PCS 3115 – Sistemas Digitais I

Circuitos Combinatórios:
Somadores e Subtratores

Suporte para EAD

Parte II:
Somador Completo – Expressões Algébricas

Aula: 14 – Data: 27/04 (S)

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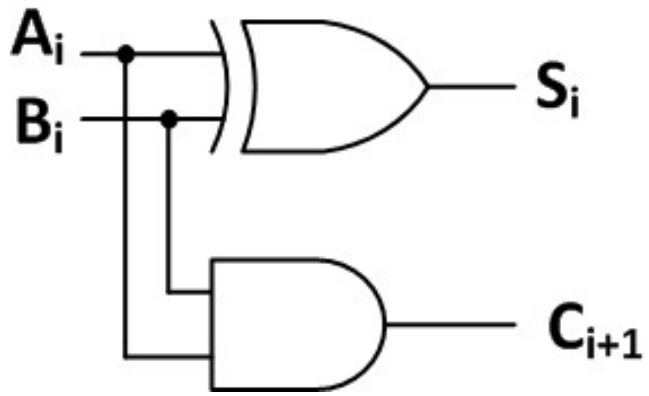
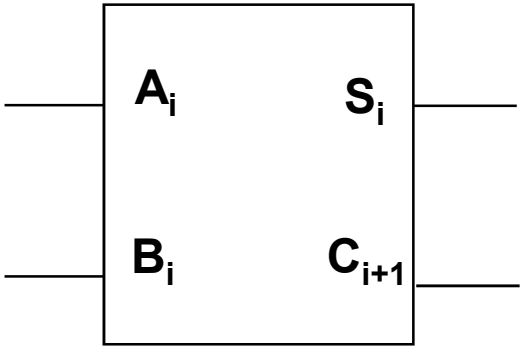
versão: 1.0 (Abril/2020)

Meio Somador – SLIDE [5]

- Qual o circuito que implementa o meio somador?





A_i	B_i	S_i	C_{i+1}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

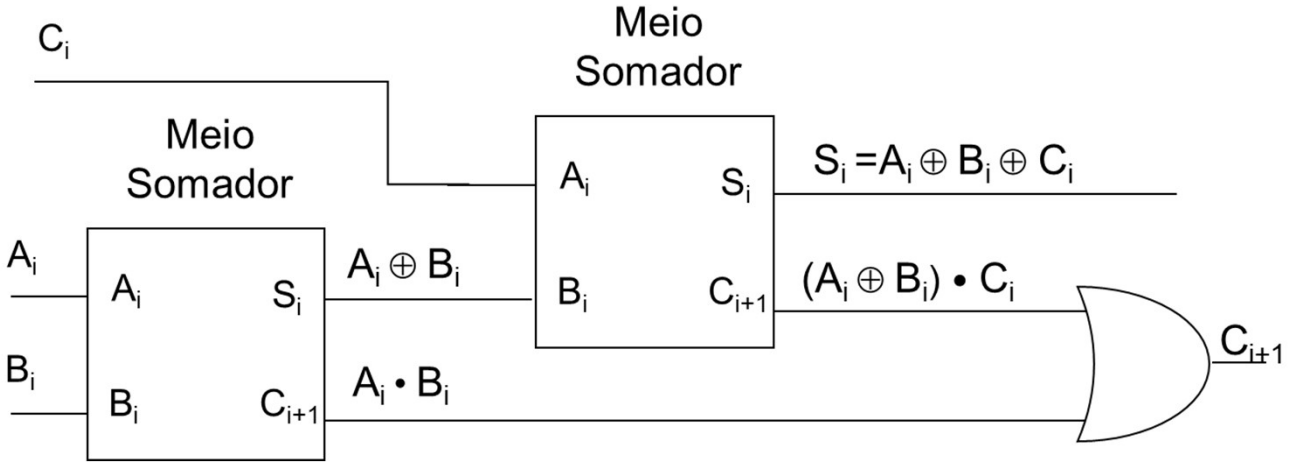
xor and

Somador completo – SLIDE [6]

