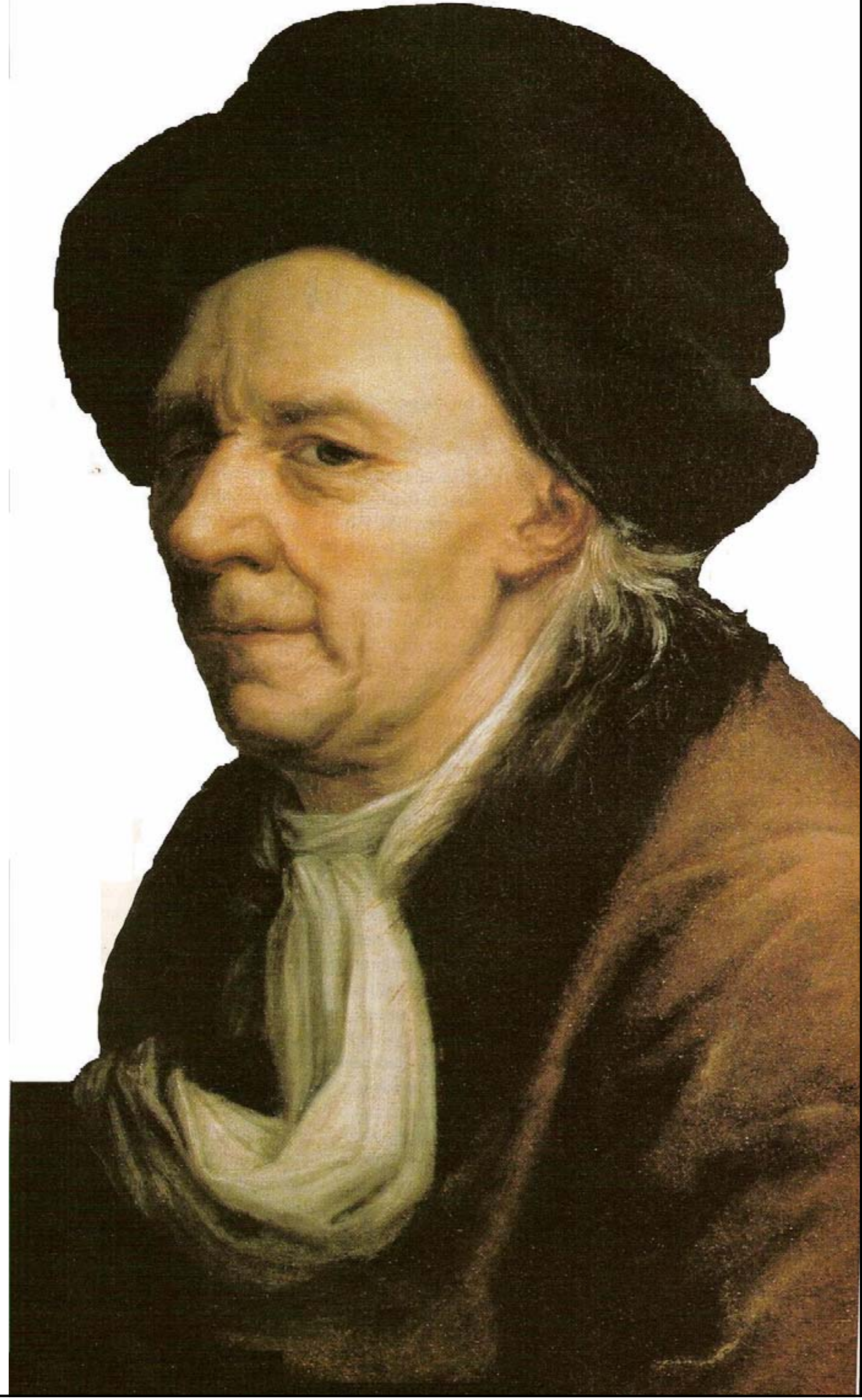


Volume de controle fixo {Euler}

Sistema em movimento {Lagrange}

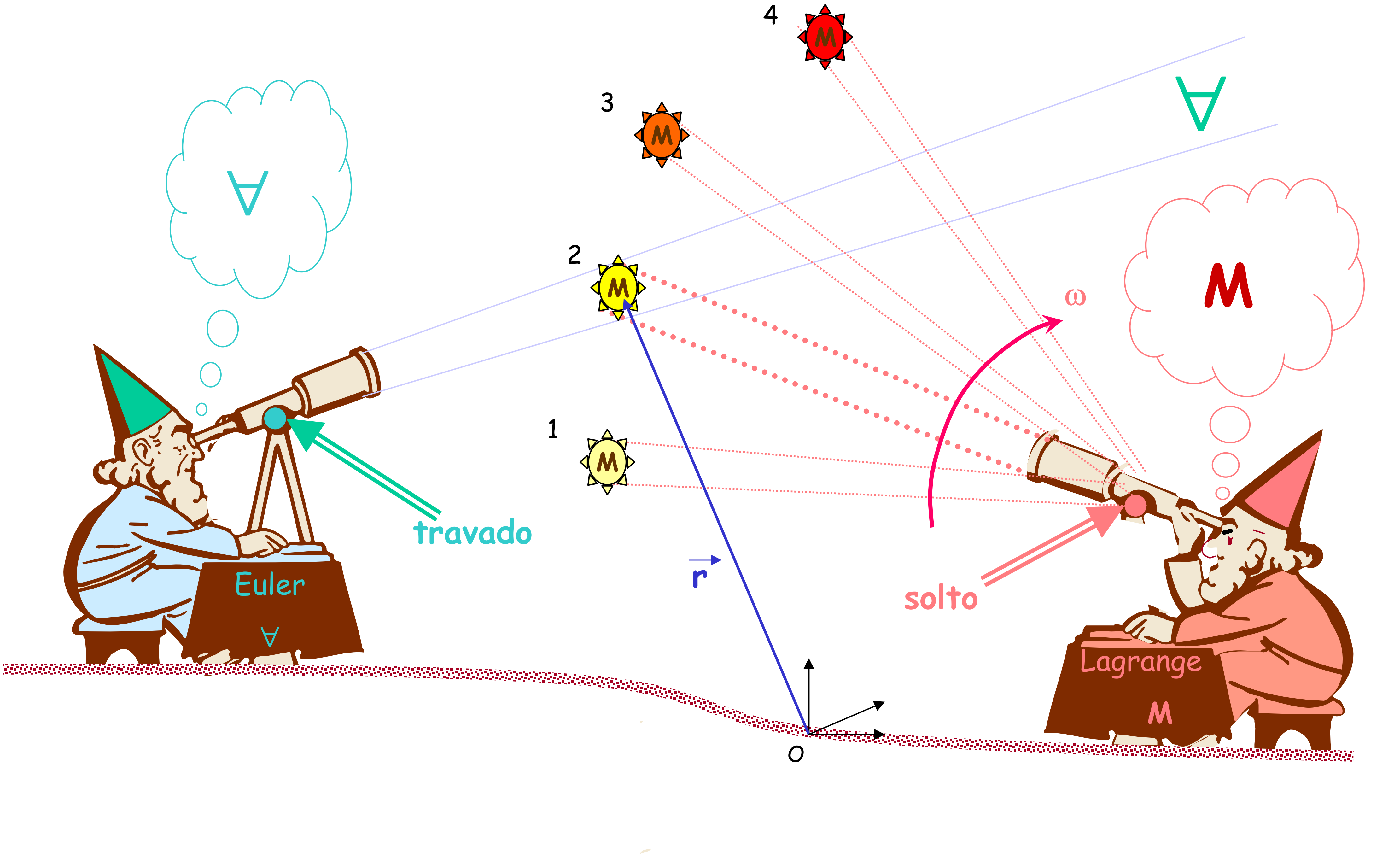
EULER

1707-1783

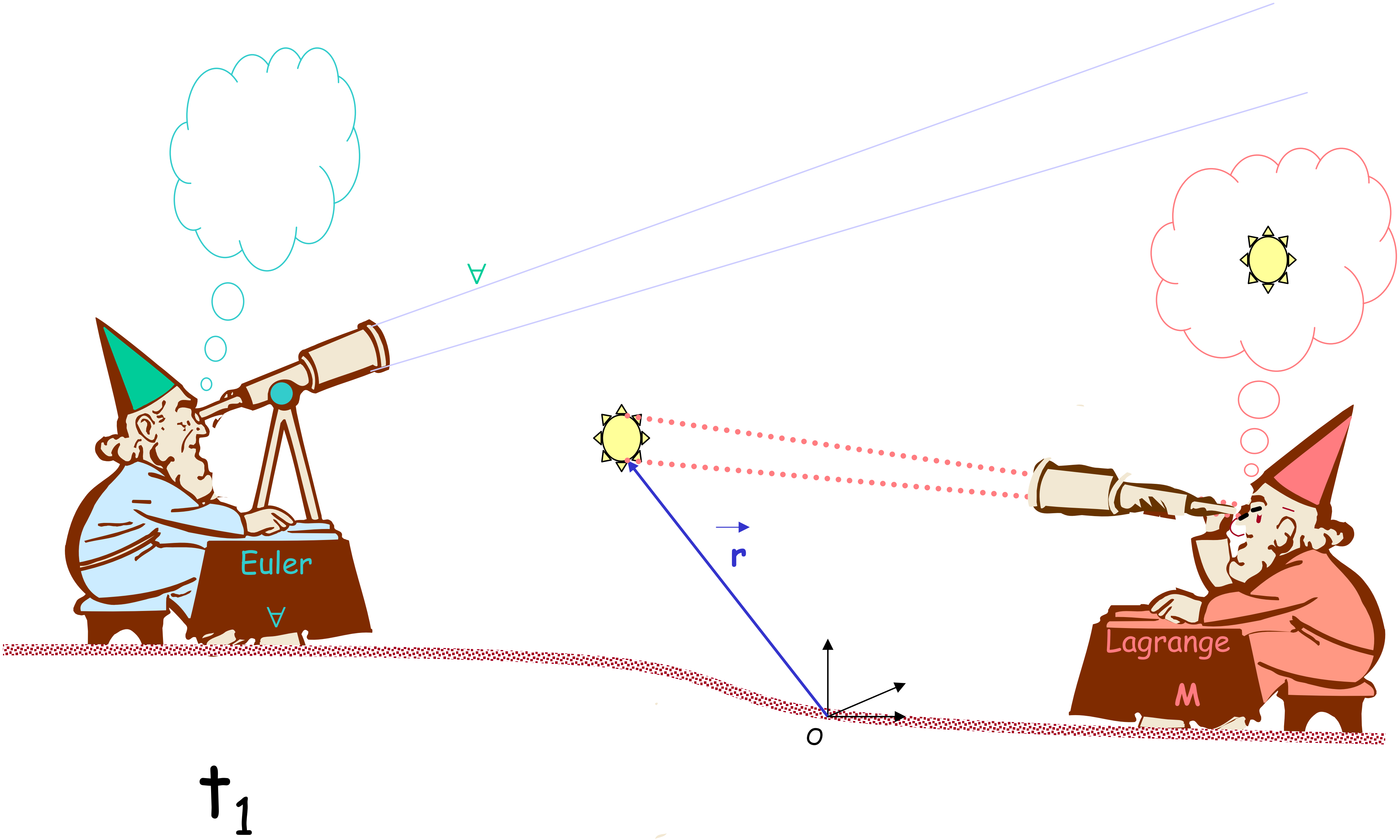


