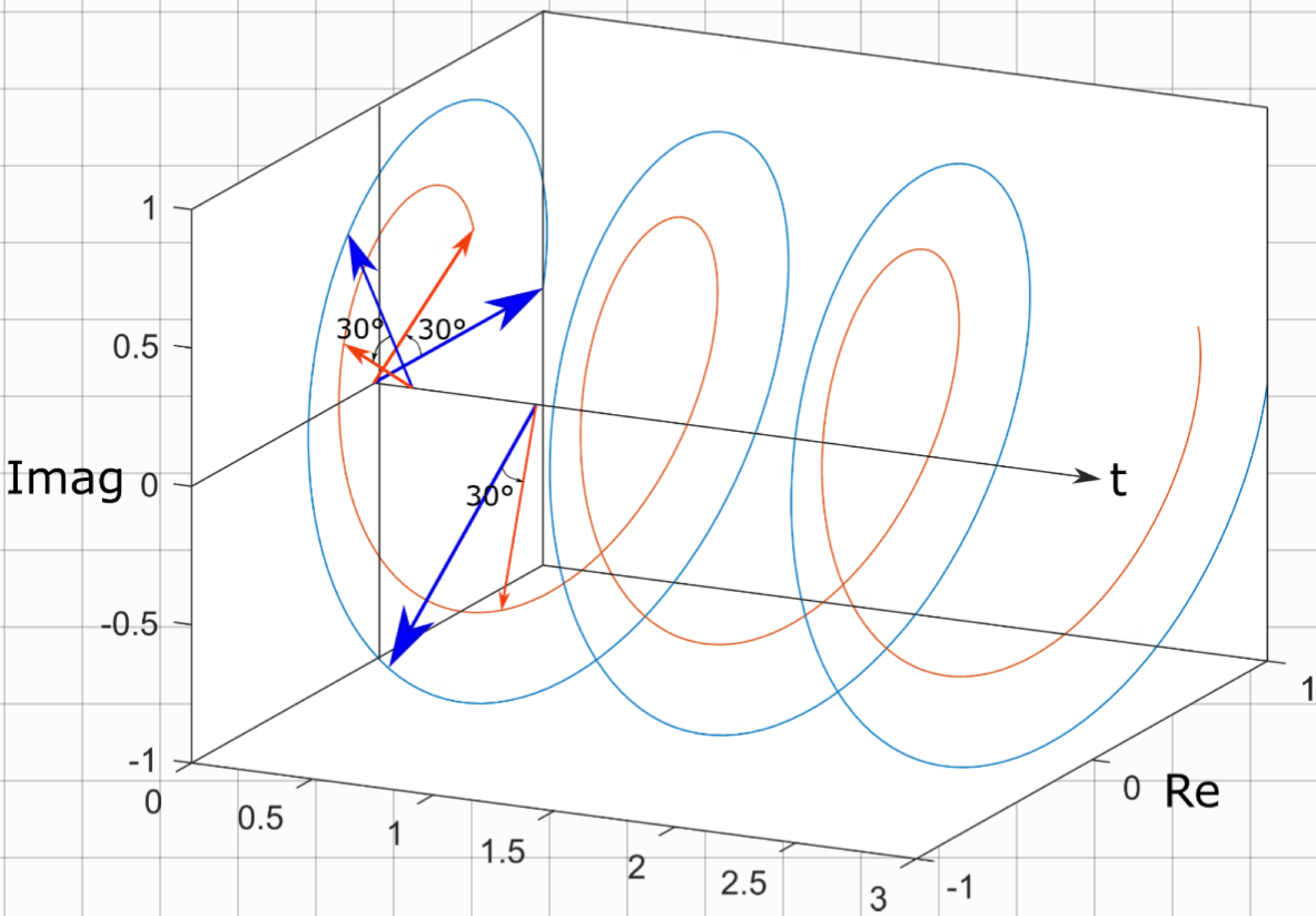
Circuitos em corrente alternada (CA)

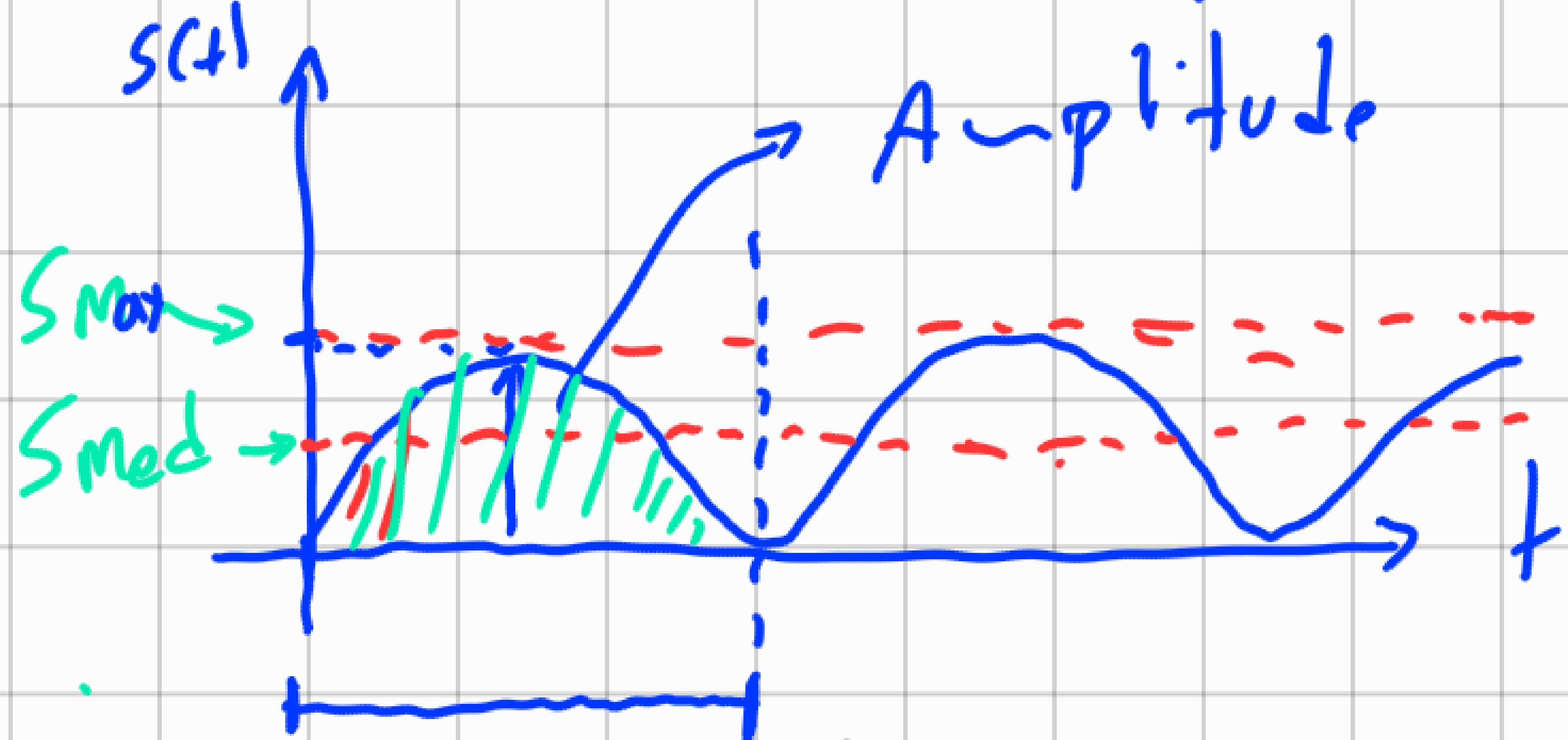
4. Senoides e fasores

4.1 Funções complexas e equação de Euler.
4.2 Fasores
4.3 Representação fasorial de bipolos



Grandezas periódicas

↳ Sinais que se repetem no tempo:



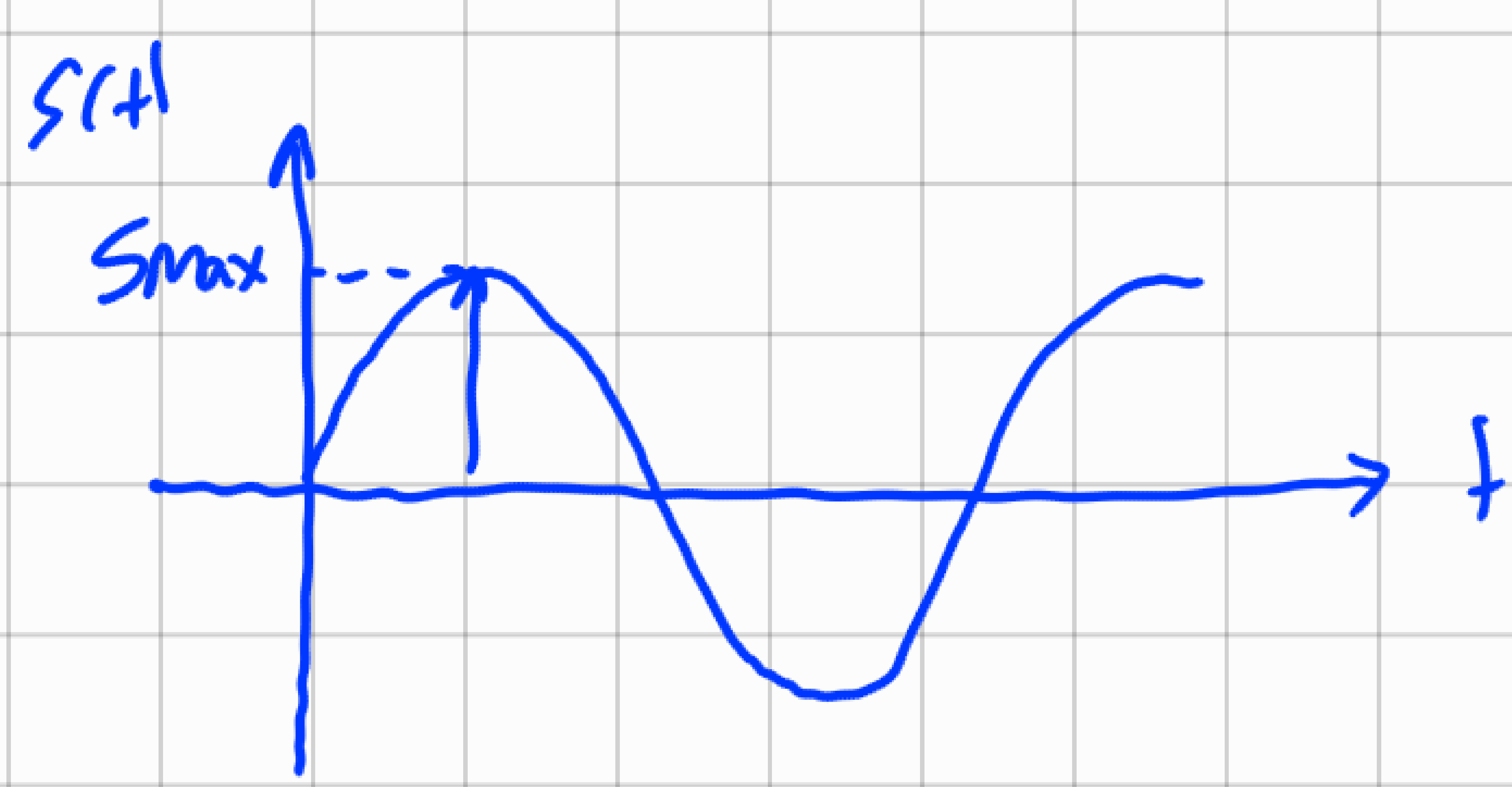
T : período [s]