- Usado para soma de operandos com 2+ bits
  - Entrada adicional para tratar o "vem-um" do bloco anterior (entrada "C_i")
- Como implementar um somador completo?
  - Pode-se combinar 2 meio somadores: meia-soma entre A_i e B_i seguida de meia-soma com C_i.
  - "Vai-um" se qualquer das somas levar a "vai-um"

A_i	B_i	C_i	S_i	C_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Somador completo – SLIDE [8]



$$S_i = A_i \oplus B_i \oplus C_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$$

$$C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i \quad \leftarrow \text{mintermos}$$

$$= (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i \quad \leftarrow \text{meio-somadores}$$

$$= (A_i + B_i) \bullet C_i + A_i \bullet B_i \quad \leftarrow \text{ANDs e ORs}$$

$$= Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi \quad \leftarrow \text{AND2}$$

Somador Completo – Dedução de Expressões Algébricas

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

$$C_{i+1}$$

{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ← mintermos

{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ← meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ← ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ← AND2

Como se pode obter {1} [1/2]? Análise de Circuitos.

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

$$C_{i+1}$$

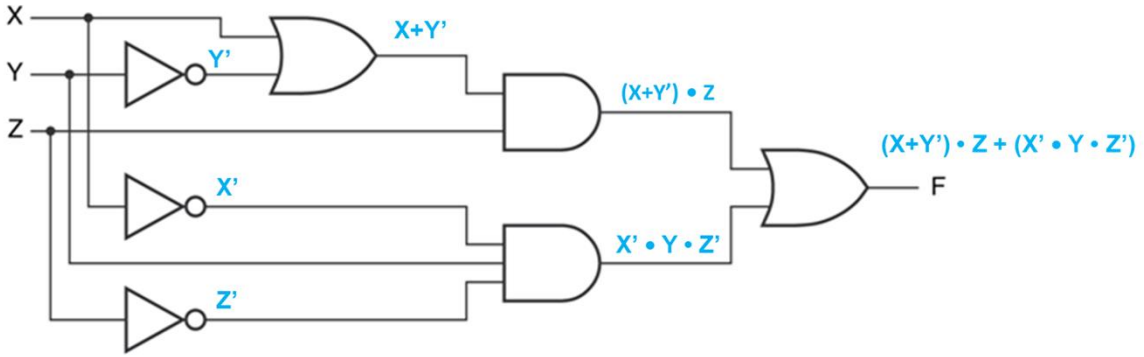
{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ← mintermos

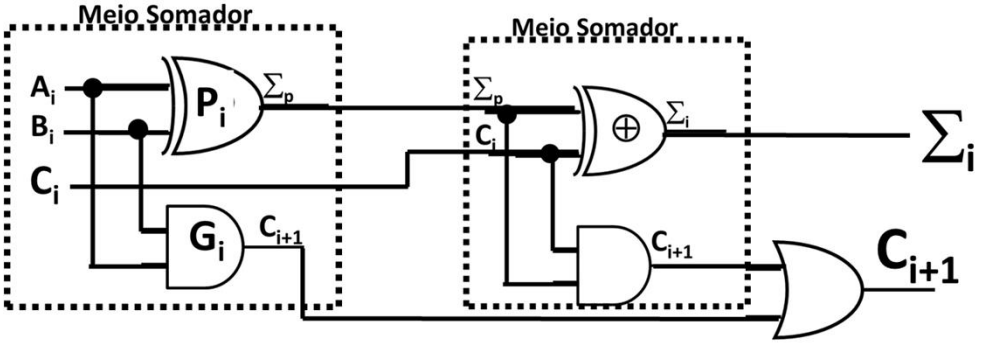
{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ← meio-somadores

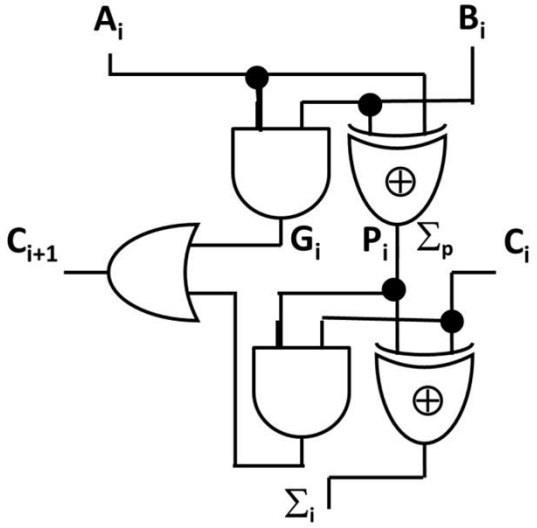
{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ← ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ← AND2