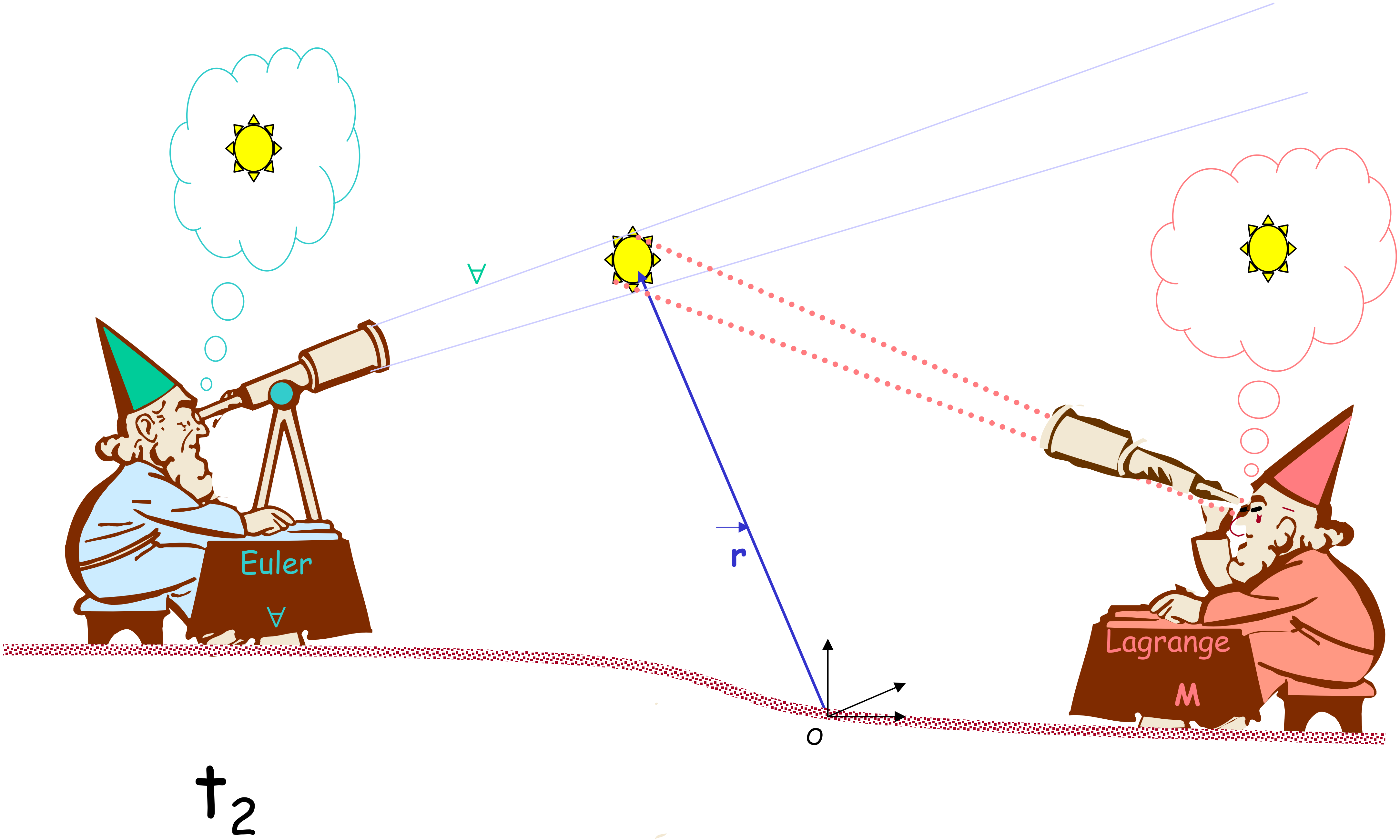
Lagrange

1736-1813

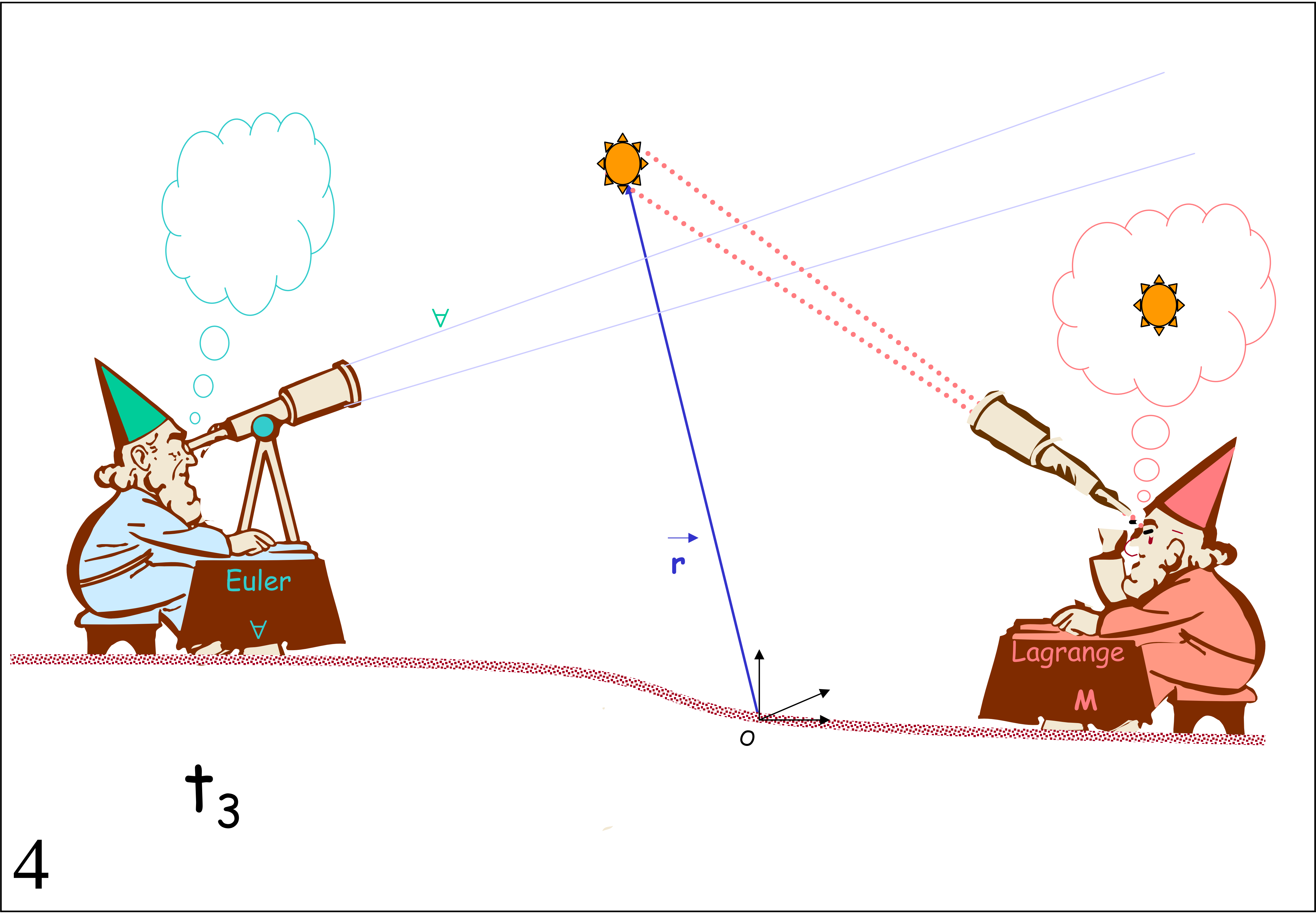


1

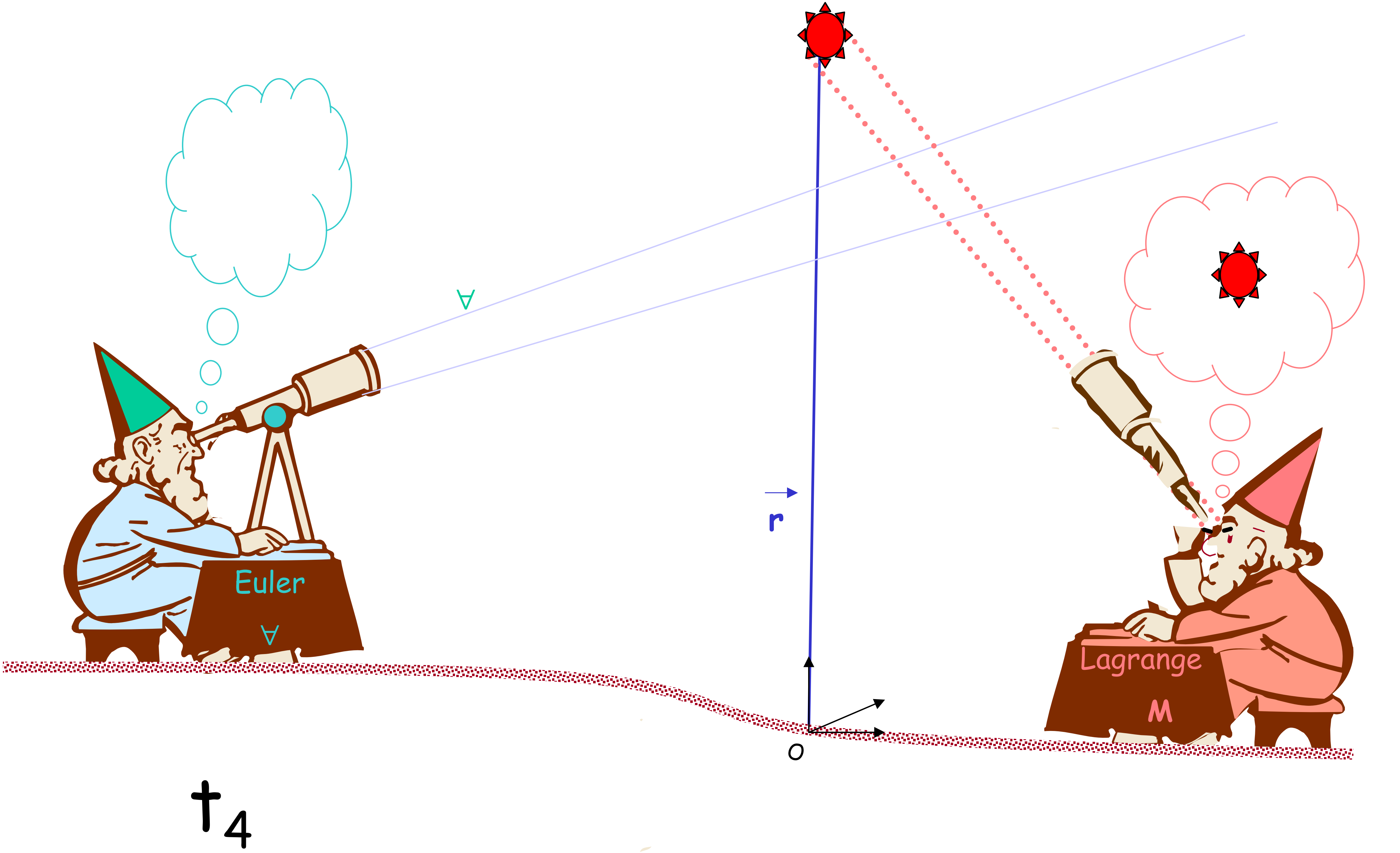

∀
Euler
∀
r
O
Lagrange
M
t1



3



∀
Euler
∀
$\vec{r}$
O
