PMR3503 Controle Avançado

Apoio à Aula

Projeto de controladores por realimentação dos estados (alocação de polos)

Alocação de polos - introdução

Sistema na forma companheira de controlabilidade

$$\dot{x} = Ax + Bu$$

$$u = -Gx$$

$$H(s) = \frac{y(s)}{u(s)} = \frac{1}{s^k + a_1 s^{k-1} + \cdots + a_k}$$

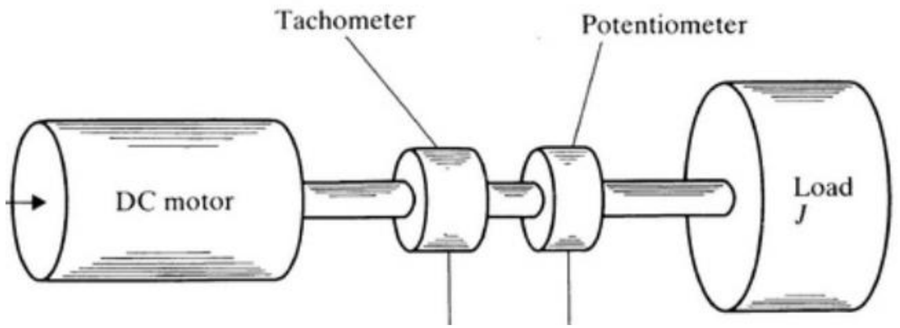
$$A = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{k-1} & -a_k \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$C = [0 \quad 0 \quad \cdots \quad 0 \quad 1]$$

$$bg' = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} [g_1, g_2, \ldots, g_k] = \begin{bmatrix} g_1 & g_2 & \cdots & g_k \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$A_c = A - bg' = \begin{bmatrix} -a_1 - g_1 & -a_2 - g_2 & \cdots & -a_k - g_k \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

Alocação de polos – exemplo 1



$$\dot{\theta} = \omega$$
$$\dot{\omega} = -\alpha\omega + \beta u$$

$\alpha = - K^2/JR$ $\beta = K/JR$

- θ e ω posição e velocidade angular

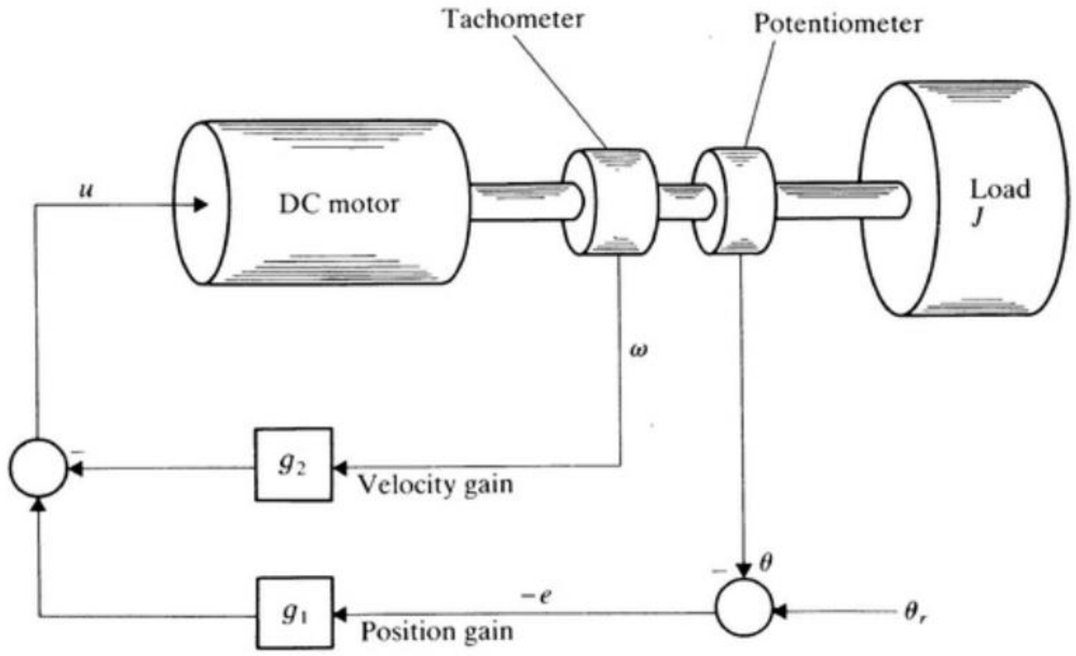
- α e β dependem dos parâmetros físicos do motor e carga

$$e = \theta - \theta_r$$

$$\dot{e} = \dot{\theta} - \dot{\theta}_r = \omega \qquad (\theta_r = \text{const})$$

$$\begin{bmatrix} \dot{e} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\alpha \end{bmatrix} \begin{bmatrix} e \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u$$