- O que faz este circuito?
  - Forma mais prática: escrever a **expressão lógica** para o circuito, **propagando sinais da entrada** até a saída







Como se pode obter {1}[2/2]? *Mapa de Karnaugh*

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

$$C_{i+1}$$

{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ←mintermos

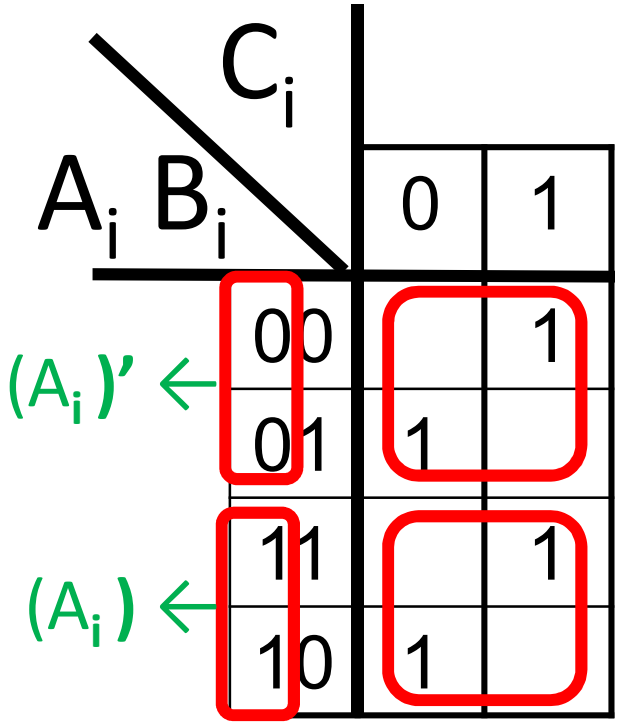
{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ←meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ←ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ←AND2

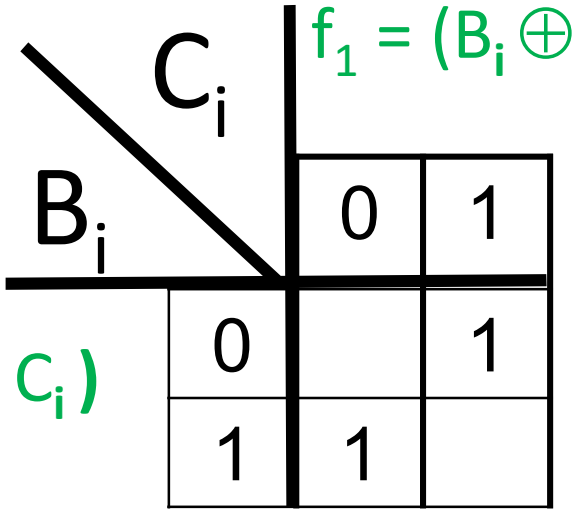
Mapas de Karnaugh

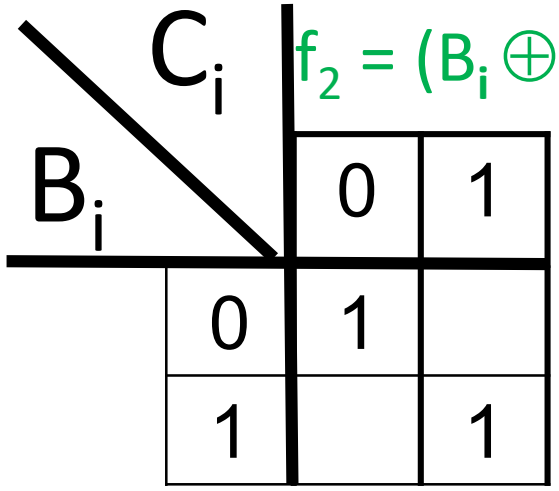
Expressão {1} [2/2] – S_i



$$S_i = (A_i)' \bullet (B_i \oplus C_i) + (A_i) \bullet (B_i \oplus C_i)'$$

$$S_i = (A_i \oplus B_i \oplus C_i)$$





Teoremas de n variáveis

(TC) X+X+...+X = X (TC') X•X• ...•X =X → idempotência generalizada

(T13) $(X_1 \bullet X_2 \bullet \ldots \bullet X_n)' = X_1' + X_2' + \ldots + X_n'$