Lagrange
M
t_3
4



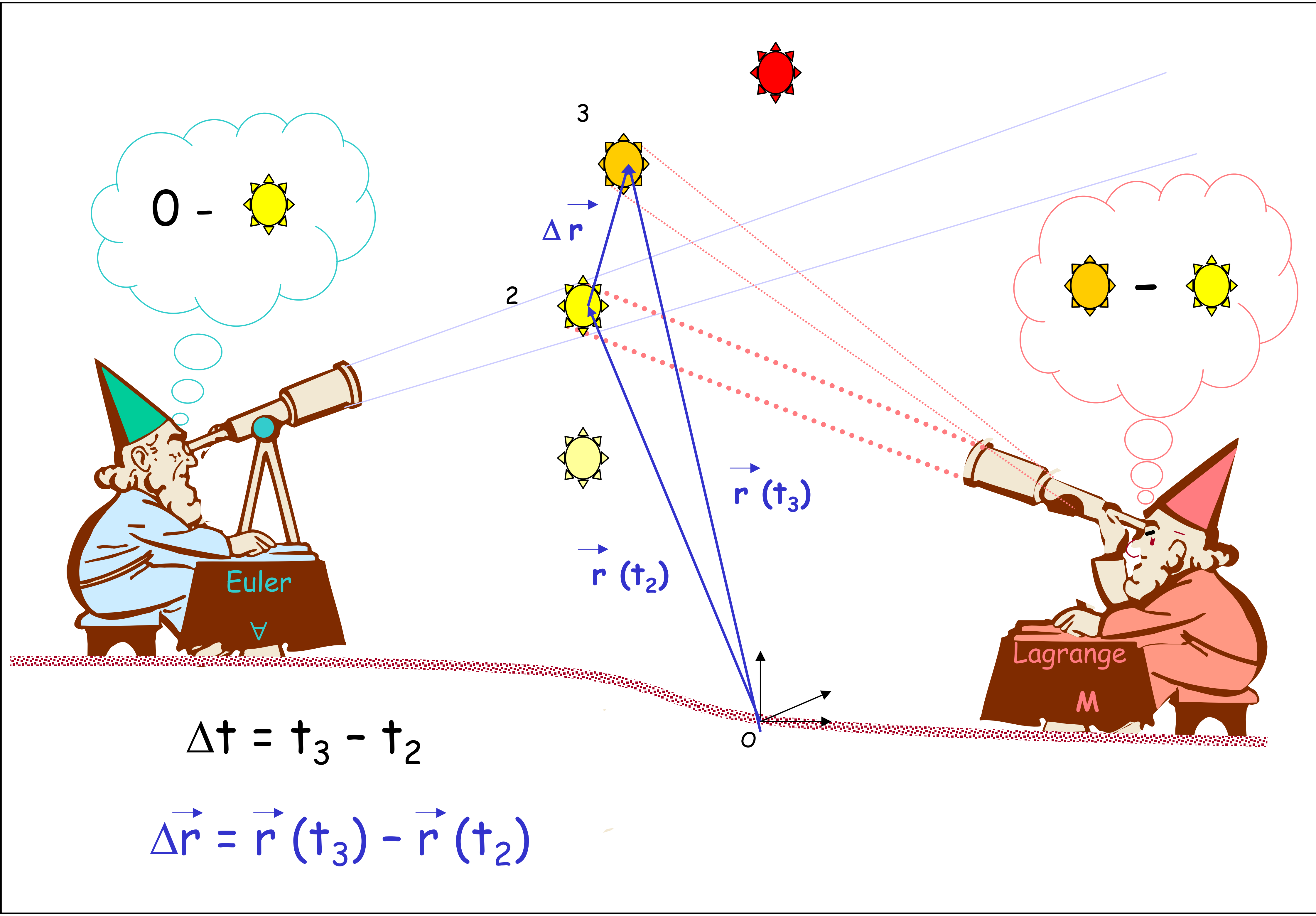
5

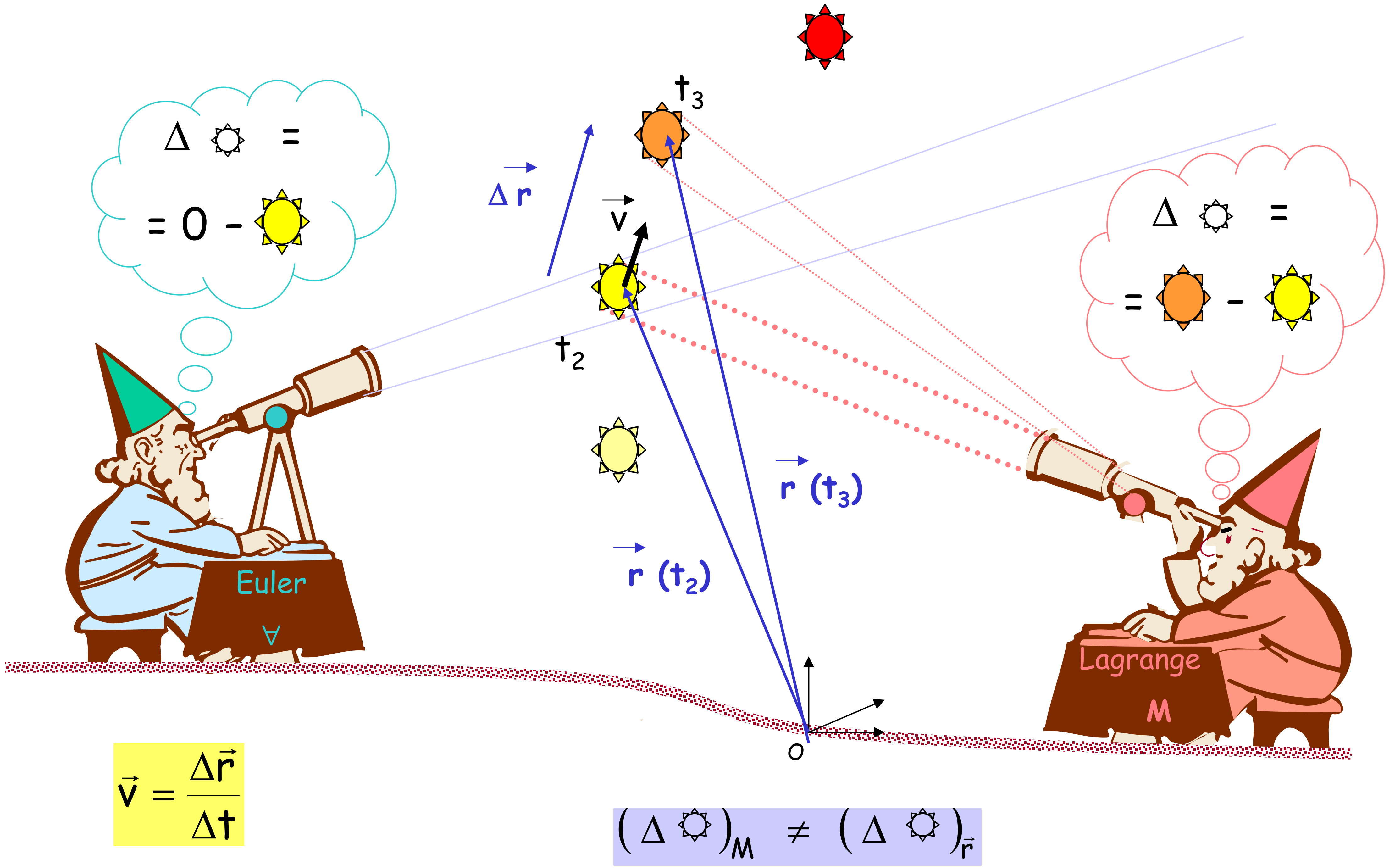
delta

$$\Delta = \text{final} - \text{inicial}$$

$$\Delta t = t(\text{final}) - t(\text{inicial})$$