Valor médio:

$$S_{med} = \frac{A}{T} = \frac{1}{T}\int_{t}^{t+T} s(t)\,dt$$

Frequência: $f = \frac{1}{T}$ [Hz] : nº ciclos / segundo

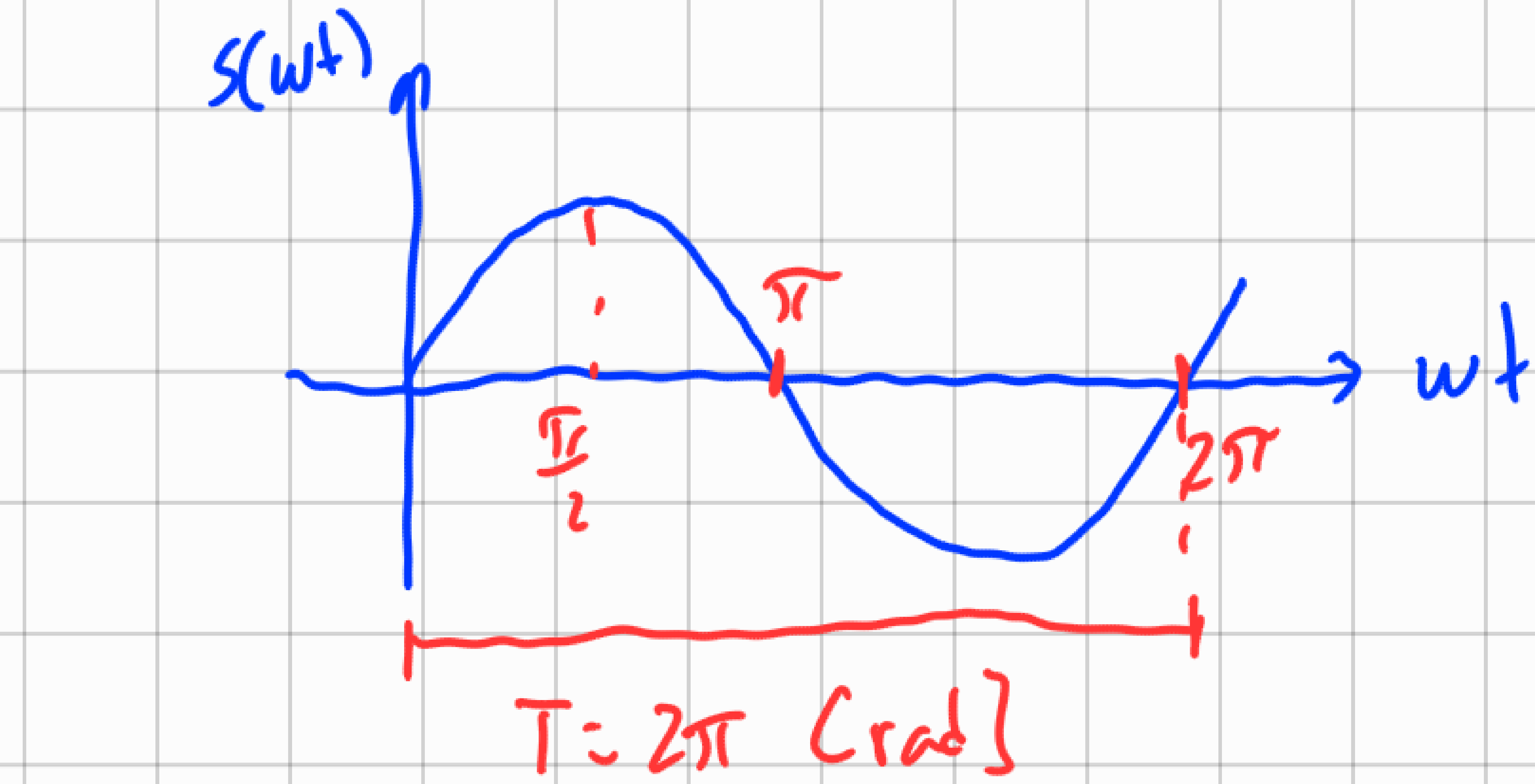
Grandezas Alternadas com valor médio = 0.



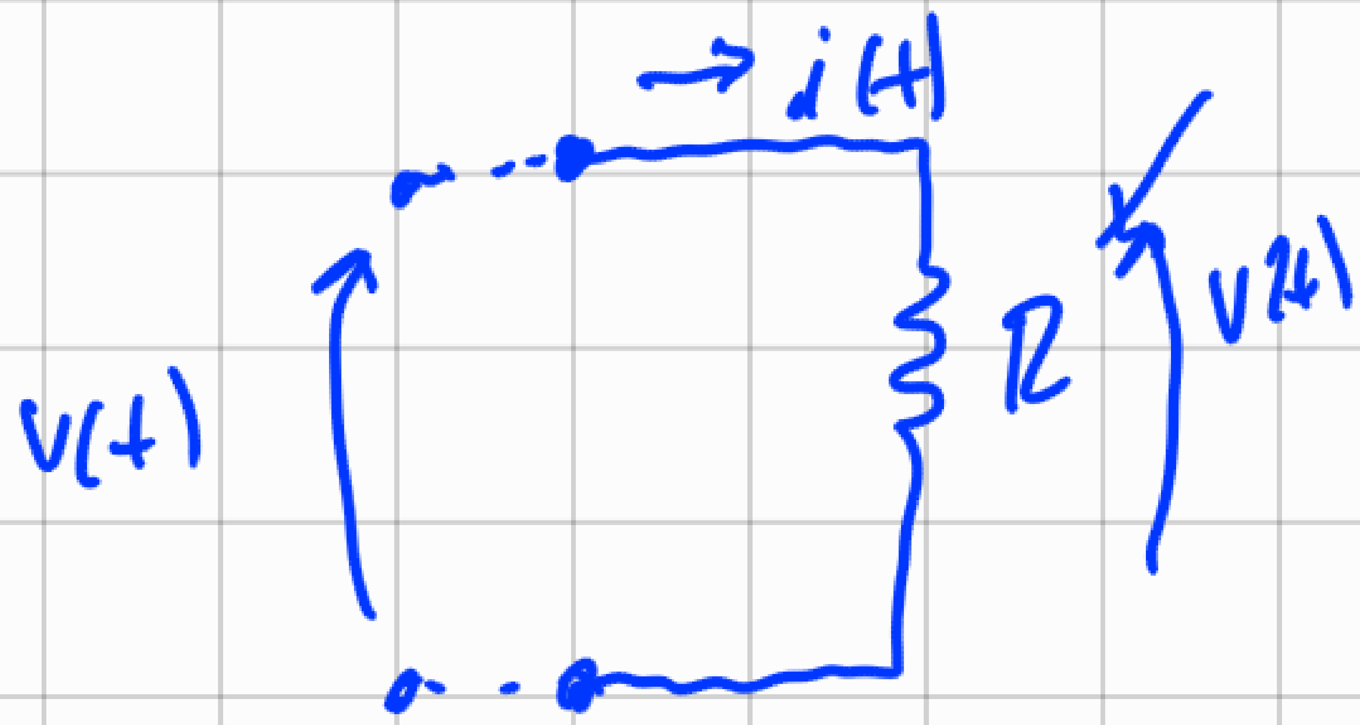
$S_{med} = 0$

$s(t) = S_m \operatorname{sen}(\omega t)$

$\omega = 2\pi f$ [rad/s]