(T13) $(X_1 + X_2 + \ldots + X_n)' = X_1' \bullet X_2' \bullet \ldots \bullet X_n'$

(T14) $[F(X_1, X_2, \ldots, X_n, +, \bullet)]' = F(X_1', X_2', \ldots, X_n', \bullet, +)$ → De Morgan generalizado

Expansão de Shannon

(T15) $\mathbf{F(X_1, X_2, \ldots, X_n) = X_1 \bullet F(1, X_2, \ldots, X_n) + X_1' \bullet F(0, X_2, \ldots, X_n)}$

(T15') F(X1,X2, ..., Xn) = [X1+F(0,X2, ..., Xn)] • [X1'+F(1,X2, ..., Xn)]

- Lição de casa:
  - Foi aplicado (T15) ou (T15') na obtenção de S_i por *Mapa de karnaugh*?
  - Explicar como foi aplicado!

Como se pode obter {2}[1/2]? A partir de {1}

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

C_{i+1}

{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ← mintermos

{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ← meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ← ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ← AND2

Como se pode obter {2} [1/2]?A partir de {1}

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i = A_i' \bullet (B_i \oplus C_i) + A_i \bullet (B_i \oplus C_i)' =$

$S_i = A_i' \bullet (B_i' \bullet C_i + B_i \bullet C_i') + A_i \bullet (B_i' \bullet C_i' + B_i \bullet C_i) =$

$S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i =$

{1} = S_i = {2} CQD

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

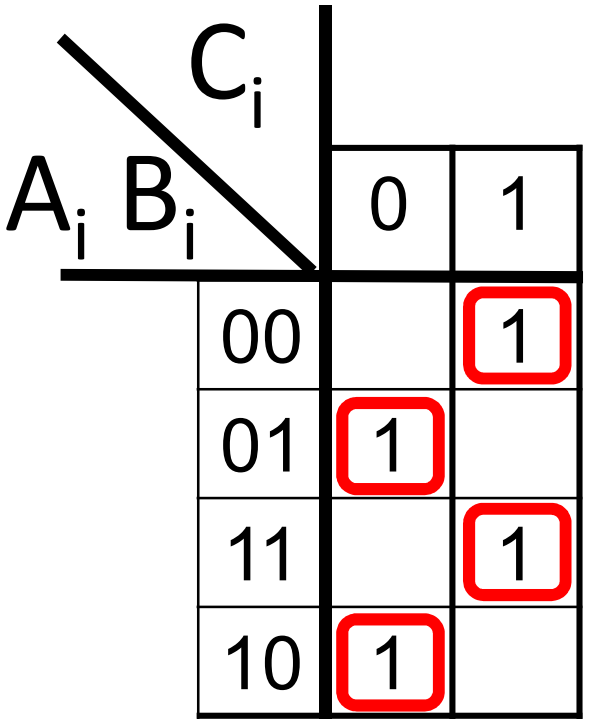
Como se pode obter {2}[2/2]? *Mapa de Karnaugh*

