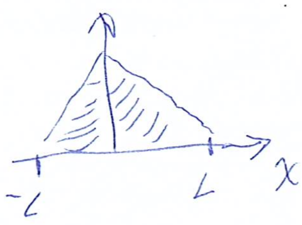
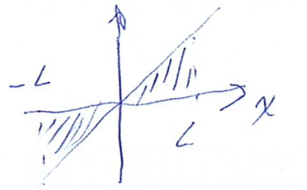
4) Se f for uma função par, então

$$\int_{-L}^{L} f(x)\,dx = 2\int_{0}^{L} f(x)\,dx$$



5) Se f for uma função ímpar, então

$$\int_{-L}^{L} f(x)\,dx = 0$$



Prova 4 : Temos $f(-x) = f(x)$ par

$$\int_{-L}^{L} f(x)\,dx = \underbrace{\int_{-L}^{0} f(x)\,dx}_{} + \int_{0}^{L} f(x)\,dx$$

$s = -x$, $x = -L \Rightarrow s = L$
$ds = -dx$, $x = 0 \Rightarrow s = 0$

$$= \int_{L}^{0} -f(-s)\,ds + \int_{0}^{L} f(x)\,dx$$

$$= \int_{0}^{L} f(-s)\,ds + \int_{0}^{L} f(x)\,dx$$

$$= \int_{0}^{L} f(s)\,ds + \int_{0}^{L} f(x)\,dx$$

$$= 2\int_{0}^{L} f(x)\,dx.$$

Prova 5 : Temos $f(-x) = -f(x)$ ímpar

$$\int_{-L}^{L} f(x)\,dx = \underbrace{\int_{-L}^{0} f(x)\,dx}_{} + \int_{0}^{L} f(x)\,dx$$

$s = -x$, $x = -L \Rightarrow s = L$
$ds = -dx$, $x = 0 \Rightarrow s = 0$

$$= \int_{L}^{0} -f(-s)\,ds + \int_{0}^{L} f(x)\,dx = \int_{0}^{L} f(-s)\,ds + \int_{0}^{L} f(x)\,dx$$

$$= -\int_{0}^{L} f(s)\,ds + \int_{0}^{L} f(x)\,dx = 0$$