Valor eficaz

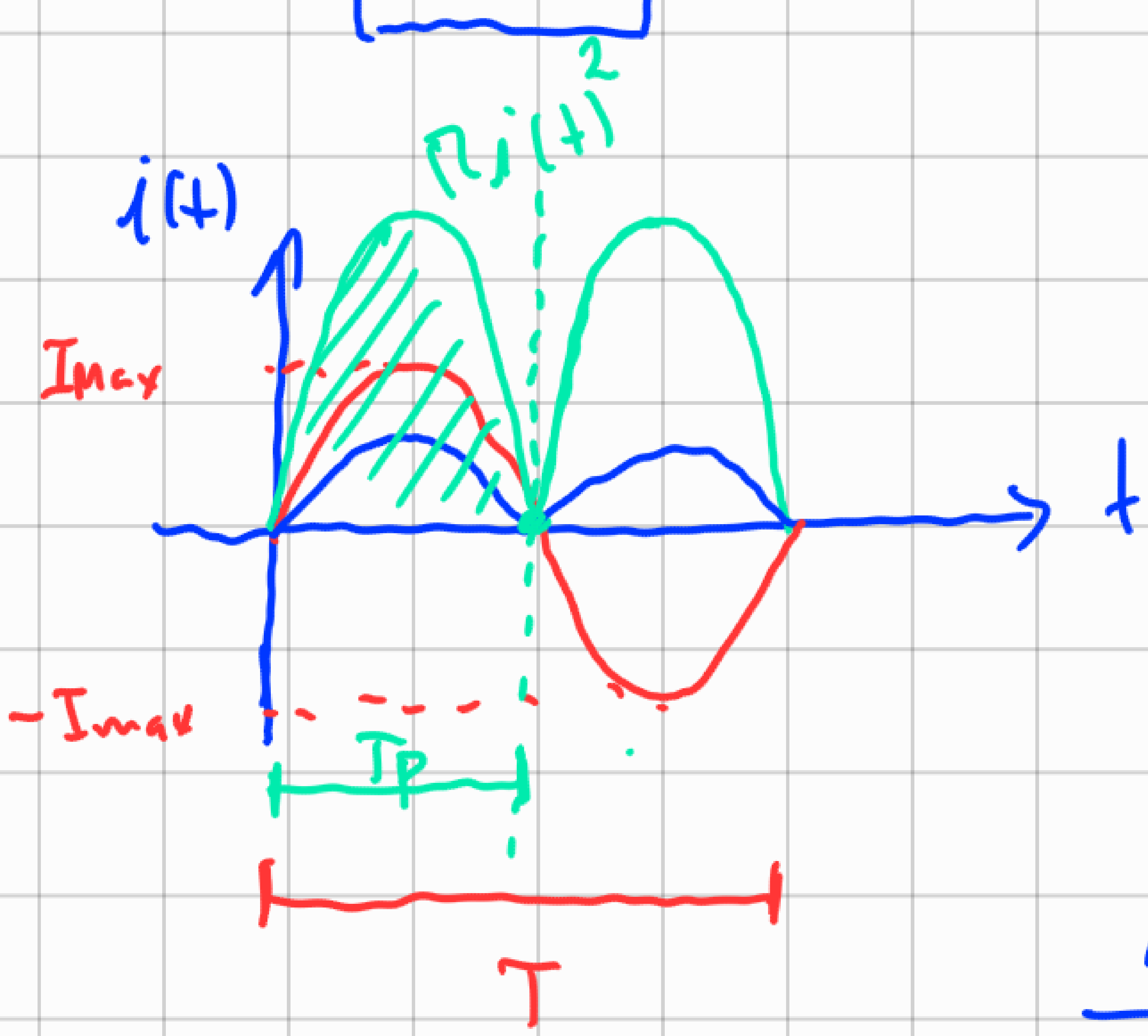


$$v(t) = R\, i(t)$$

$$p(t) = R\, i(t)^2$$

↳ potência instantânea

$$\boxed{P_{cc}} = R\, I_{cc}^2$$



$$P_{med} = \frac{A}{T_p} = \frac{1}{T_p}\int_0^{T_p} R\, i^2(t)\, dt$$

$$P_{med} = P_{cc}$$

$$\frac{1}{T_p}\int_0^{T_p} \cancel{R}\, i^2(t)\, dt = \cancel{R}\, I_{cc}^2$$

$$I_{cc}^2 = \frac{1}{T_p}\int_0^{T_p} i^2(t)\,dt$$

$$I_{cc} = \sqrt{\frac{1}{T_p}\int_0^{T_p} i^2(t)\,dt} = I_{ef}$$

→ valor eficaz da corrente $i(t)$

p/ tensão:

$$\boxed{V_{cc} = \sqrt{\frac{1}{T_p}\int_0^{T_p} v^2(t)\,dt}} = V_{ef}$$

→ valor eficaz da tensão $v(t)$

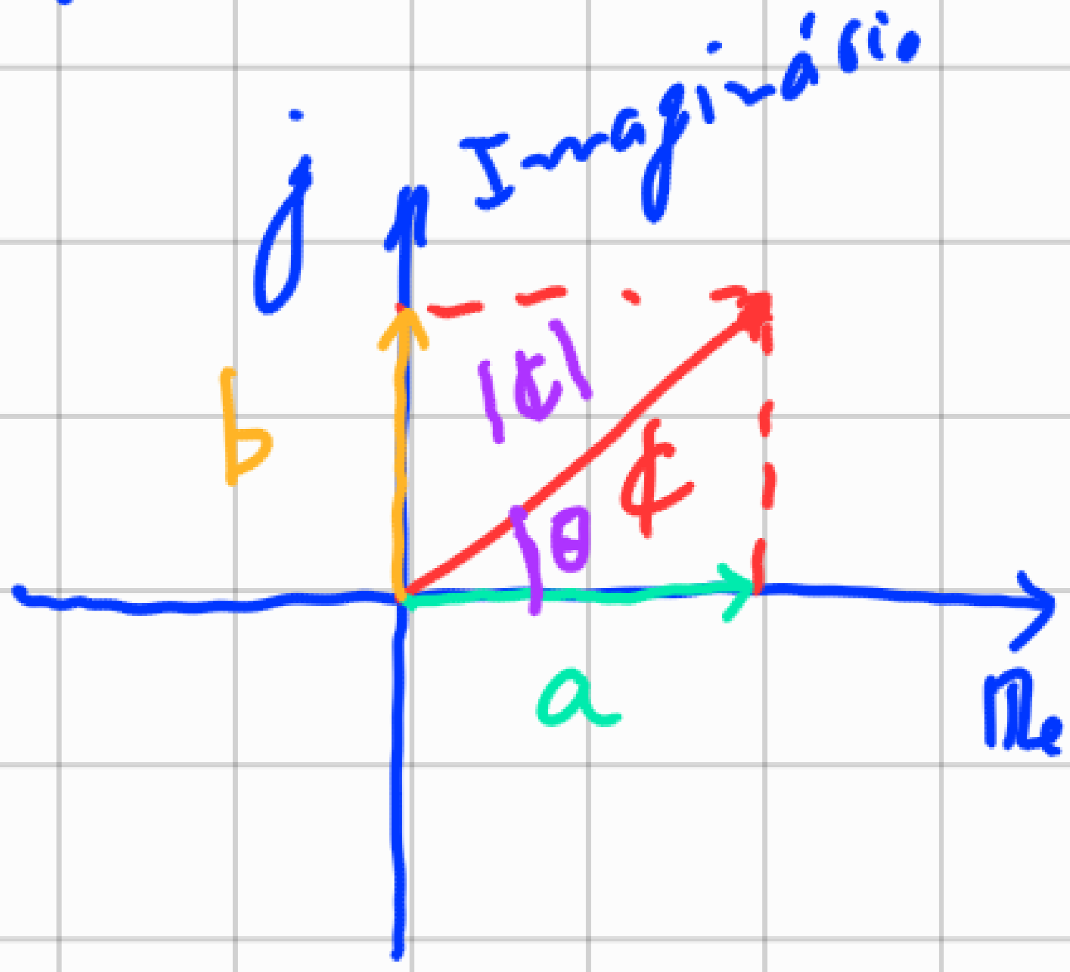
No caso de $i(t)$ e $v(t)$ puramente senoidais:

ex: $i(t) = I_{max}\,\mathrm{sen}(\omega t)$

ex: $v(t) = V_{max}\,\mathrm{sen}(\omega t)$

$$\boxed{I_{ef} = \frac{I_{max}}{\sqrt{2}}} \quad e \quad \boxed{V_{ef} = \frac{V_{max}}{\sqrt{2}}}$$

Funções complexas



$$\boxed{\mathbb{C} = a + jb}$$

$$x^2 + 1 = 0$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1} = \pm j \rightarrow \text{nº complexo}$$

a: parte real de $\mathbb{C}$
b: parte imaginária de $\mathbb{C}$

Equação de Euler:

$$\mathbb{C} = \underbrace{|\mathbb{C}|\cos\theta}_{a} + j\,\underbrace{|\mathbb{C}|\,\text{sen}\,\theta}_{b}$$ (forma retangular)

$$\boxed{\mathbb{C} = |\mathbb{C}|\,e^{j\theta}}$$ (equação de Euler)
(forma polar)

complexo conjugado

$$\mathbb{C}^* = |\mathbb{C}|\cos\theta - j\,|\mathbb{C}|\,\text{sen}\,\theta$$ ou

$$\mathbb{C}^* = |\mathbb{C}|\,e^{-j\theta}$$

$$|\mathbb{C}| = \sqrt{a^2 + b^2}$$

Ainda: $\boxed{\mathbb{C}} \cdot \boxed{\mathbb{C}^*} = |\mathbb{C}|^2$

Fasores

Suponha os sinais:

$V_1(t) = \boxed{V_{1max} \cos(\omega t + \theta_1)}$ e

$V_2(t) = V_{2max} \cos(\omega t + \theta_2)$

Podemos escrevê-los como:

$V_1(t) = \mathbb{R}e \left\{ V_{1max}\, e^{j(\omega t + \theta_1)} \right\}$

$V_2(t) = \mathbb{R}e \left\{ V_{2max}\, e^{j(\omega t + \theta_2)} \right\}$

Ou:

$V_1(t) = \mathbb{R}e \left\{ \boxed{V_{1max}\, e^{j\theta_1}}\, e^{j\omega t} \cdot \frac{\sqrt{2}}{\sqrt{2}} \right\}$