$$\Delta r = r(\text{final}) - r(\text{inicial})$$


0 -
3
$\vec{\Delta r}$
2
$\vec{r}\,(t_3)$
$\vec{r}\,(t_2)$
Euler
∀
Lagrange
M
O
$\Delta t = t_3 - t_2$
$\vec{\Delta r} = \vec{r}\,(t_3) - \vec{r}\,(t_2)$

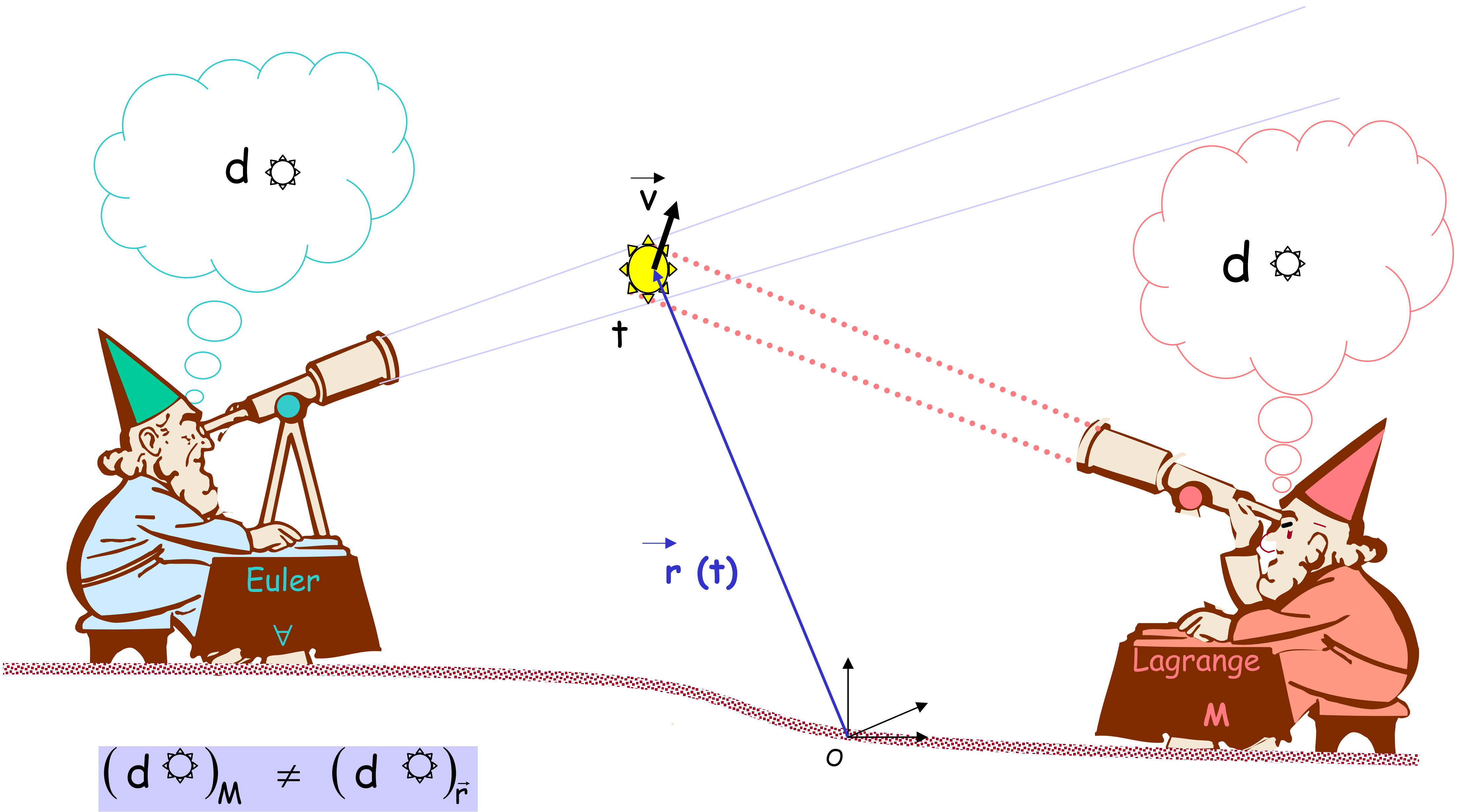

$\Delta \vec{r}$
$\vec{v}$
t_3
t_2
$\vec{r}\,(t_3)$
$\vec{r}\,(t_2)$
Euler
∀
Lagrange
M
O
$\vec{v} = \dfrac{\Delta \vec{r}}{\Delta t}$
$(\Delta)_M \neq (\Delta)_{\vec{r}}$

