Homework II

Suggested reading:

1. Pokorski sections 2.6 and 5.1.
2. Coleman (Aspects of Symmetry) sections 3.3 to 3.7 of chapter 5.
3. Nash (Relativistic Quantum Fields) chapter 1.
4. Kaku (Quantum Field Theory) section 8.7.
5. Huang (Quantum Field theory) sections 15.7 and 15.8.

due date October 4th

1. Consider the a model containing a real scalar φ and a spin-1/2 fermion Ψ whose lagrangian density is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m_\varphi^2\varphi^2 + \bar{\Psi}(i\gamma^\mu\partial_\mu - m_\Psi)\Psi - \frac{\lambda}{4!}\varphi^4 - g\varphi\bar{\Psi}\Psi \ ,$$

where m_φ, m_Ψ, λ, and g are constants. Obtain the renormalized effective potential $V_{\text{eff}}(\varphi)$ within the one-loop approximation.

2. Let us consider the QED. Work in the class of covariant gauges

$$-\frac{1}{a_B}(\partial_\mu A_B^\mu)^2 \ .$$

Derive the following Ward identities:

$$\frac{1}{a}\partial^2_{(y)}\partial^{(y)}_\mu A^\mu(y) - \partial^{(y)}_\mu\frac{\delta\Gamma}{\delta A_\mu(y)} + iqe\frac{\delta\Gamma}{\delta\Psi(y)}\Psi(y) + iqe\bar{\Psi}(y)\frac{\delta\Gamma}{\delta\bar{\Psi}(y)} = 0 \ ,$$

$$q_\mu\tilde{\Gamma}^{(3)}(p,q) = -iqe\tilde{\Gamma}^{(2)}(p+q) + iqe\tilde{\Gamma}^{(2)}(p) \ .$$

For details in the notation, see Pokorski section 5.1.

3. Consider the $\lambda\varphi^4$ model. Obtain the Schwinger-Dyson equation for $\Gamma^{(4)}$.