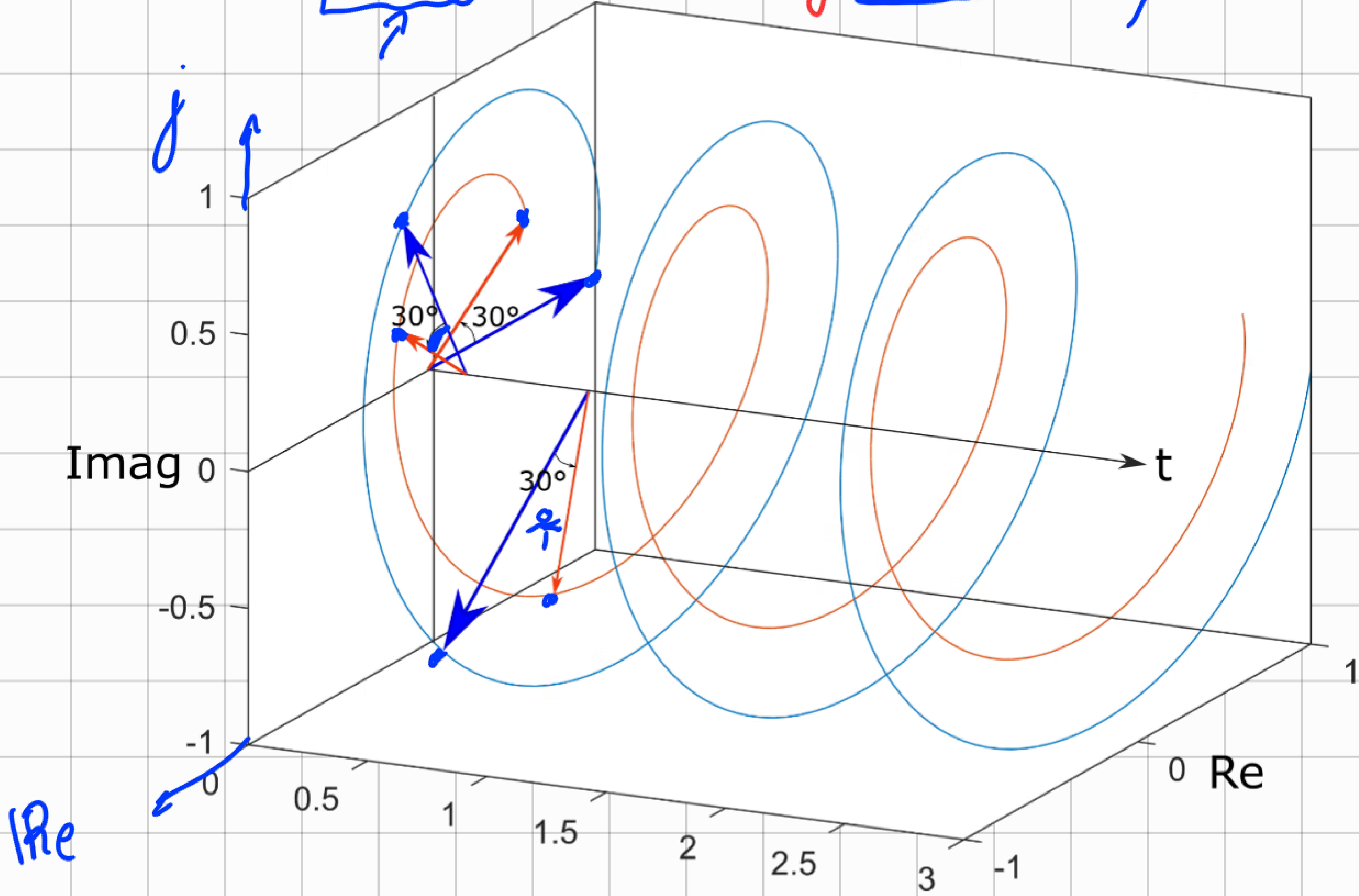
$V_2(t) = \mathbb{R}e \left\{ V_{2max}\, e^{j\theta_2} \cdot e^{j\omega t} \cdot \frac{\sqrt{2}}{\sqrt{2}} \right\}$

Considere o exemplo:

$V_{1max} = 1$ e $\theta_1 = \phi$

$V_{2max} = 0{,}7$ e $\theta_2 = 30°$

$\phi = V_{1max}\cos(\omega t + \theta_1) + j\, V_{1max}\,\mathrm{sen}(\omega t + \theta_1)$



Fasor de $v_1(t)$:

$$\boxed{\dot{V}_1 = \frac{V_{1max}}{\sqrt{2}} e^{j\theta_1}}$$

não se altera com o tempo.

Fasor de $v_2(t)$:

$$\boxed{\dot{V}_2 = \frac{V_{2max}}{\sqrt{2}} e^{j\theta_2}}$$

$$v_1(t) = \mathrm{Re}\left\{ \underbrace{\boxed{\frac{V_{1max}}{\sqrt{2}} e^{j\theta_1}}}_{\dot{V}_1} \cdot \sqrt{2} \cdot e^{j\omega t} \right\}$$

Ex: $v_1(t) = 10\cos(\omega t + 10°)$
$v_2(t) = 5\cos(\omega t - 20°)$

$v(t) = v_1(t) + v_2(t)$

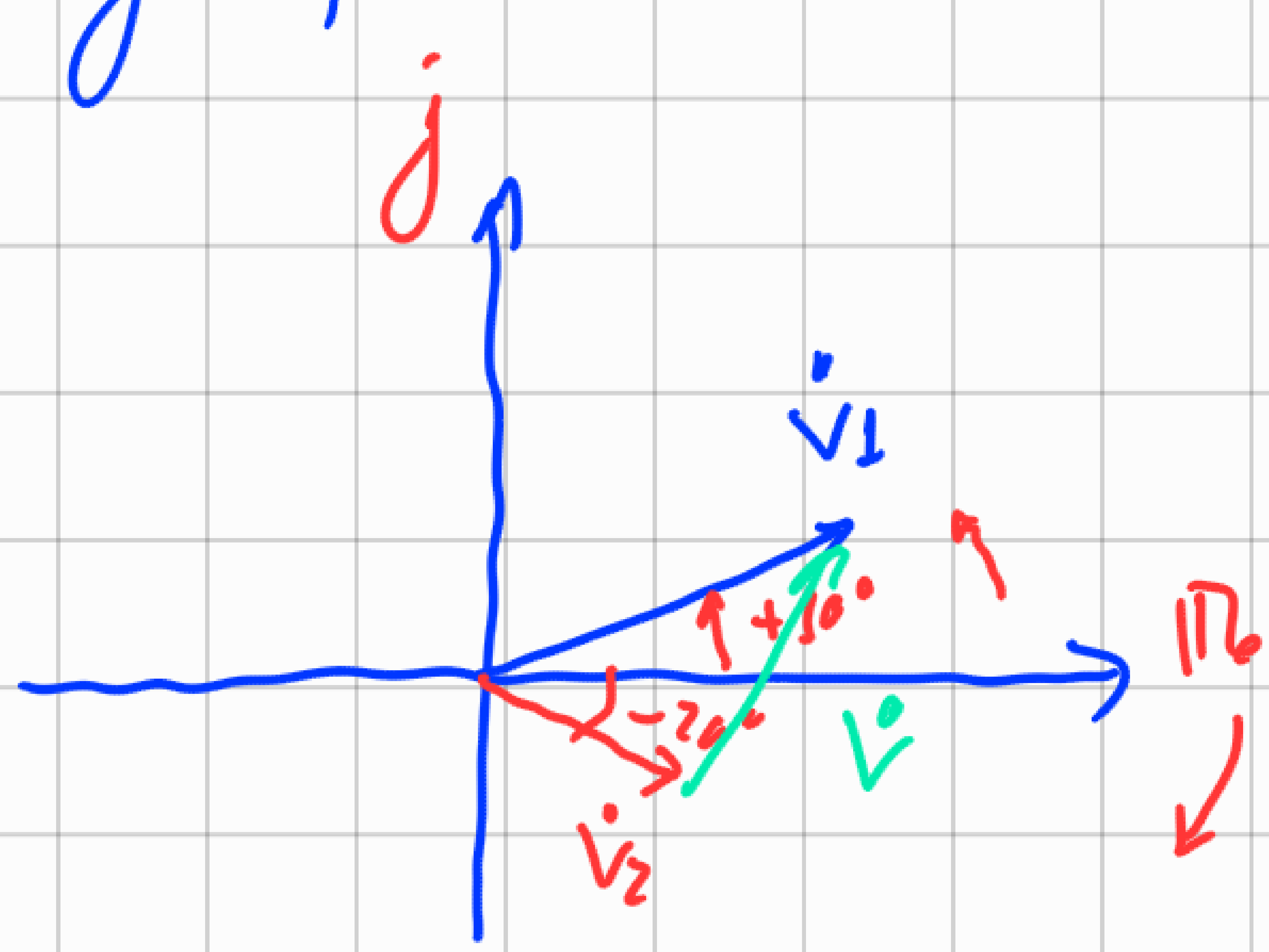
$v(t) = 10\cos(\omega t + 10°) + 5\cos(\omega t - 20°)$

$v(t) = \mathbb{R}e\left\{\frac{10}{\sqrt{2}} e^{j10°} \cdot \sqrt{2}\, e^{j\omega t}\right\} + \mathbb{R}e\left\{\frac{5}{\sqrt{2}} e^{-j20°} \cdot \sqrt{2}\, e^{j\omega t}\right\}$

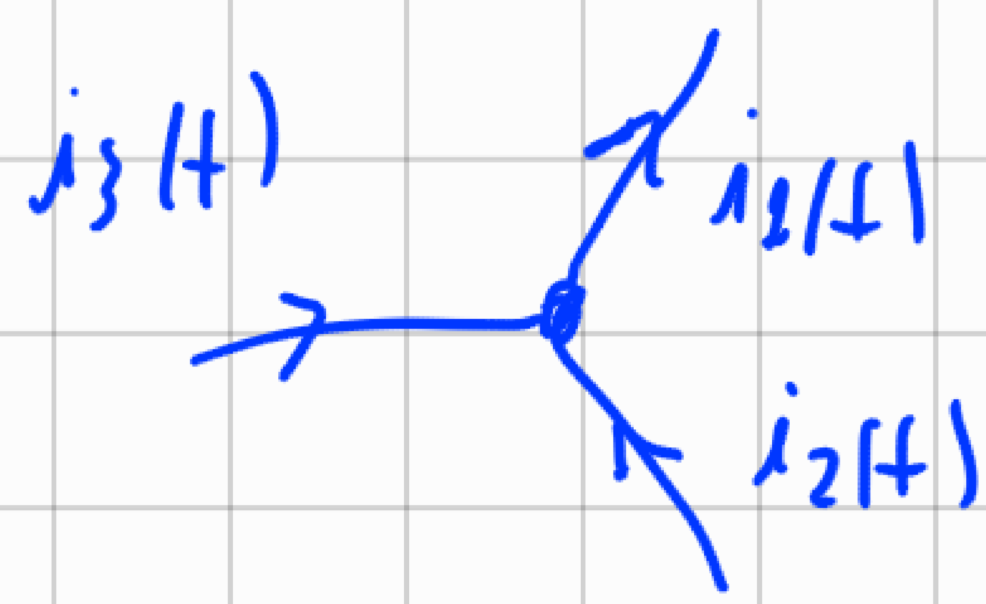
$v(t) = \mathbb{R}e\left\{\overbrace{\left[\underbrace{\frac{10}{\sqrt{2}} e^{j10°}}_{\dot{V}_1} + \underbrace{\frac{5}{\sqrt{2}} e^{-j20°}}_{\dot{V}_2}\right]}^{\dot{V}} \cdot \boxed{\sqrt{2}\, e^{j\omega t}}\right\}$