fazendo

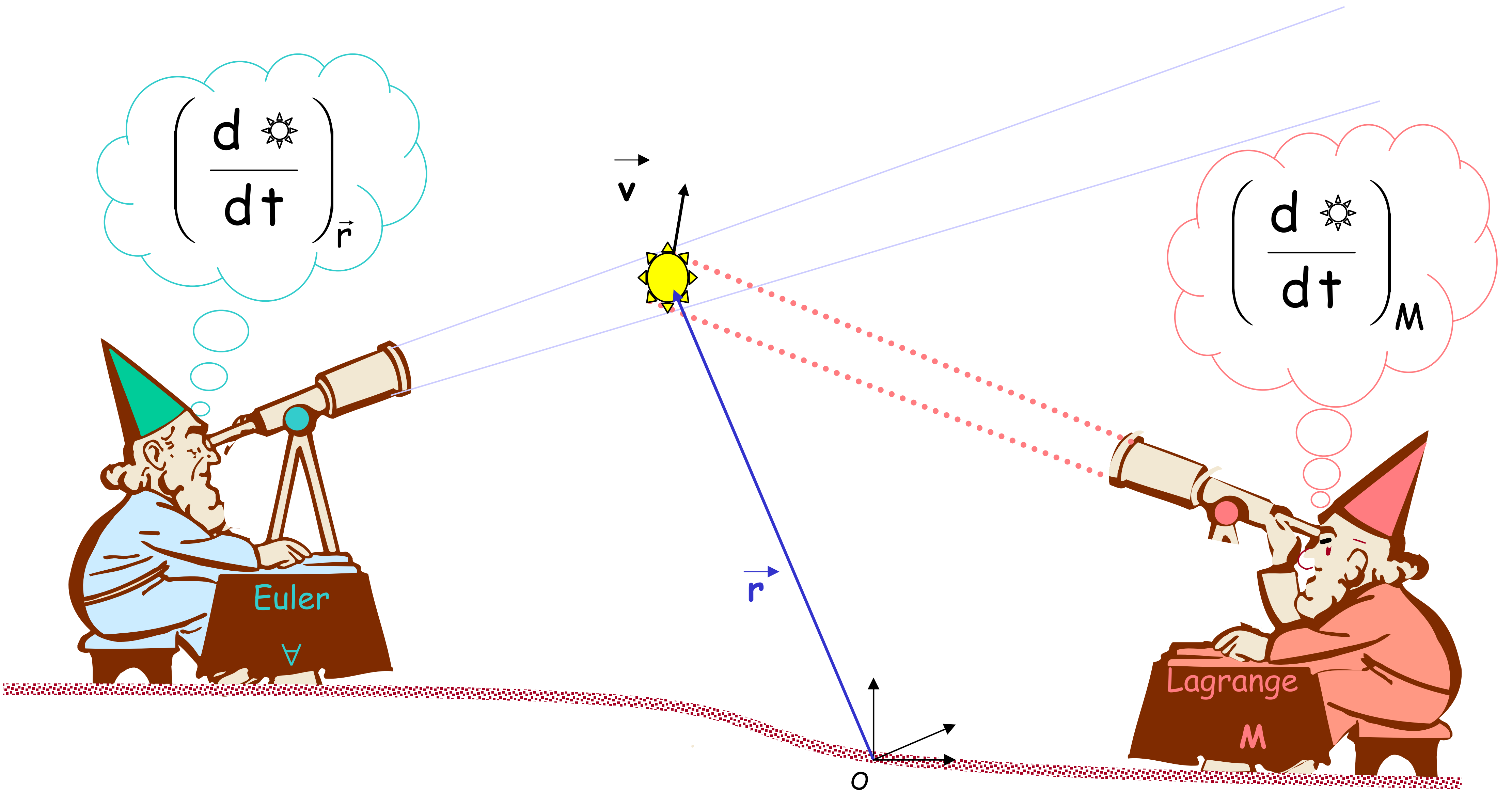
t(final) -> t(inicial)

(à procura de um instante)

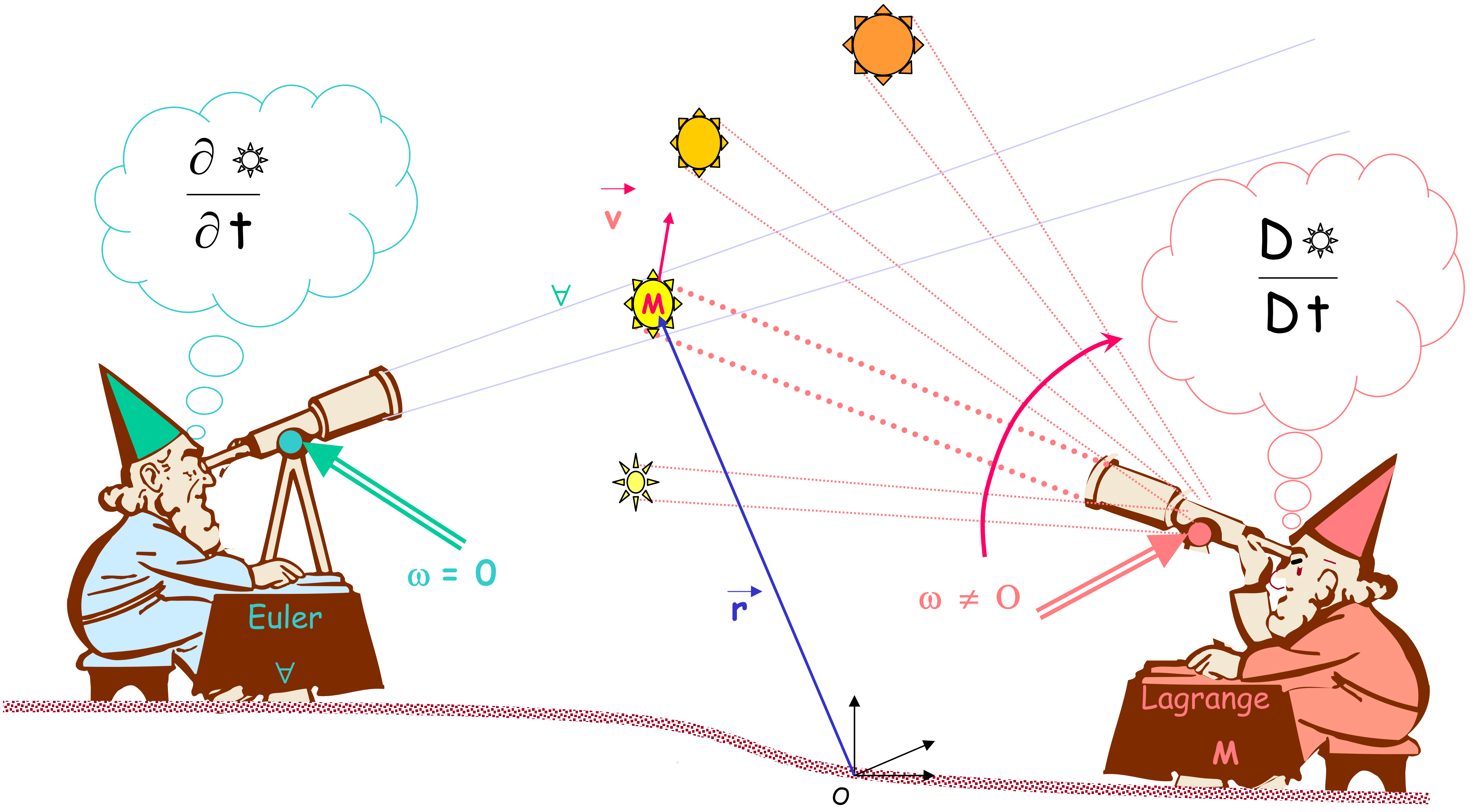
Δ t -> dt



d ☼
$\vec{v}$
t
d ☼
$\vec{r}$ (t)
Euler
∀
Lagrange
M
O
$(d ☼)_M \neq (d ☼)_{\vec{r}}$



