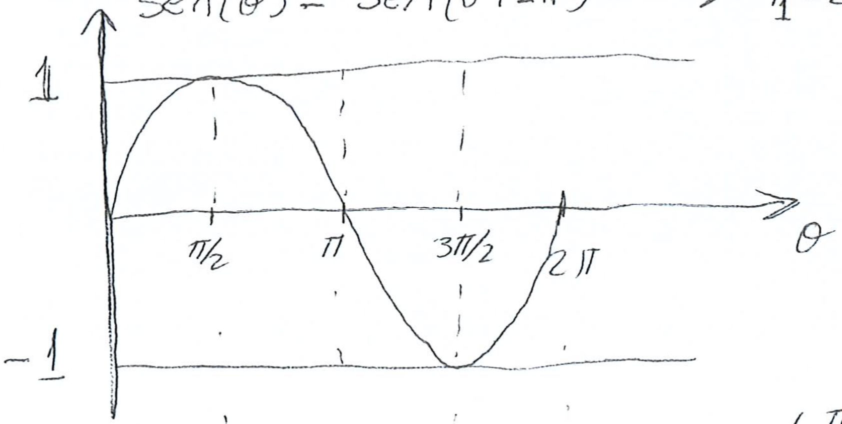
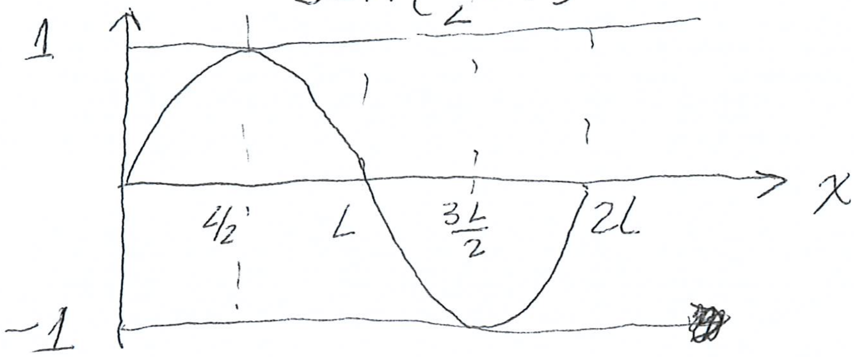
$\operatorname{sen}(\theta) = \operatorname{sen}(\theta + 2\pi) \Rightarrow T_1 = 2\pi$



$\operatorname{sen}\left(\frac{\pi}{L} x\right) = \operatorname{sen}\left(\frac{\pi}{L}(x + 2L)\right) = \operatorname{sen}\left(\frac{\pi x}{L} + 2\pi\right)$

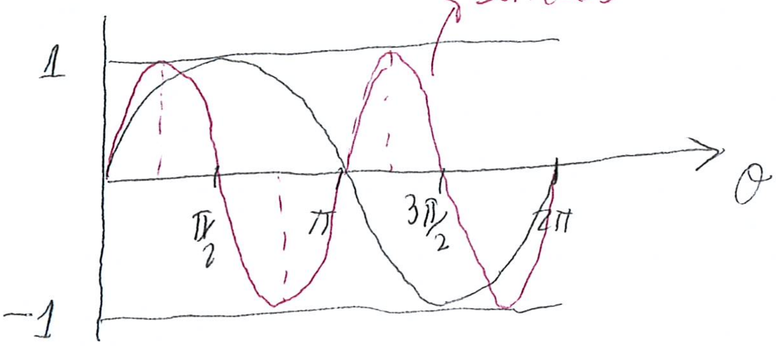
$T_1 = 2L$



$\operatorname{sen}(2\theta) = \operatorname{sen}\left(2\left(\theta + \frac{2\pi}{2}\right)\right) = \operatorname{sen}(2\theta + 2\pi)$

$T_2 = \frac{2\pi}{2} = \pi$

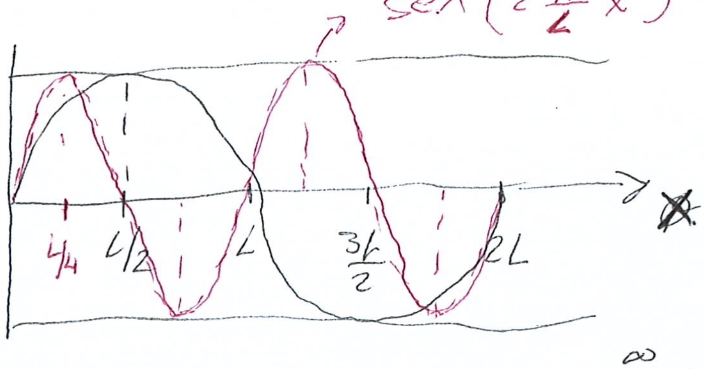
$\boxed{T_n = \frac{2\pi}{n}}$



$\operatorname{sen}\left(2 \frac{\pi}{L} x\right) = \operatorname{sen}\left(2 \frac{\pi}{L}\left(x + \frac{2L}{2}\right)\right) = \operatorname{sen}\left(2 \frac{\pi}{L} x + 2\pi\right.$

$T_2 = \frac{2L}{2} = L$

$\boxed{T_n = \frac{2L}{n}}$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(n \frac{\pi}{L} x\right) + \sum_{n=1}^{\infty} b_n \operatorname{sen}\left(n \frac{\pi}{L} x\right)$$

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\, dx \quad , \quad a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(n \frac{\pi}{L} x\right) dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \operatorname{sen}\left(n \frac{\pi}{L} x\right) dx$$