Logo: $\boxed{\dot{V} = \dot{V}_1 + \dot{V}_2}$ → Lei de Kirchhoff das tensões

Então, graficamente:



Lei de Kirchhoff p/ correntes:

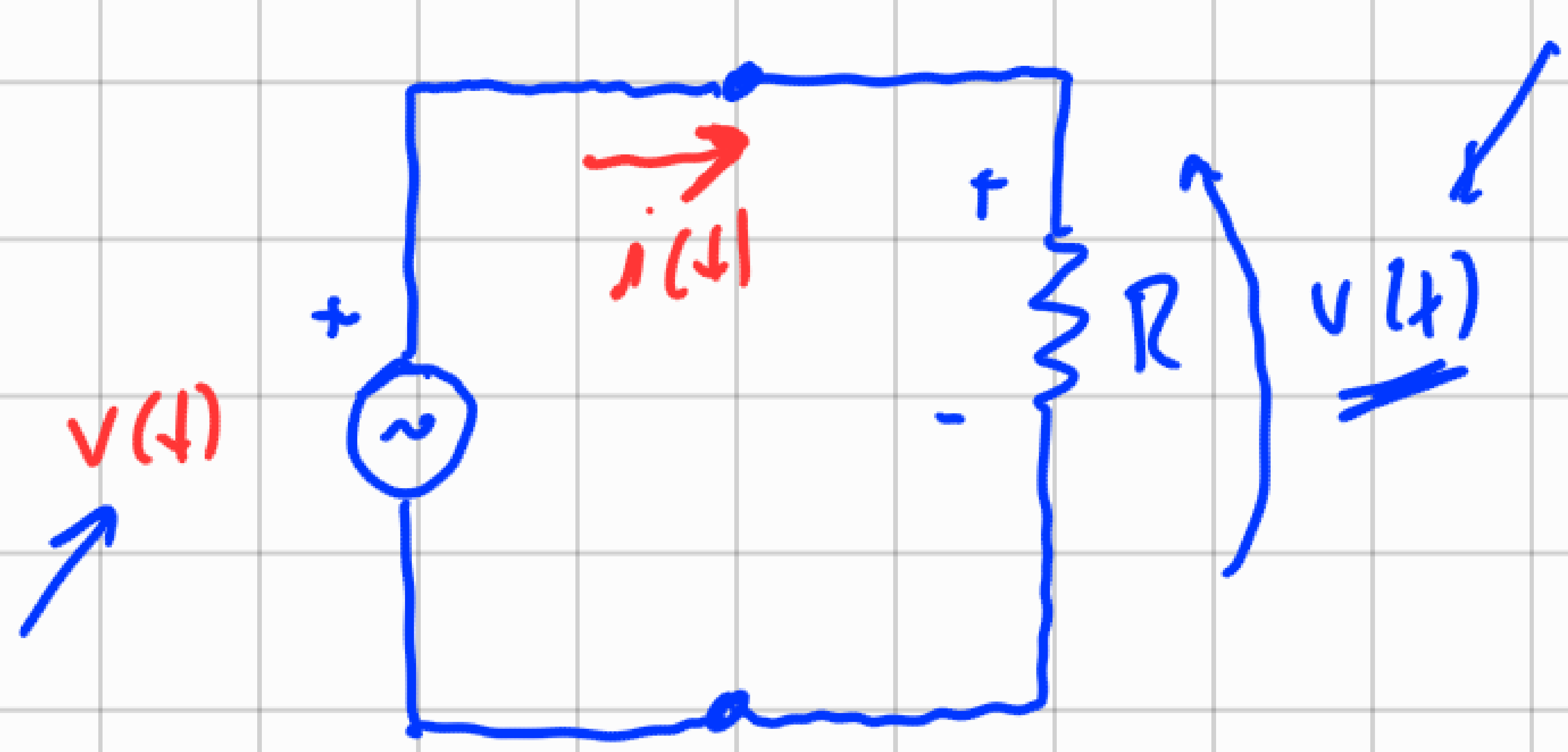


$$-i_1(t) + i_2(t) + i_3(t) = 0$$

$$\boxed{-\dot{I}_1 + \dot{I}_2 + \dot{I}_3 = 0}$$

<u>Lei de Ohm</u> (representação de bipolos passivos em fasores).

1) <u>Resistor</u>:



$$v(t) = R\, i(t)$$

Suponha: $v(t) = V_{max}\cos(\omega t + \theta)$

$i(t) = I_{max}\cos(\omega t + \delta)$

$$v(t) = \Re e\left\{ \underbrace{R\,\underbrace{\frac{I_{max}}{\sqrt{2}}\, e^{j\delta}}_{\dot{I}}}_{\dot{V}} \cdot \sqrt{2}\, e^{j\omega t} \right\}$$