Alocação de polos – exemplo 1



$$u = -g_2\omega - g_1 e$$

$$\dot{e} = \omega$$

$$\dot{\omega} = -g_1\beta e - (\alpha + \beta g_2)\omega$$

Polinômio característico do sistema

$$|sI - A| = \begin{vmatrix} s & -1 \\ 0 & s+\alpha \end{vmatrix} = s^2 + \alpha s$$

$$a = \begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$

Matrizes Q e W

$$Q = [b, Ab] = \begin{bmatrix} 0 & \beta \\ \beta & -\alpha\beta \end{bmatrix} \qquad W = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

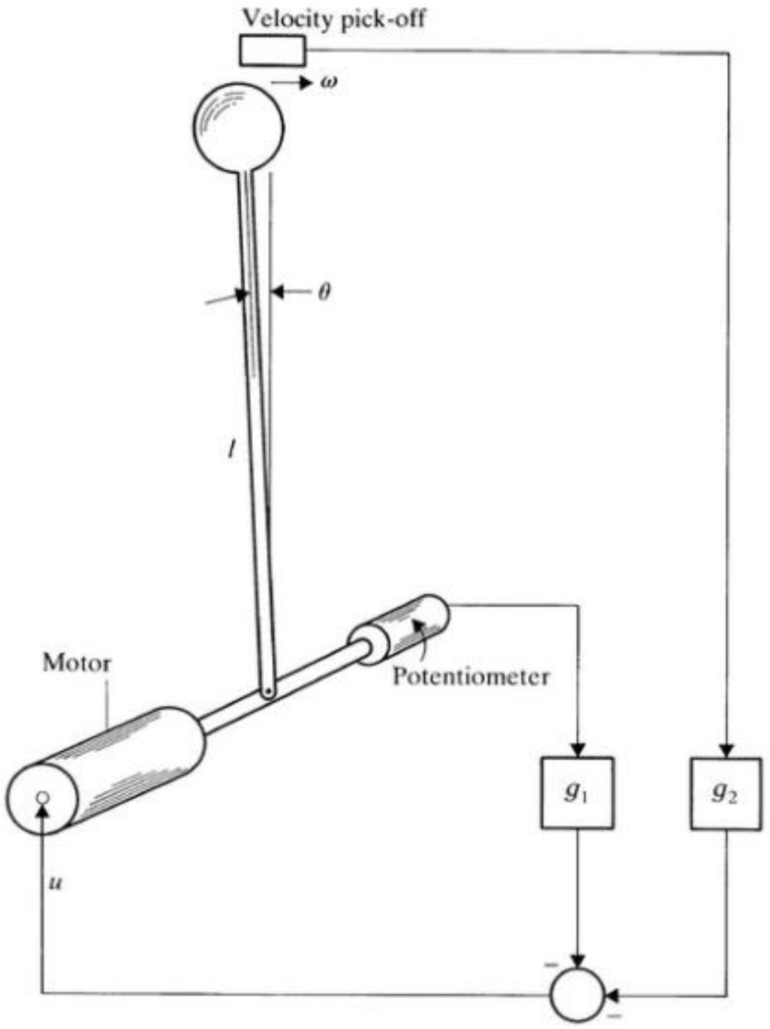
$$[(QW)']^{-1} = \begin{bmatrix} 0 & 1/\beta \\ 1/\beta & 0 \end{bmatrix}$$

Controlador

$$g = [(QW)']^{-1}(\hat{a} - a)$$

$$g = \begin{bmatrix} 0 & 1/\beta \\ 1/\beta & 0 \end{bmatrix} \begin{bmatrix} \bar{a}_1 - \alpha \\ \bar{a}_2 \end{bmatrix} = \begin{bmatrix} \bar{a}_2/\beta \\ (\bar{a}_1 - \alpha)/\beta \end{bmatrix}$$

Alocação de polos – exemplo 2



$$\dot{\theta} = \omega$$

$$\dot{\omega} = \Omega^2\theta - \alpha\omega + \beta u$$

$\alpha = -K^2/JR \; \beta = K/JR$ $\quad J = J_m + ml^2$ $\quad \Omega^2 = \dfrac{mgl}{J + ml^2} = \dfrac{g}{l + J/ml}$

$$A = \begin{bmatrix} 0 & 1 \\ \Omega^2 & -\alpha \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ \beta \end{bmatrix}$$

$$|sI - A| = \begin{vmatrix} s & -1 \\ -\Omega^2 & s + \alpha \end{vmatrix} = s^2 + \alpha s - \Omega^2 \quad \Rightarrow \quad a_1 = \alpha, \quad a_2 = -\Omega^2$$

$$Q = \begin{bmatrix} 0 & \beta \\ \beta & -\alpha\beta \end{bmatrix} \qquad W = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \quad \Rightarrow \quad [(QW)']^{-1} = \begin{bmatrix} 0 & 1/\beta \\ 1/\beta & 0 \end{bmatrix}$$