(1)

$$\begin{cases} sen(a+b) = sen(a)\cos(b) + sen(b)\cos(a) \\ \cos(a+b) = \cos(a)\cos(b) - sen(a)sen(b) \end{cases}$$

(I) $\cos(a+b) = \cos(a)\cos(b) - sen(a)sen(b)$

(II) $\cos(a-b) = \cos(a)\cos(b) + sen(a)sen(b)$

(II) − (I)

$$\boxed{sen(a)sen(b) = \frac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]}$$

$$\int_{-L}^{L} sen\left(\frac{m\pi x}{L}\right) sen\left(\frac{n\pi x}{L}\right) dx = \frac{1}{2}\int_{-L}^{L}\left[\cos\left(\frac{(m-n)\pi x}{L}\right) - \cos\left(\frac{(m+n)\pi x}{L}\right)\right]dx$$

se $m \neq n$

$$= \frac{1}{2}\left[\frac{L}{(m-n)\pi} sen\left(\frac{(m-n)\pi x}{L}\right) - \frac{L}{(m+n)\pi} sen\left(\frac{(m+n)\pi x}{L}\right)\right]\Bigg|_{-L}^{L}$$

$$= \frac{L}{2\pi}\left[\frac{sen\left(\frac{(m-n)\pi x}{L}\right)}{m-n} - \frac{sen\left(\frac{(m+n)\pi x}{L}\right)}{m+n}\right]_{-L}^{L}$$

$$= \frac{L}{2\pi}\left[\frac{sen((m-n)\pi)}{m-n} - \frac{sen((m+n)\pi)}{m+n} - \frac{sen(-(m-n)\pi)}{m-n} + \frac{sen(-(m+n)\pi)}{m+n}\right]$$

$sen(\lambda\pi) = 0 \quad \forall \lambda \in \mathbb{N}$

$$\boxed{\int_{-L}^{L} sen\left(\frac{m\pi x}{L}\right) sen\left(\frac{n\pi x}{L}\right) dx = 0 \quad \text{se} \quad m \neq n}$$

No caso $m = n$ temos que

$$\boxed{sen(a)sen(a) = \frac{1}{2}\left[1 - \cos(2a)\right]} = sen^2(a).$$

logo

(2)

$$\int_{-L}^{L} \operatorname{sen}^2\left(\frac{m\pi x}{L}\right) dx = \frac{1}{2}\int_{-L}^{L}\left[1-\cos\left(\frac{2m\pi x}{L}\right)\right] dx =$$

$$= \frac{1}{2}\left[x - \frac{L}{2m\pi}\operatorname{sen}\left(\frac{2m\pi x}{L}\right)\right]\Big|_{-L}^{L}$$

$$= \frac{1}{2}\left[L - \frac{L}{2m\pi}\cancelto{0}{\operatorname{sen}(2m\pi)} + L + \cancelto{0}{\operatorname{sen}(2m\pi)}\right]$$

$$= \frac{2L}{2} = L$$

Somando (I) e (II) temos

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$

(III) $\operatorname{sen}(a+b) = \operatorname{sen}(a)\cos(b) + \operatorname{sen}(b)\cos(a)$

(IV) $\operatorname{sen}(a-b) = \operatorname{sen}(a)\cos(b) - \operatorname{sen}(b)\cos(a)$

Somando (III) e (IV) temos

$$\operatorname{sen}(a)\cos(b) = \frac{1}{2}\left[\operatorname{sen}(a+b) + \operatorname{sen}(a-b)\right]$$

$$I = \int_{-L}^{L} \operatorname{sen}\left(m\frac{\pi}{L}x\right)\cos\left(n\frac{\pi}{L}x\right) dx = \frac{1}{2}\int_{-L}^{L}\left[\operatorname{sen}\left((m+n)\frac{\pi}{L}x\right) + \operatorname{sen}\left((m-n)\frac{\pi}{L}x\right)\right] dx$$

se $m \neq n$ temos

$$I = \frac{1}{2}\left[\frac{-L}{(m+n)\pi}\cos\left((m+n)\frac{\pi}{L}x\right) - \frac{L}{(m-n)\pi}\cos\left(\frac{(m-n)\pi x}{L}\right)\right]\Big|_{-L}^{L}$$

$$I = \frac{-L}{2\pi}\left[\frac{\cos\left((m+n)\frac{\pi}{L}x\right)}{m+n} - \frac{\cos\left(\frac{(m-n)\pi x}{L}\right)}{m-n}\right]_{-L}^{L}$$

$$I = \frac{-L}{2\pi}\left[\cancel{\frac{\cos((m+n)\pi)}{m+n}} - \cancel{\frac{\cos((m+n)\pi)}{m+n}} - \cancel{\frac{\cos((m-n)\pi)}{m-n}} + \cancel{\frac{\cos((m-n)\pi)}{m-n}}\right]$$

$$I = 0$$

Se $m = n$

$$\boxed{\operatorname{sen}(a)\cos(a) = \frac{1}{2}\operatorname{sen}(2a)}$$

$$\int_{-L}^{L} \operatorname{sen}\left(m\frac{\pi}{L}x\right)\cos\left(m\frac{\pi}{L}x\right)dx = \frac{1}{2}\int_{-L}^{L} \underbrace{\operatorname{sen}\left(\frac{2m\pi}{L}x\right)}_{\text{ímpar}} dx = 0$$

Limites de Integração Simétricos