Laplaciano ou
Operador de Laplace

denotado por $\Delta = (\nabla \cdot \nabla) = \nabla^2 = \nabla_x^2 + \nabla_y^2 + \nabla_z^2 = \frac{\partial}{\partial x}\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\frac{\partial}{\partial z} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla\varphi = grad\ \varphi = \hat{\imath}\frac{\partial\varphi}{\partial x} + \hat{y}\frac{\partial\varphi}{\partial y} + \hat{k}\frac{\partial\varphi}{\partial z}$$

$$\vec{b}(\vec{b}\varphi) = b^2\varphi$$

$$\text{div grad}\ \varphi = \nabla(\nabla\varphi) =$$

$$\nabla^2\varphi = \left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2}\right)$$

$$\nabla\vec{a} = \text{div}\ \vec{a} = \nabla_x a_x + \nabla_y a_y + \nabla_z a_z = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$\nabla(\nabla\vec{a}) = \nabla(\text{div}\ \vec{a}) =$$

$$\text{grad div}\ \vec{a} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}\right)$$

$$\vec{b}(\vec{b}\vec{a}) \neq b^2\vec{a}$$

$$\nabla^2\vec{a} = (\nabla\nabla)\vec{a} = \frac{\partial^2\vec{a}}{\partial x^2} + \frac{\partial^2\vec{a}}{\partial y^2} + \frac{\partial^2\vec{a}}{\partial z^2}$$

$$(\nabla^2\vec{a})_x = (\nabla\nabla)a_x = \frac{\partial^2 a_x}{\partial x^2} + \frac{\partial^2 a_x}{\partial y^2} + \frac{\partial^2 a_x}{\partial z^2}$$

$$\left[\vec{b} \times [\vec{b} \times \vec{a}]\right] = \vec{b}(\vec{b} \cdot \vec{a}) - (\vec{b} \cdot \vec{b})\vec{a}$$

$$[\nabla \times [\nabla \times \vec{a}]] = \nabla(\nabla \cdot \vec{a}) - \nabla^2\vec{a}$$

$$\text{rot rot}\ \vec{a} = \text{grad div}\ \vec{a} - \nabla^2\vec{a}$$

$$\text{rot rot}\ \vec{a} = \text{grad div}\ \vec{a} - \Delta\vec{a}$$