$$\left(\frac{d\odot}{dt}\right)_{M} = \left(\frac{d\odot}{dt}\right)_{\vec r} + \left(\frac{d\odot}{d\vec r}\right)_{t} \cdot \left(\frac{d\vec r}{dt}\right)_{M}$$



$$\frac{D\text{☼}}{Dt} = \frac{\partial \text{☼}}{\partial t} + \left(\frac{d\text{☼}}{d\vec{r}}\right)_t \cdot \vec{v}$$

$$\left(\frac{d\,\circledast}{dt}\right)_M = \left(\frac{d\,\circledast}{dt}\right)_{\vec{r}} + \left(\frac{d\,\circledast}{d\vec{r}}\right)_t \cdot \left(\frac{d\vec{r}}{dt}\right)_M$$

$$\frac{D\circledast}{Dt} = \frac{\partial\circledast}{\partial t} + \left(\frac{\partial\circledast}{\partial\vec{r}}\right)_t \cdot \vec{v}$$

☼ = cor

✳ = brilho

☼ = freqüência

☼ = forma

☼ = tamanho

☼ = densidade ρ

☼ = temperatura T

☼ = etc ...

☼ = φ (genericamente)

$$\frac{D\varphi}{Dt} = \frac{\partial\varphi}{\partial t} + \vec{v} \cdot \left(\frac{\partial\varphi}{\partial\vec{r}}\right)_t$$

$$\frac{D\varphi}{Dt} = \frac{\partial \varphi}{\partial t} + \vec{v} \cdot \left(\frac{\partial \varphi}{\partial \vec{r}}\right)_t$$

Pergunta: Como φ varia com r a t constante ?

Resposta: congelemos o tempo e agora

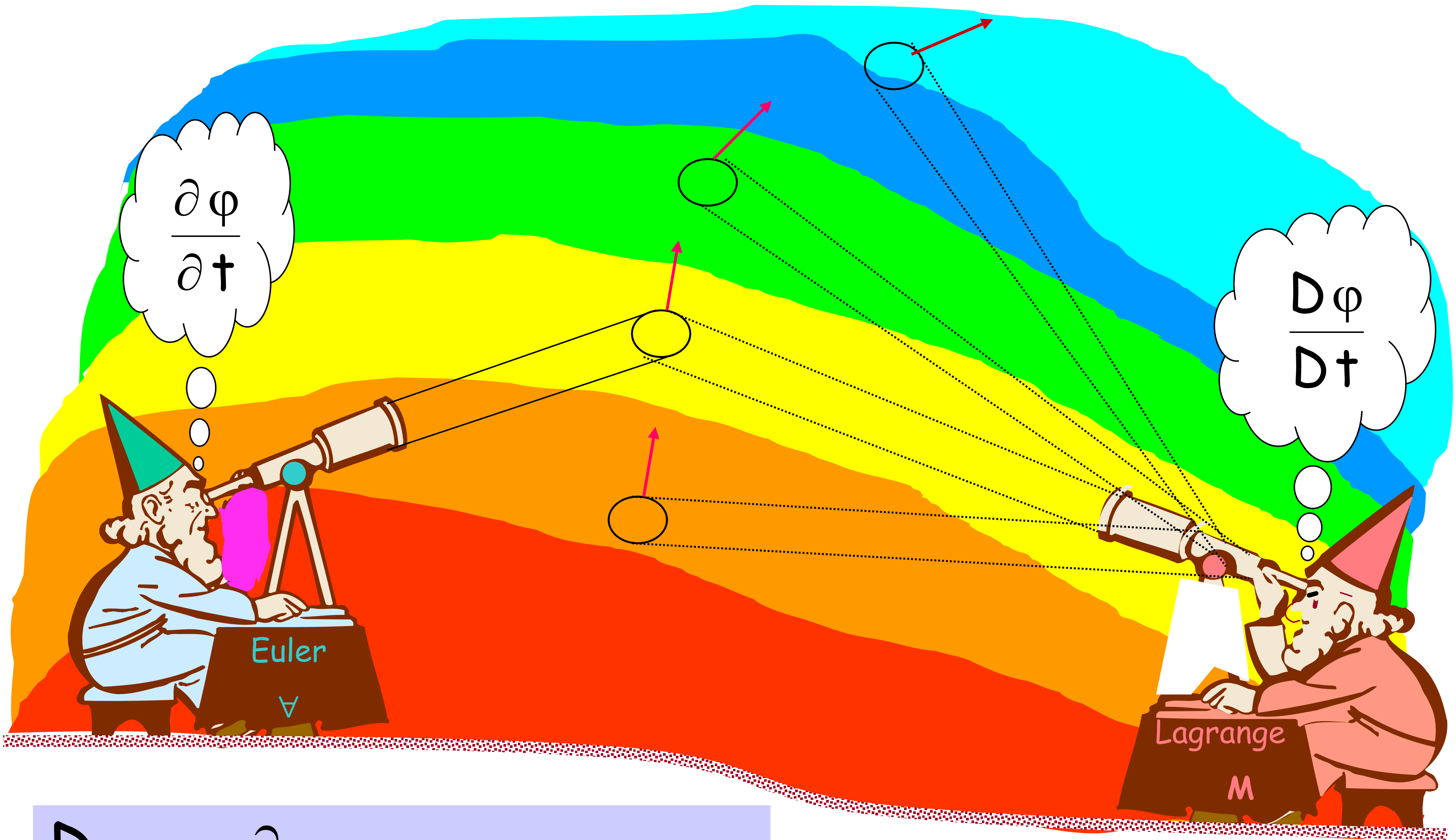
imaginemos uma variação **contínua**

de φ no espaço.