$$S_i = \sum_i$$

$$\{2\}\ S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$$

Mapas de Karnaugh

Expressão {2} [2/2] – S_i



$$S_i$$

$$=$$

$$= (A_i)' \bullet (B_i)' \bullet (C_i) +$$

$$+ (A_i)' \bullet (B_i) \bullet (C_i)' +$$

$$+ (A_i) \bullet (B_i) \bullet (C_i) +$$

$$+ (A_i) \bullet (B_i)' \bullet (C_i)'$$

$\{2\}\ S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

Como obter {3} *[1/1]? Mapa de Karnaugh*

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

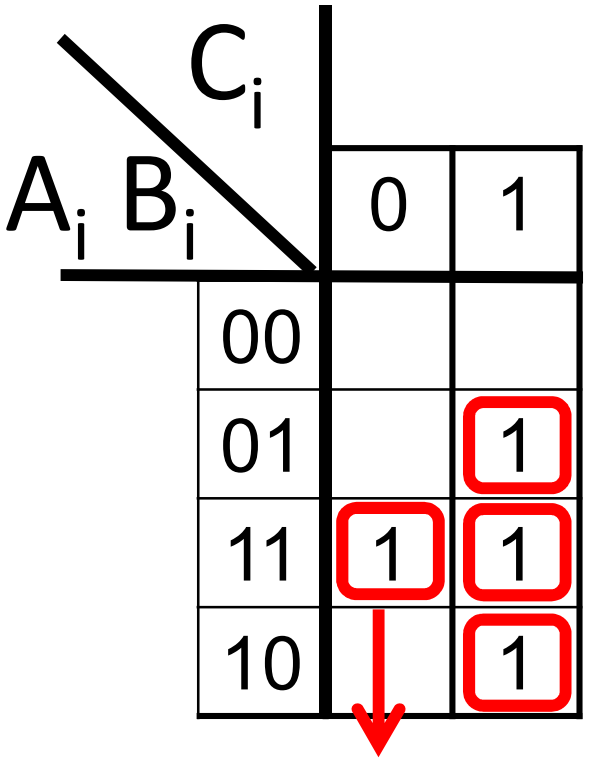
$$C_{i+1}$$

{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ← mintermos

{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ← meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ← ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ← AND2



$$C_{i+1} =$$

$$= (A_i)' \bullet (B_i) \bullet (C_i) +$$
$$+ (A_i) \bullet (B_i) \bullet (C_i) +$$
$$+ (A_i) \bullet (B_i)' \bullet (C_i) +$$
$$+ (A_i) \bullet (B_i) \bullet (C_i)'$$

$$\mathbf{C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i'}$$

$$\mathbf{\{3\}\ C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i}$$

Como obter {4} *[1/2]? Análise de circuitos*

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

C_{i+1}

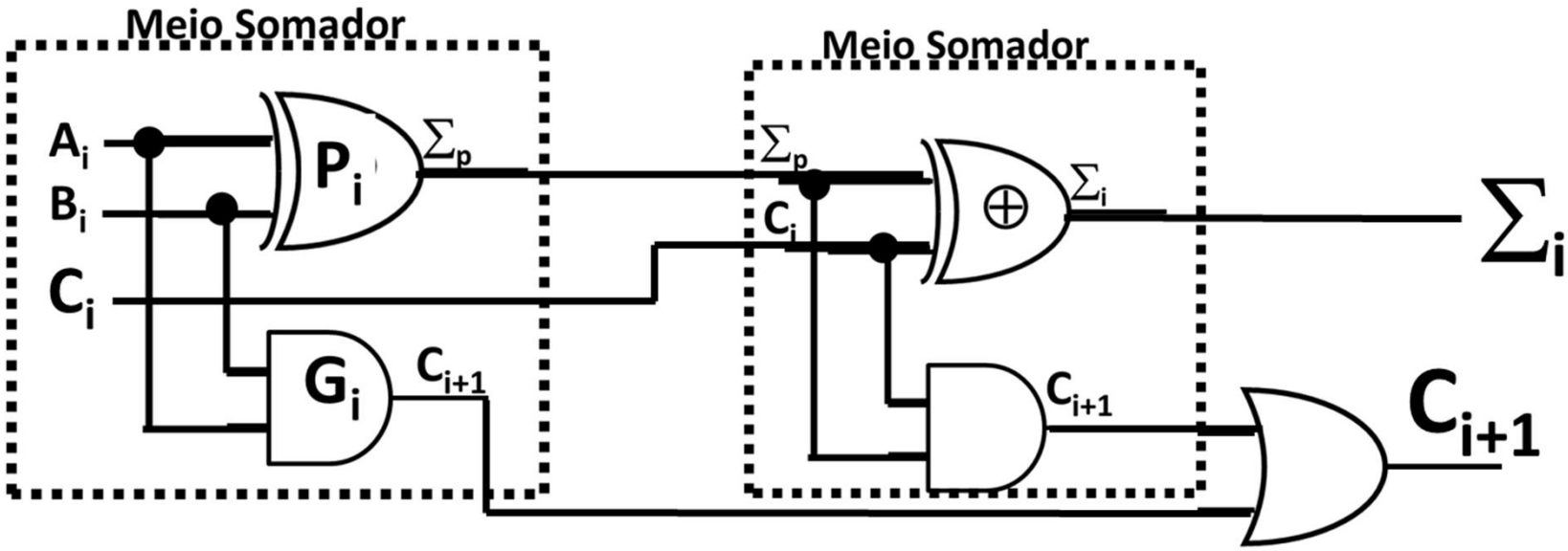
{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ← mintermos