Alocação de polos – exemplo 2

$$\dot{\theta} = \omega$$

$$\dot{\omega} = \Omega^2\theta - \alpha\omega + \beta u$$

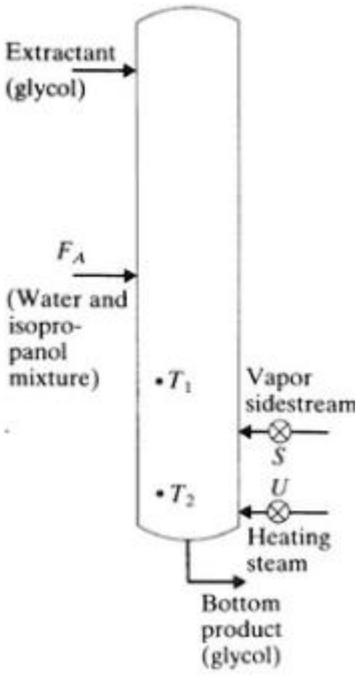
$$Q = \begin{bmatrix} 0 & \beta \\ \beta & -\alpha\beta \end{bmatrix} \qquad W = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \quad \Rightarrow \quad [(QW)']^{-1} = \begin{bmatrix} 0 & 1/\beta \\ 1/\beta & 0 \end{bmatrix}$$

Controlador

$$g = [(QW)']^{-1}(\hat{a} - a)$$

$$g = \begin{bmatrix} 0 & 1/\beta \\ 1/\beta & 0 \end{bmatrix} \begin{bmatrix} (\bar{a}_1 - \alpha) \\ \bar{a}_2 + \Omega^2 \end{bmatrix} = \begin{bmatrix} (\bar{a}_2 + \Omega^2)/\beta \\ (\bar{a}_1 - \alpha)/\beta \end{bmatrix}$$

Alocação de polos – exemplo MIMO



$$x = \begin{bmatrix} \Delta Q_l \\ \Delta V_l \\ \Delta z_1 \\ \Delta z_2 \end{bmatrix} \quad u = \begin{bmatrix} \Delta u_1 \\ \Delta S \end{bmatrix} \quad x_0 = \begin{bmatrix} \Delta x_{FA1} \\ \Delta F_A \end{bmatrix} \quad y = \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \end{bmatrix}$$

ΔQ_l = heat flow to reboiler "holdup"
ΔV_l = vapor flow rate
Δu_1 = steam flow rate

where ΔS = flow rate of vapor side stream
Δx_{FA1} = feed composition
ΔF_A = feed flow rate

$$A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & a_{32} & 0 & 0 \\ 0 & a_{42} & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & 0 \\ 0 & 0 \\ 0 & b_{32} \\ 0 & b_{42} \end{bmatrix} \quad E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ f_{31} & f_{32} \\ 0 & f_{42} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & c_{13} & 0 \\ 0 & 0 & 0 & c_{24} \end{bmatrix}$$

Alocação de polos – exemplo MIMO

Δu_1 depends on x_1, x_2, and x_3,

Δu_2 depends on x_4.

ΔQ_l = heat flow to reboiler "holdup"
ΔV_l = vapor flow rate
Δu_1 = steam flow rate

where ΔS = flow rate of vapor side stream
Δx_{FA1} = feed composition
ΔF_A = feed flow rate

$$G = \begin{bmatrix} g_1 & g_2 & g_3 & 0 \\ 0 & 0 & 0 & g_4 \end{bmatrix}$$

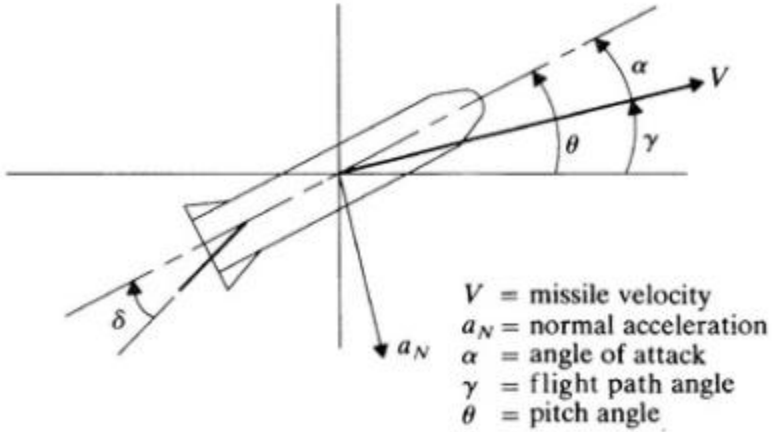
$$A_c = A - BG = \begin{bmatrix} a_{11} - b_{11}g_1 & -b_{11}g_2 & -b_{11}g_3 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & a_{32} & 0 & -b_{32}g_4 \\ 0 & 0 & 0 & -b_{42}g_4 \end{bmatrix}$$