Veremos uma gradação de φ

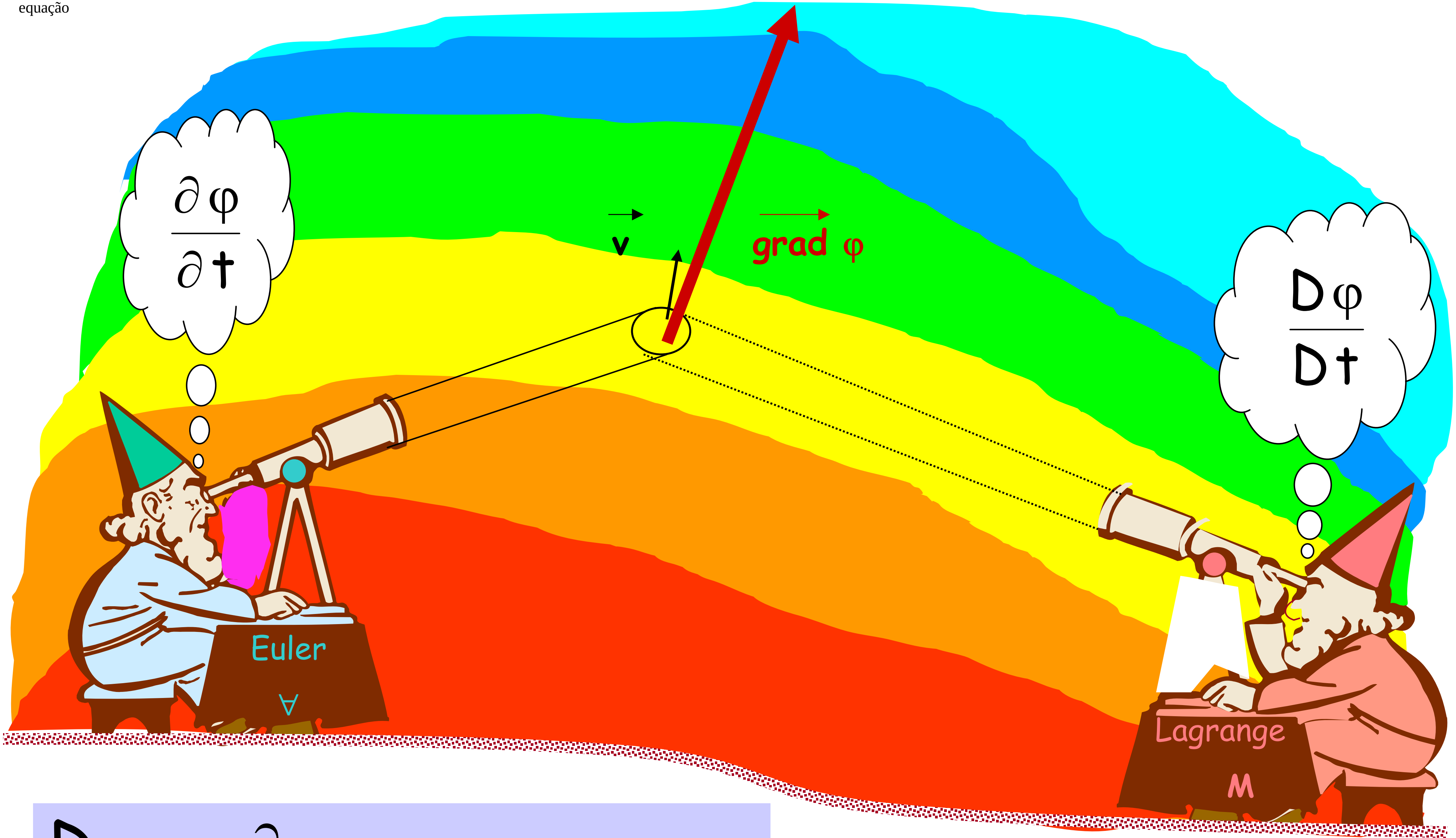
que denominaremos gradiente de φ

$$\left(\frac{\partial \varphi}{\partial \vec{r}}\right)_t = \mathrm{gr\vec{a}d}\ \varphi$$



$$\frac{D\varphi}{Dt} = \frac{\partial \varphi}{\partial t} + \vec{v}.\overleftarrow{grad}\,\varphi$$

equação



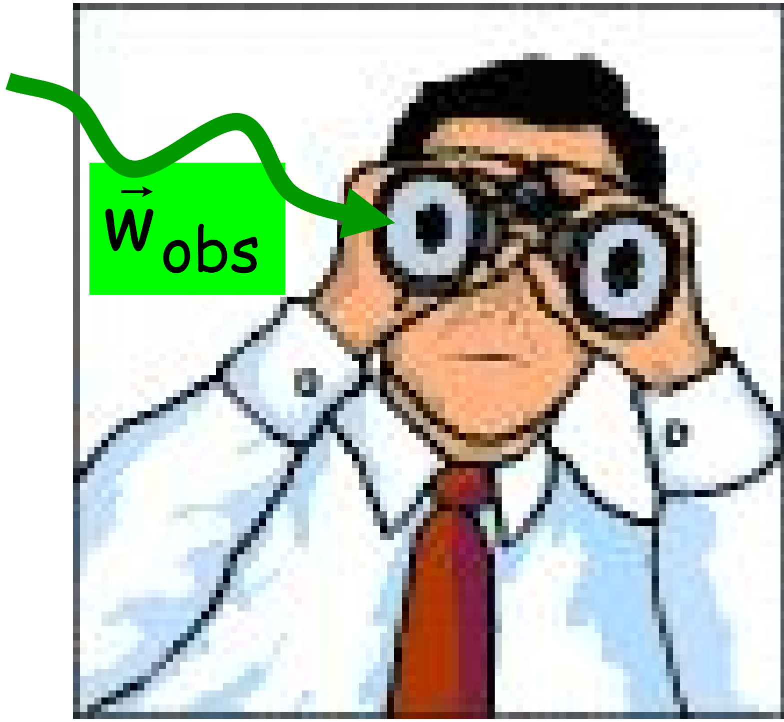
$$\frac{D\varphi}{Dt} = \frac{\partial \varphi}{\partial t} + \vec{v}\,.\,\overrightarrow{\mathrm{grad}}\;\varphi$$

dedução

Ponto observado: $\varphi = \varphi\,(x,y,z,t) = \varphi\,(t,\vec{r}\,)$

Definição de diferencial: $$d\varphi = \frac{\partial \varphi}{\partial t}\,dt + \frac{\partial \varphi}{\partial \vec{r}}\,d\vec{r}$$

Dividindo por d t : $$\frac{d\varphi}{dt} = \frac{\partial \varphi}{\partial t}\frac{dt}{dt} + \frac{\partial \varphi}{\partial \vec{r}}\frac{d\vec{r}}{dt}$$



Definição da velocidade do ponto observado $$\frac{d\varphi}{dt} = \frac{\partial \varphi}{\partial t} + \frac{\partial \varphi}{\partial \vec{r}} \cdot \vec{w}_{obs}$$

Se $w_{obs} = 0$ -> Euler $$\boxed{\frac{d\varphi}{dt} = \frac{\partial \varphi}{\partial t}}$$

Definição de gradiente:

$d\,\varphi = grad\,\varphi \,.\, dr$ $$\frac{d\varphi}{dt} = \frac{\partial \varphi}{\partial t} + \vec{w}_{obs} \cdot \vec{grad}\,\varphi$$

Se $W_{obs} = V$ -> Lagrange $$\boxed{\frac{D\varphi}{Dt} = \frac{\partial \varphi}{\partial t} + \vec{v} \cdot \vec{grad}\,\varphi}$$