{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ← meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ← ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ← AND2

- O que faz este circuito?
  - Forma mais prática: escrever a **expressão lógica** para o circuito, **propagando sinais da entrada** até a saída



Como obter {4} *[2/2]? Mapa de Karnaugh*

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

$$C_{i+1}$$

{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ← mintermos

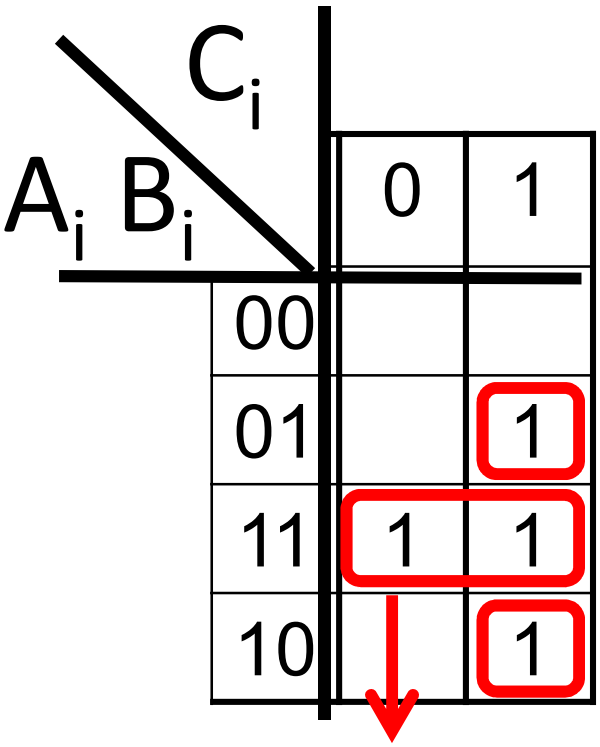
{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ← meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ← ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ← AND2