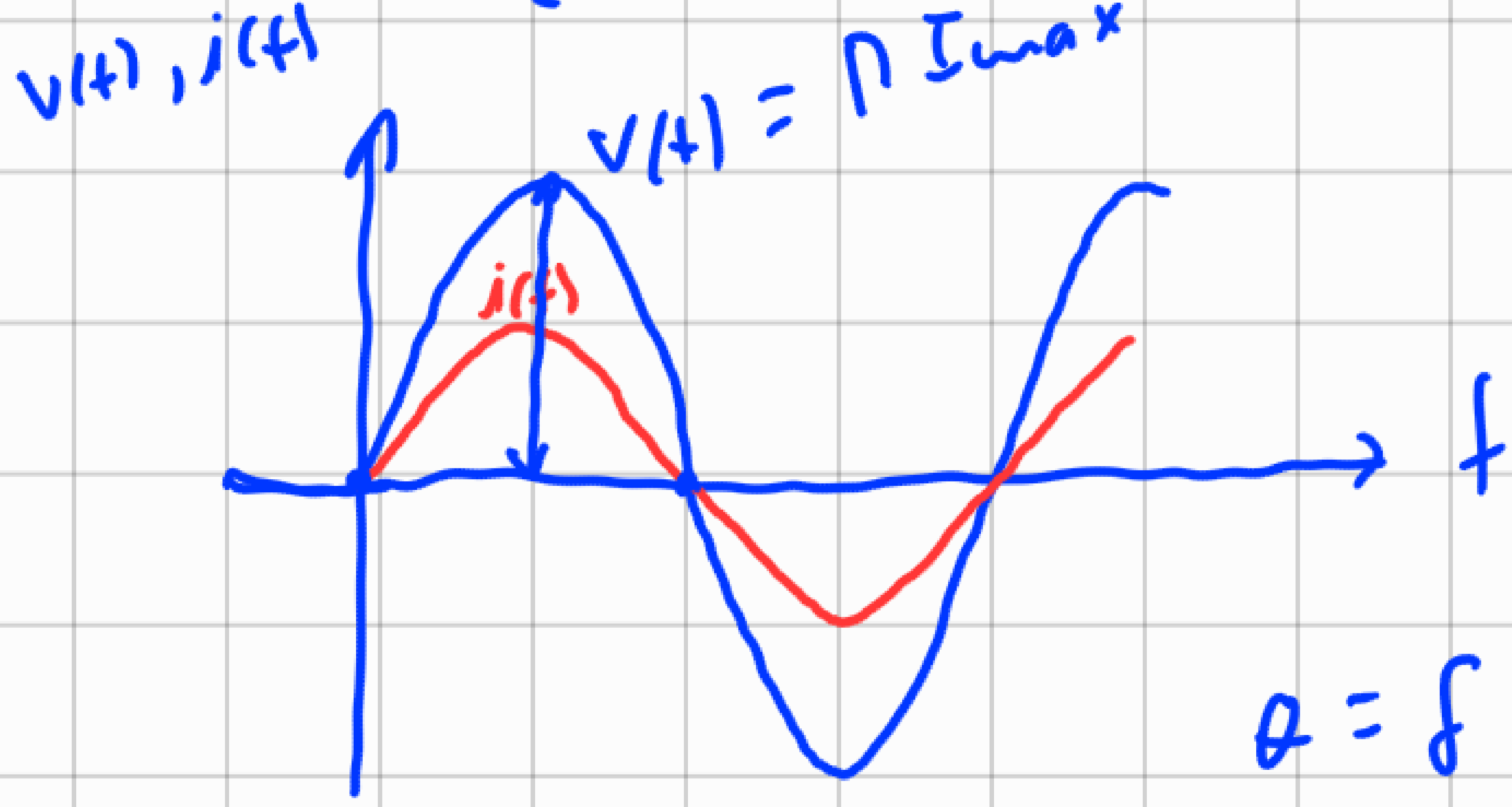
$$\boxed{\dot{V} = R\,\dot{I}}$$

$$\dot{V} = R\,\frac{I_{max}}{\sqrt{2}}\,e^{j\delta}$$

$$\dot{V} = \frac{V_{max}}{\sqrt{2}}\,e^{j\theta}$$

$V_{max} = R\,I_{max}$

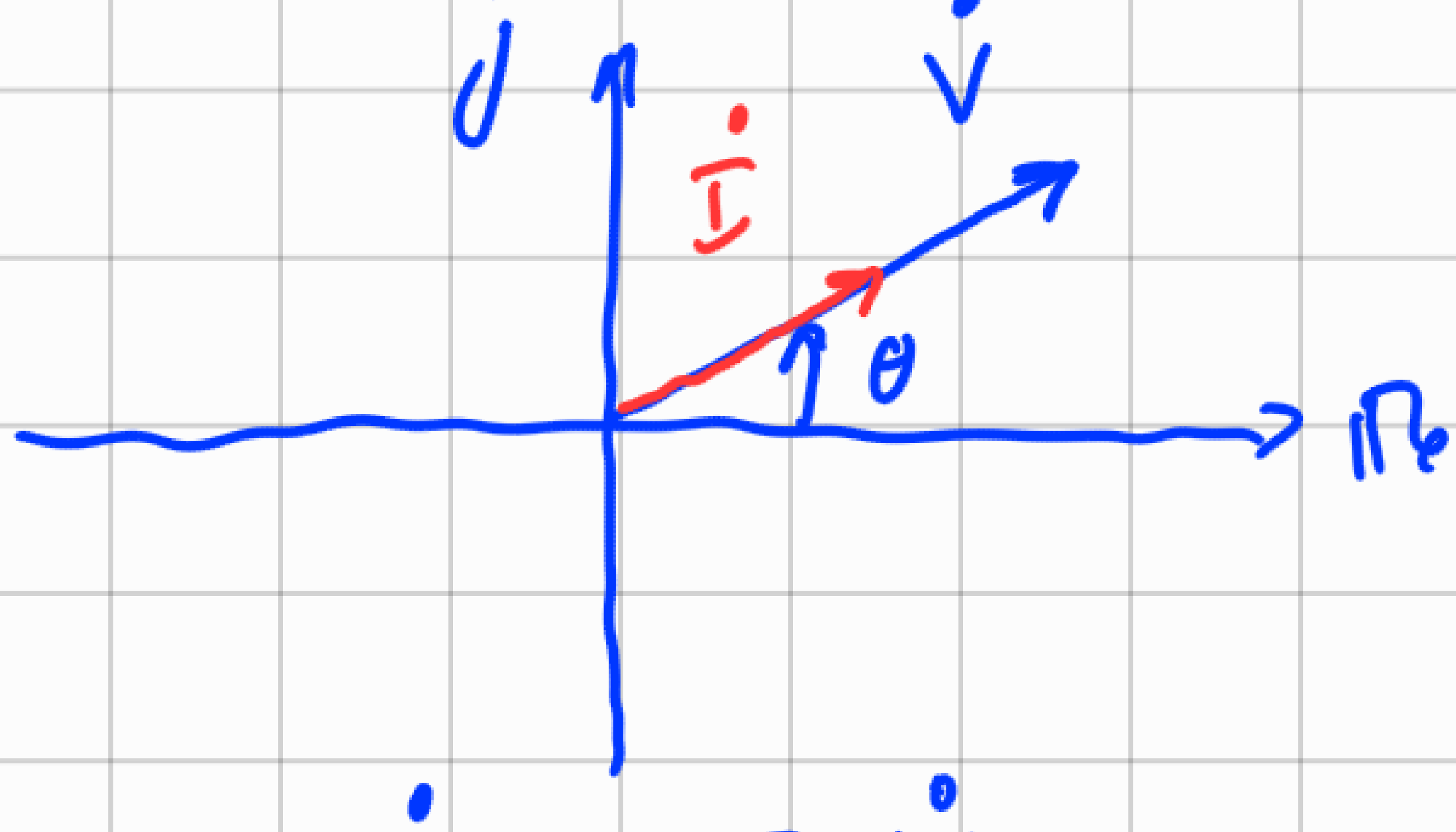
$\theta = \delta$



$\theta = \delta$

No resistor, a corrente está em fase com a queda de tensão sobre ele.

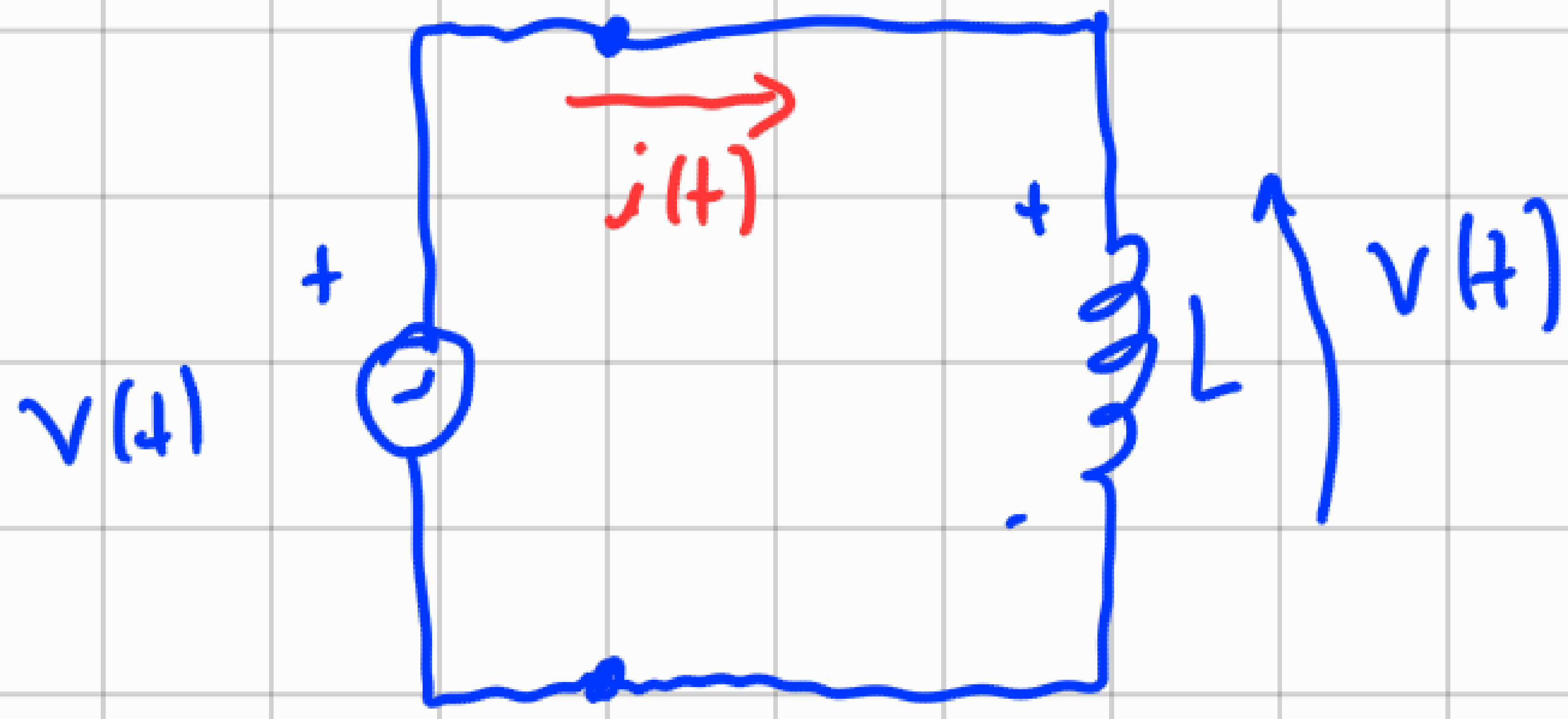
Diagrama de fasores:



$\dot{V} = R\,\dot{I}$

↳ impedância [Ω]

2) Indutor



$v(t) = V_{max} \cos(wt + \theta)$

$i(t) = I_{max} \cos(wt + \delta) = \mathrm{Re}\left\{ \underline{\frac{I_{max}}{\sqrt{2}} e^{j\delta}} \cdot \sqrt{2}\, e^{jwt} \right\}$

Da Lei de Faraday-Lenz: $v(t) = L \dfrac{di(t)}{dt}$

$v(t) = -L\, I_{max}\, w\, \mathrm{sen}(wt + \delta)$

$v(t) = -wL\, I_{max} \cos\left(wt + \delta - \frac{\pi}{2}\right)$

$v(t) = \underline{wL\, I_{max} \cos\left(wt + \delta + \frac{\pi}{2}\right)}$

Em fasores:

$v(t) = \mathrm{Re}\left\{ \overline{wL \frac{I_{max}}{\sqrt{2}} e^{j\left(\delta + \frac{\pi}{2}\right)}} \cdot \sqrt{2}\, e^{jwt} \right\}$

$v(t) = \mathrm{Re}\Big\{$

$$\boxed{e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\,\mathrm{sen}\,\frac{\pi}{2} = j}$$

$$v(t) = \mathrm{Re}\left\{ \underbrace{j\omega L \underbrace{\frac{I_{max}}{\sqrt{2}} e^{j\delta}}_{\dot{I}}}_{\dot{V}} \cdot \sqrt{2}\, e^{j\omega t} \right\}$$

Então: $\boxed{\dot{V} = j\omega L\, \dot{I}} \Rightarrow \dot{V} = \boxed{\omega L e^{j\frac{\pi}{2}}}\, \dot{I}$

impedância do indutor [Ω]

$$\boxed{\bar{Z} = j\omega L} = \omega L e^{j\frac{\pi}{2}}$$

$$v(t) = V_{max} \cos(\omega t + \theta)$$

$$V_{max} = \omega L\, I_{max}$$

$X_L = \omega L$: reatância indutiva [Ω]

$$\theta = \delta + \frac{\pi}{2}$$

No indutor, a corrente está atrasada de 90° em relação à tensão

No tempo:

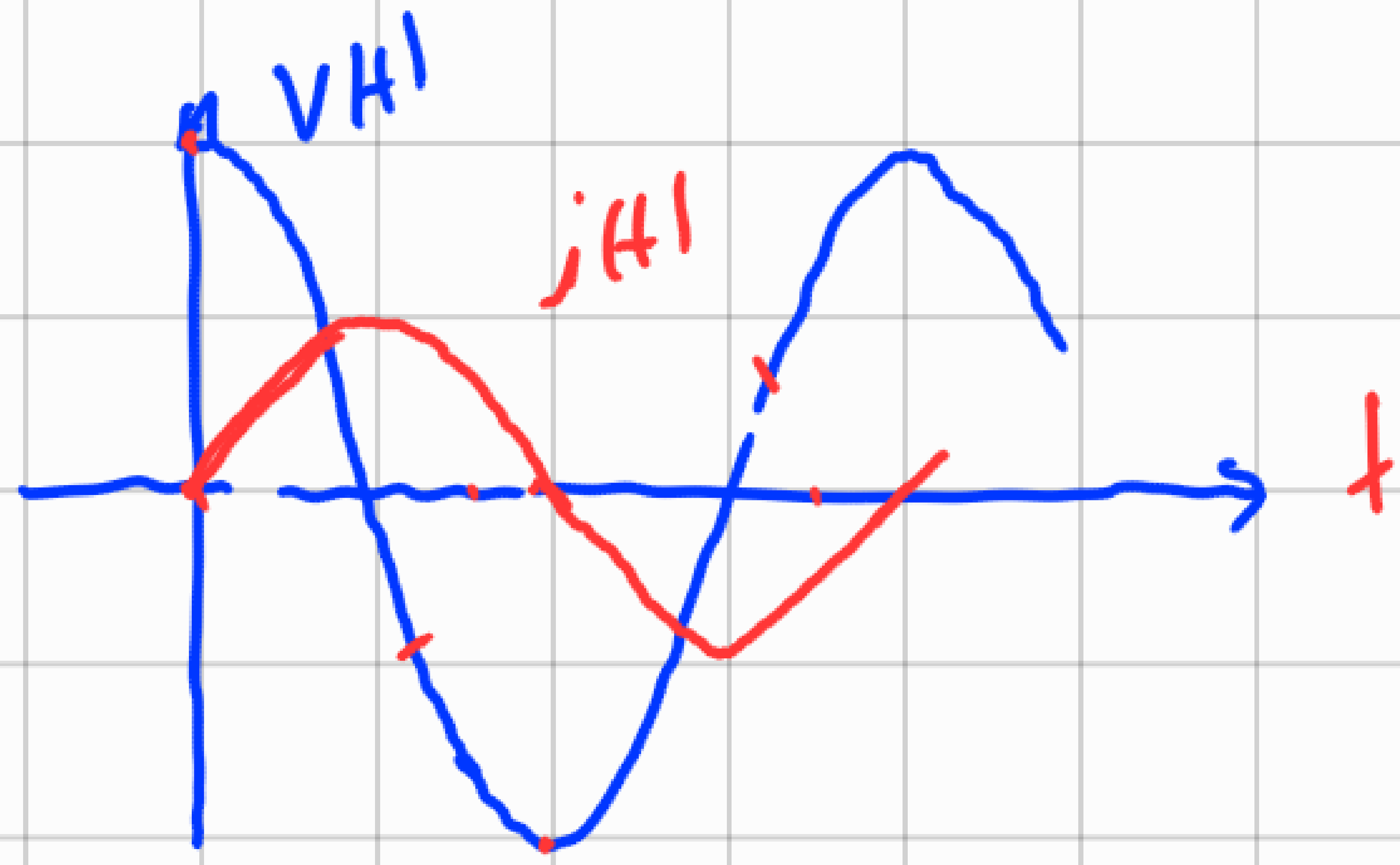
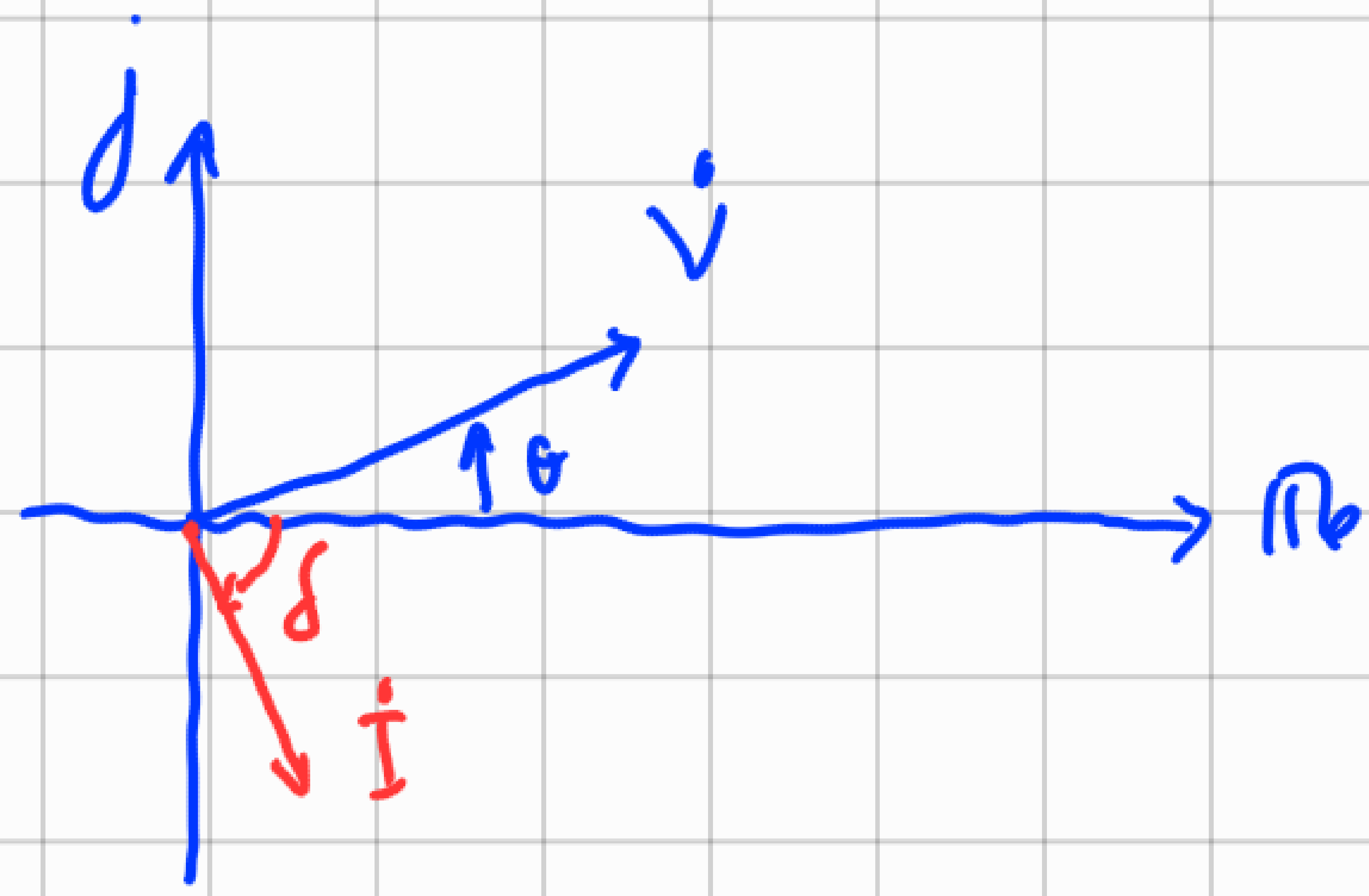
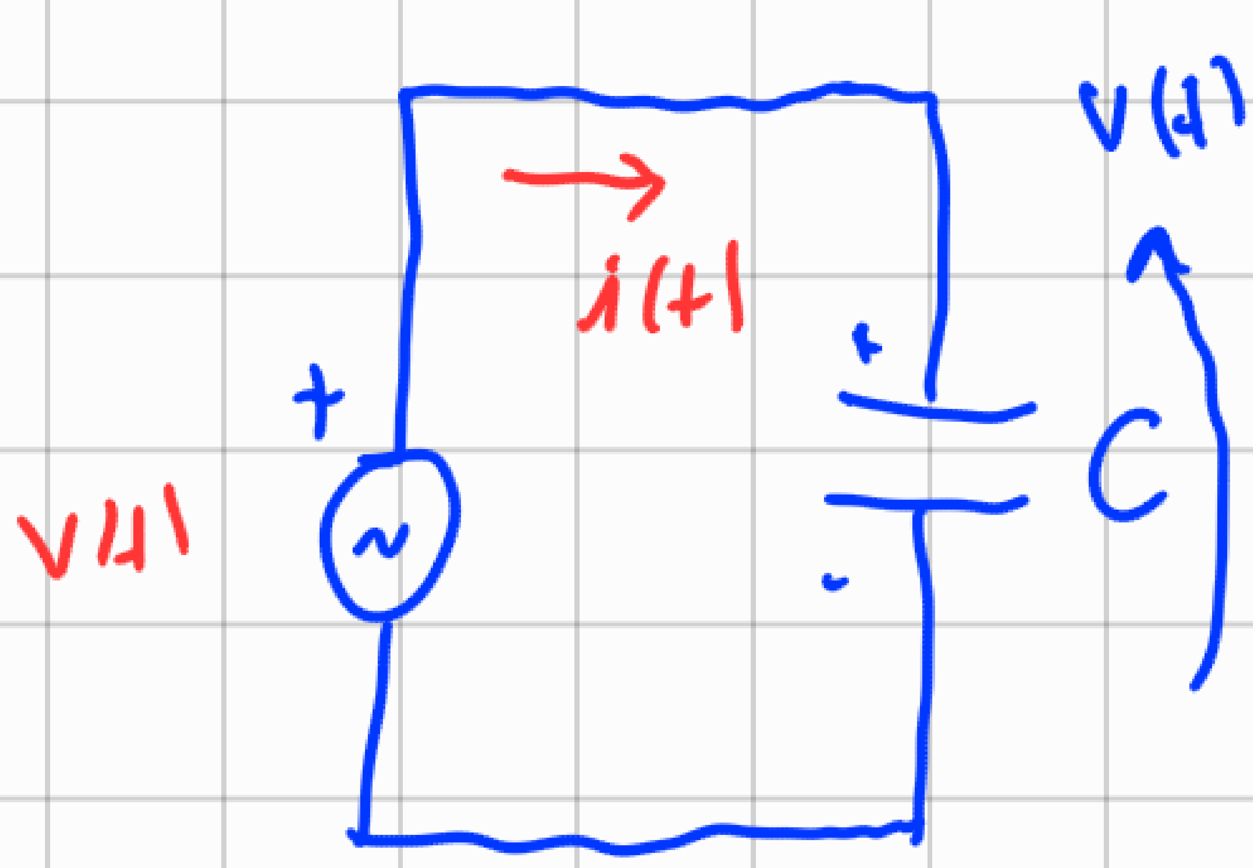