$$|sI - A_c| = \begin{vmatrix} s - a_{11} + b_{11}g_1 & b_{11}g_2 & b_{11}g_3 & 0 \\ -a_{21} & s - a_{22} & 0 & 0 \\ 0 & -a_{32} & s & 0 \\ 0 & 0 & 0 & s + b_{42}g_4 \end{vmatrix}$$

$$= (s + b_{42}g_4) \begin{vmatrix} s - a_{11} + b_{11}g_1 & b_{11}g_2 & b_{11}g_3 \\ -a_{21} & s - a_{22} & 0 \\ 0 & -a_{32} & s \end{vmatrix}$$

$$= (s + b_{42}g_4)(s^3 + \bar{a}_1 s^2 + \bar{a}_2 s + \bar{a}_3)$$

Alocação de polos – exemplo MIMO



$$\dot{\alpha} = q + \frac{Z_\alpha}{V}\alpha + \frac{Z_\delta}{V}\delta$$

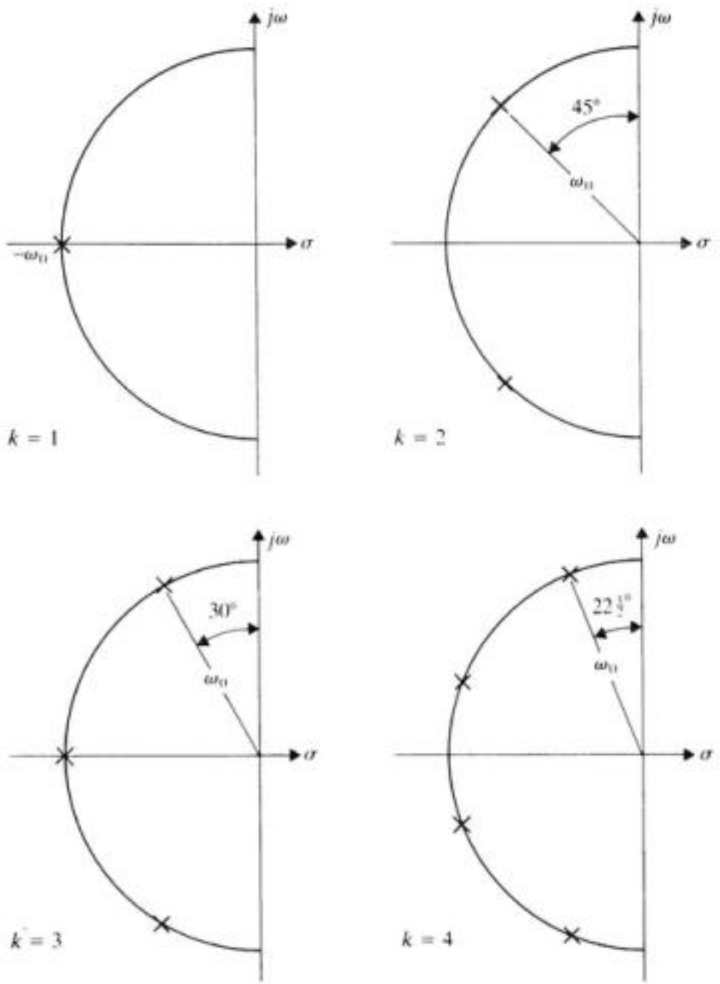
$$\dot{q} = M_\alpha \alpha + M_\delta \delta \qquad \text{(assuming } M_q \approx 0\text{)}$$

$$\dot{\delta} = \frac{1}{\tau}(u - \delta)$$

$$e = a_{NC} - a_N$$

$$A = \begin{bmatrix} Z_\alpha/V & -Z_\alpha & Z_\delta/\tau \\ -M_\alpha/Z_\alpha & M_q & \bar{M}_\delta \\ 0 & 0 & -1/\tau \end{bmatrix} \qquad B = \begin{bmatrix} -Z_\delta/\tau \\ 0 \\ 1/\tau \end{bmatrix} \qquad E = \begin{bmatrix} -Z_\alpha/V \\ M_\alpha/Z_\alpha \\ 0 \end{bmatrix}$$

Alocação de polos – posicionamento de polos



$$\left(\frac{s}{\omega_0}\right)^{2k} = (-1)^{k+1}$$

[z,p,k]=buttap(4)