Mapas de Karnaugh

Expressão {4} [2/2] – C_{i+1}



C_{i+1}

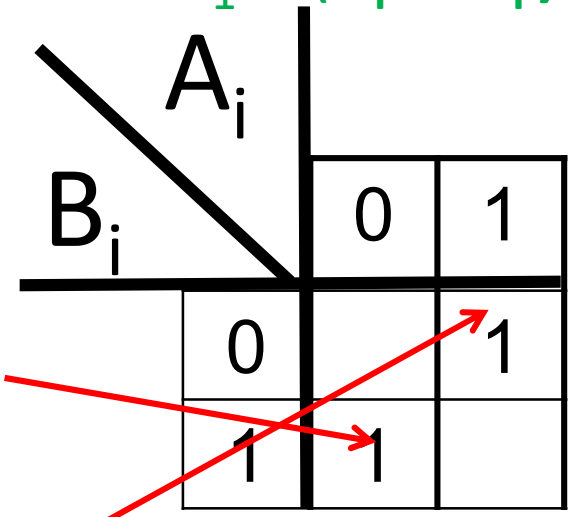
=

$\leftarrow (C_i) \bullet (A_i)' \bullet (B_i)$

+

$\leftarrow (C_i) \bullet (A_i) \bullet (B_i)'$

$f_1 = (A_i \oplus B_i)$



$(A_i) \bullet (B_i) + (C_i) \bullet (A_i \oplus B_i)$

$C_{i+1} = (A_i \bullet B_i) + (C_i) \bullet (A_i \oplus B_i)$

Como obter {5} *[1/1]? Mapa de Karnaugh*

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

C_{i+1}

{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ←mintermos

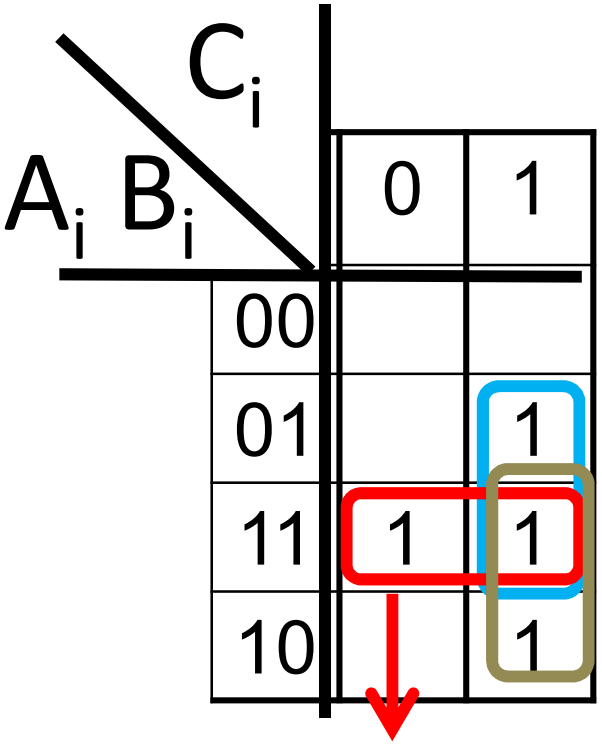
{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ←meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ←ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ←AND2

Mapas de Karnaugh

Expressão {5} [1/1] – C_{i+1}



$$C_{i+1} =$$

$$\leftarrow (C_i) \bullet (B_i)$$

$$+$$

$$\leftarrow (C_i) \bullet (A_i)$$

$$(A_i) \bullet (B_i) + (C_i) \bullet (A_i + B_i)$$

$$C_{i+1} = (A_i \bullet B_i) + (C_i) \bullet (A_i + B_i)$$

Como se pode obter {6}[1/3]? A partir de {5}

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

C_{i+1}

{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ←mintermos

{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ←meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ←ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ←AND2

Como se pode obter {6}[2/3]? A partir de {3}

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

C_{i+1}

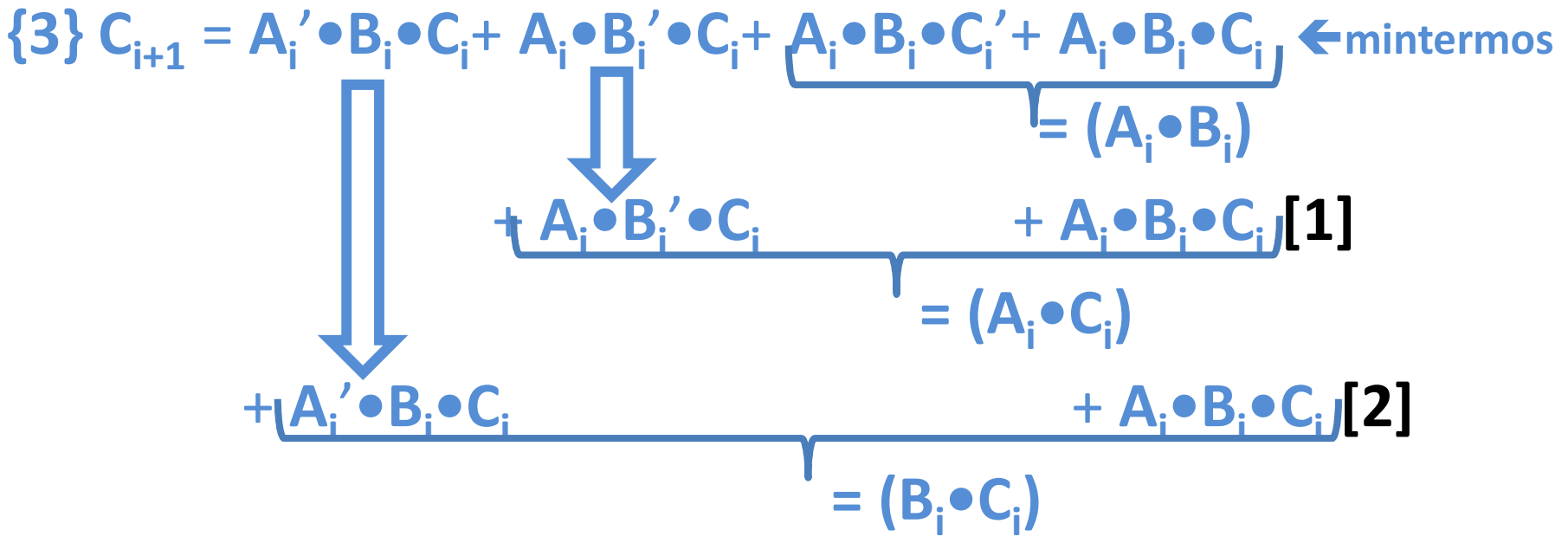
{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ←mintermos