Diagrama de fasores:



Impedância: $\bar{Z} = jX_L = X_L e^{j\frac{\pi}{2}} = X_L \angle 90° \, [\Omega]$

3) Capacitor:



$v(t) = V_{max} \cos(\omega t + \theta)$

$i(t) = I_{max} \cos(\omega t + \delta)$

$\dot{I} = \frac{I_{max}}{\sqrt{2}} e^{j\delta}$

$$v(t) = \frac{1}{C} \int i(t)\, dt$$

$$v(t) = \frac{1}{C} I_{max} \frac{1}{\omega} \operatorname{sen}(\omega t + \delta)$$

$$v(t) = \frac{1}{\omega C} I_{max} \cos\left(\omega t + \delta - \frac{\pi}{2}\right)$$

$$v(t) = \operatorname{Re}\left\{ \frac{1}{\omega C} \frac{I_{max}}{\sqrt{2}} e^{j(\delta - \frac{\pi}{2})} \cdot \sqrt{2}\, e^{j\omega t} \right\}$$

$$v(t) = \mathrm{Re}\left\{ \boxed{\frac{1}{wC} e^{-j\frac{\pi}{2}} \underbrace{\boxed{\frac{I_{max}}{\sqrt{2}} e^{j\delta}}}_{\dot{I}}} \cdot \sqrt{2}\, e^{jwt} \right\}$$

Então: $\boxed{\dot{V} = \frac{1}{wC} e^{-j\frac{\pi}{2}} \cdot \dot{I}}$

$-j\frac{\pi}{2}$ → modifica o ângulo da corrente

$\frac{1}{wC}$ → modifica o I_{max}

$$\bar{Z} = \frac{1}{wC} e^{-j\frac{\pi}{2}} = -j\frac{1}{wC} = \frac{1}{jwC} \; [\Omega]$$

impedância do capacitor

→ $X_C = \frac{1}{wC}$: reatância capacitiva $[\Omega]$

No capacitor, a corrente está adiantada de 90° em relação à queda de tensão nele.

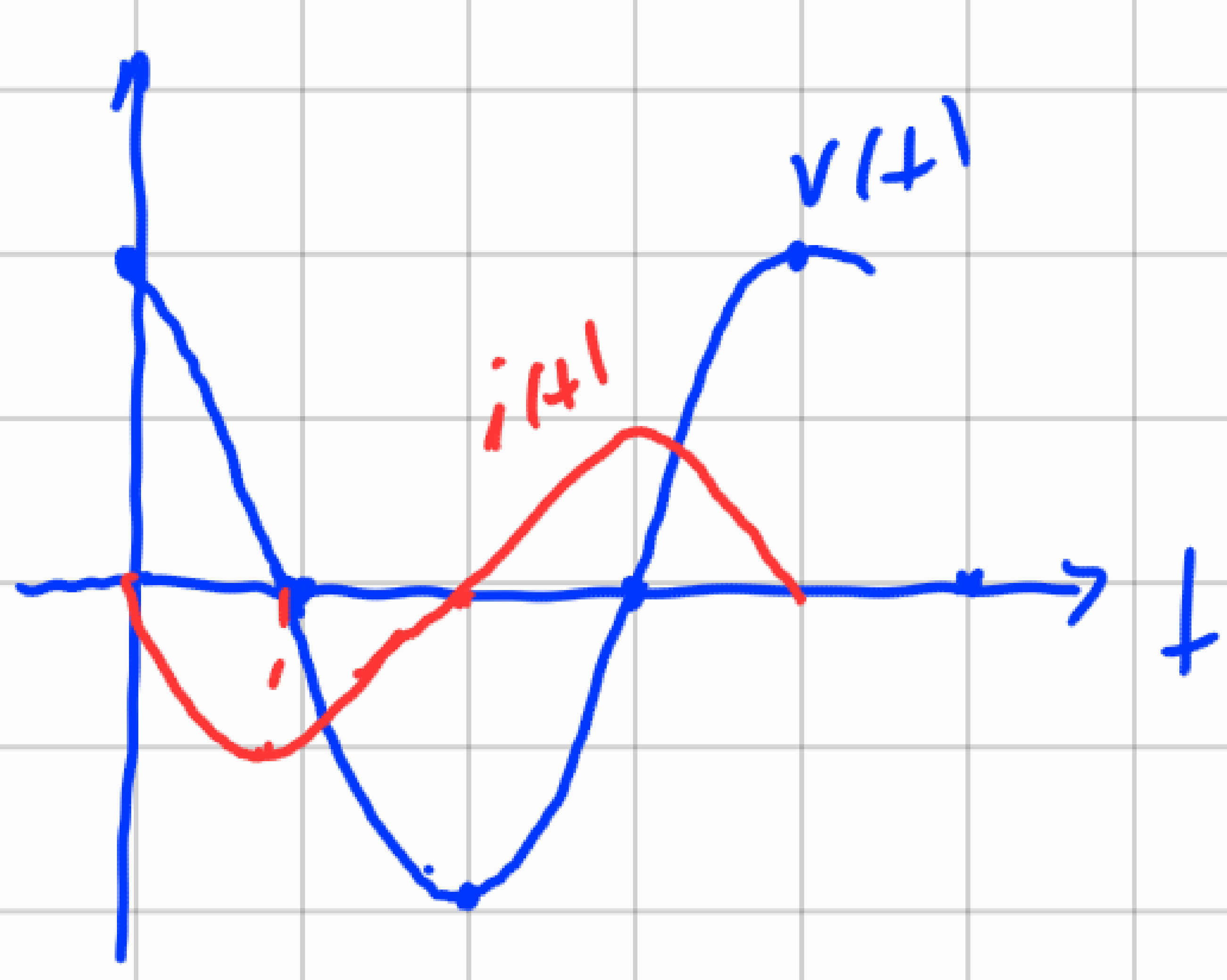
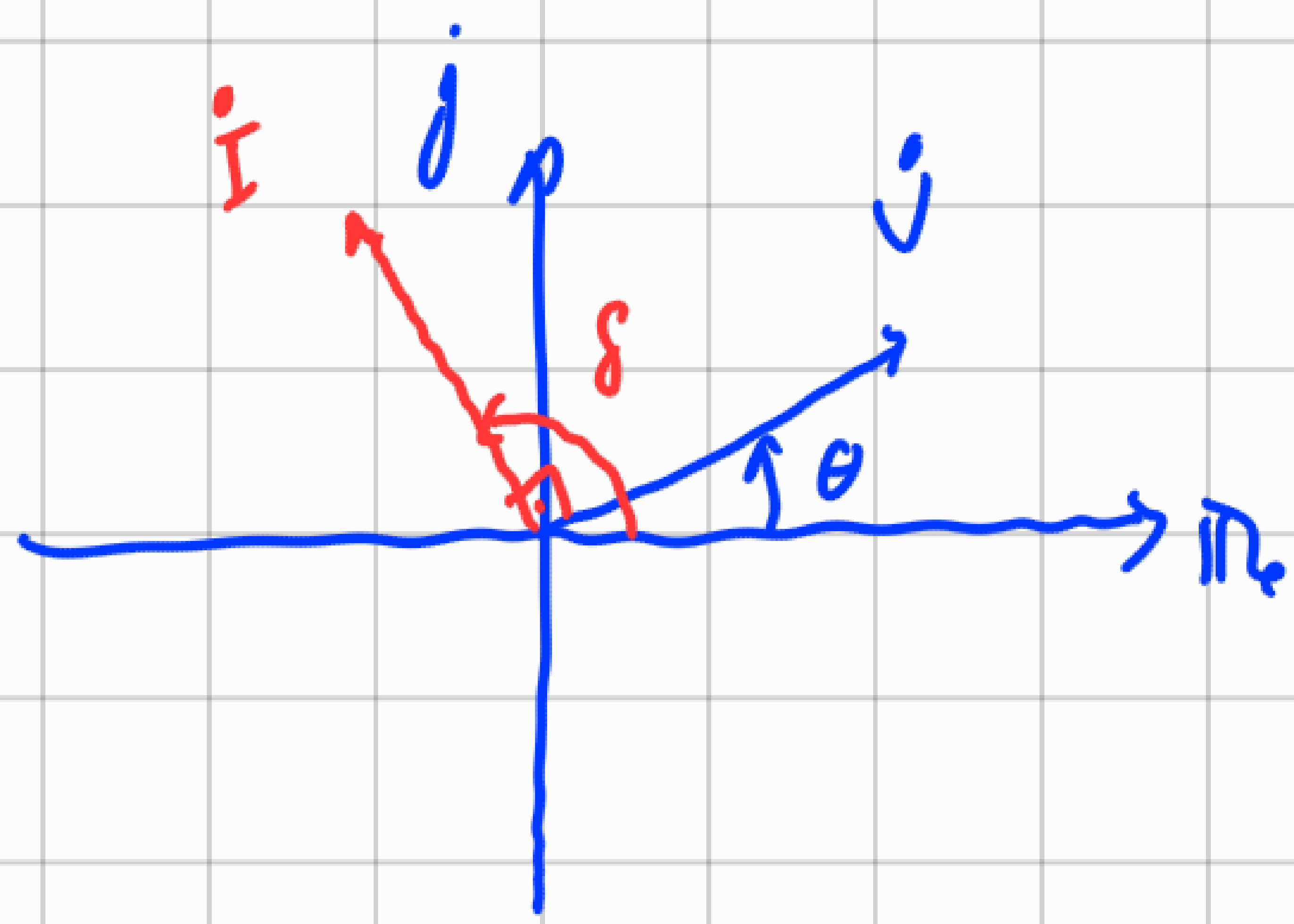


Diagrama de fasores:



Resumindo:

No resistor: $\dot{V} = R\dot{I}$ (v e i estão em fase)

No indutor: $\dot{V} = wL e^{j\frac{\pi}{2}} \dot{I} = jwL\dot{I}$

(i está atrasada de 90° em relação à tensão)

No capacitor: $\dot{V} = \frac{1}{wC} e^{-j\frac{\pi}{2}} \dot{I} = -j\frac{1}{wC}\dot{I} = \frac{1}{jwC}\dot{I}$

(i está adiantada de 90° em relação à tensão.)