{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ←meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ←ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ←AND2

Como se pode obter {6}[2/3]? A partir de {3}



$C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ←AND2

Como se pode obter {6}[3/3]? *Mapa de Karnaugh*

$$S_i = \sum_i$$

{1} $S_i = A_i \oplus B_i \oplus C_i$

{2} $S_i = A_i' \bullet B_i' \bullet C_i + A_i' \bullet B_i \bullet C_i' + A_i \bullet B_i' \bullet C_i' + A_i \bullet B_i \bullet C_i$

C_{i+1}

{3} $C_{i+1} = A_i' \bullet B_i \bullet C_i + A_i \bullet B_i' \bullet C_i + A_i \bullet B_i \bullet C_i' + A_i \bullet B_i \bullet C_i$ ← mintermos

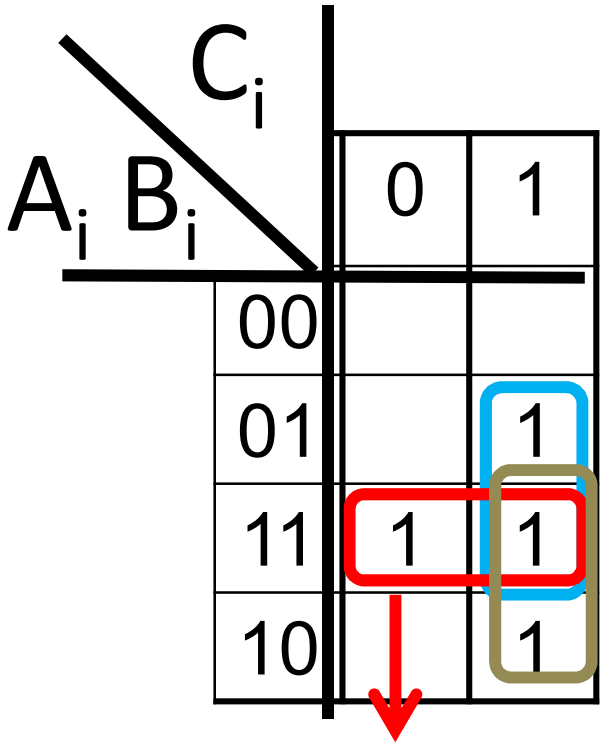
{4} $C_{i+1} = (A_i \oplus B_i) \bullet C_i + A_i \bullet B_i$ ← meio-somadores

{5} $C_{i+1} = (A_i + B_i) \bullet C_i + A_i \bullet B_i$ ← ANDs e ORs

{6} $C_{i+1} = Ai \bullet Ci + Bi \bullet Ci + Ai \bullet Bi$ ← AND2

Mapas de Karnaugh

Expressão {6} [3/3] – C_{i+1}



C_{i+1} =

←$(B_i) \bullet (C_i)$

+

←$(A_i) \bullet (C_i)$

$(A_i) \bullet (B_i) + (A_i) \bullet (C_i) + (B_i \bullet C_i)$

{6} C_{i+1} = Ai•Ci+ Bi•Ci + Ai•Bi ←AND2