Recuperação de Cálculo II, 2016

1) Resolva o problema de valor inicial $y'' + 4y = 0$,

$y(\pi/6) = 1$, $y'(\pi/6) = 0$.

Sol.: $y'' + 4y = 0$ Eq. Dif.

$y'' \to r^2$, $y' \to r$, $y \to 1$ ↓ $r^2 + 4 = 0$ Eq. característica

$r_{1,2} = \pm 2i$ TIPO III

$\alpha = 0$ e $\beta = 2$

$$y_{gh}(x) = C_1 \cos(2x) + C_2 \operatorname{sen}(2x)$$

$$y'_{gh}(x) = -2C_1 \operatorname{sen}(2x) + 2C_2 \cos(2x)$$

- Usando a 1ra restrição: $y(\pi/6) = 1$

$$1 = C_1 \cos(2 \cdot \pi/6) + C_2 \operatorname{sen}(2 \cdot \pi/6)$$

$\pi/3 = 60°$

$$\boxed{1 = \frac{1}{2} C_1 + C_2 \frac{\sqrt{3}}{2}} \quad (I)$$

- Usando a 2da restrição: $y'(\pi/6) = 0$

$$0 = -2C_1 \operatorname{sen}(\pi/3) + 2C_2 \cos(\pi/3)$$

$$\boxed{0 = -\frac{\sqrt{3}}{2} C_1 + \frac{1}{2} C_2} \quad (II)$$

$$\begin{cases} \frac{1}{2}C_1 + \frac{\sqrt{3}}{2}C_2 = 1 \\ -\frac{\sqrt{3}}{2}C_1 + \frac{1}{2}C_2 = 0 \end{cases} \xrightarrow{①} C_2 = \sqrt{3}\,C_1$$

②

③

$$\frac{1}{2}C_1 + \frac{\sqrt{3}}{2}\sqrt{3}\,C_1 = 1$$

$$4C_1 = 2$$

$$C_1 = 1/2$$

$$C_2 = \frac{\sqrt{3}}{2}$$

$$y_{P.V.I}(x) = \frac{1}{2}\cos(2x) + \frac{\sqrt{3}}{2}\operatorname{sen}(2x)$$

Alternativamente

$$y_{gh}(x) = D_1\cos(2(x-\pi/6)) + D_2\operatorname{sen}(2(x-\pi/6))$$

$$y'_{gh}(x) = -2D_1\operatorname{sen}(2(x-\pi/6)) + 2D_2\cos(2(x-\pi/6))$$

1ª restrição: $y(\pi/6) = 1$

$$1 = D_1$$

2ª restrição: $y'(\pi/6) = 0$

$$0 = 2D_2 \Rightarrow D_2 = 0$$

$$y_{P.V.I}(x) = 1\cdot\cos(2x - \pi/3)$$

Usando $\cos(a+b) = \cos(a)\cos(b) - \operatorname{sen}(a)\operatorname{sen}(b)$

$$\cos(2x - \pi/3) = \cos(2x)\cos(-\pi/3) - \operatorname{sen}(2x)\operatorname{sen}(-\pi/3)$$

$$\cos(-\pi/3) = \cos(\pi/3) = \frac{1}{2} \qquad \operatorname{sen}(-\pi/3) = -\operatorname{sen}(\pi/3) = -\frac{\sqrt{3}}{2}$$

se mostra que as soluções são equivalentes

2) Encontre uma equação cartesiana para a curva polar $r^2\cos(2\theta) = 1$.

Sol.: $r^2\cos(2\theta) = 1$

$$r^2\left[\cos^2(\theta) - \operatorname{sen}^2(\theta)\right] = 1$$

mas $x = r\cos(\theta) \longrightarrow \cos(\theta) = \frac{x}{r}$

$y = r\operatorname{sen}(\theta) \longrightarrow \operatorname{sen}(\theta) = \frac{y}{r}$

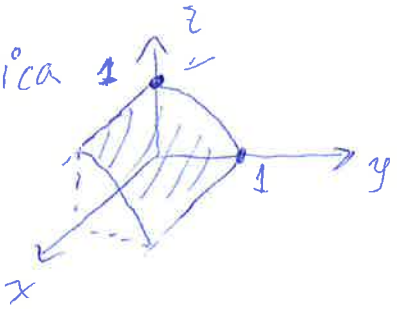
logo $r^2\left[\frac{x^2 - y^2}{r^2}\right] = 1$

$\boxed{x^2 - y^2 = 1}$ Hipérbole

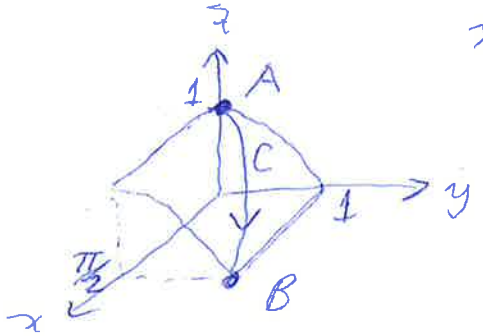
3) Esboce o gráfico da curva

$\vec{r}(t) = \langle t, \operatorname{sen}(t), \cos(t)\rangle$

Sol.: $C: \begin{cases} x = t \\ y = \operatorname{sen}(t) \\ z = \cos(t) \end{cases}$ $\Rightarrow$ superfície cilíndrica $z^2 + y^2 = 1 \Longrightarrow$



t	x	y	z	Ponto
0	0	0	1	A
$\pi/2$	$\pi/2$	1	0	B



4) Determine o domínio da função $f(x,y,z) = \dfrac{1}{\sqrt{16-x^2-y^2-z^2}}$. Faça um esboço do domínio.

Sol.: $D = \{(x,y,z) \in \mathbb{R}^3 \,/\, 16-x^2-y^2-z^2 > 0\}$

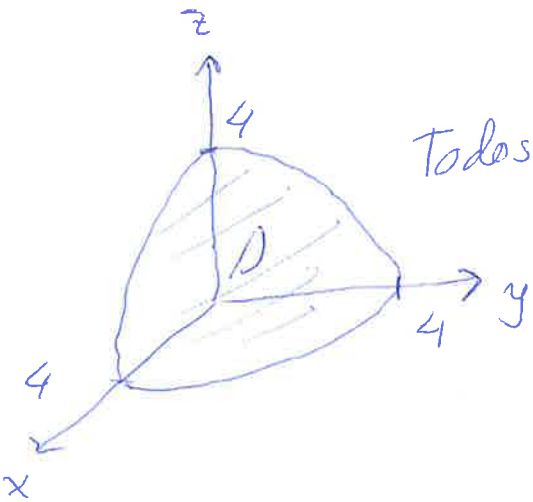
Se $16-x^2-y^2-z^2 = 0$ (Fronteira não incluída no domínio)

$x^2+y^2+z^2 = 16 \rightarrow$ Esfera de raio 4 e centrada na origem

Ponto de Teste $(x_0, y_0, z_0) = (0,0,0)$

$16-0^2-0^2-0^2 > 0$

$16 > 0$ verdadeiro



Todos os pontos no interior da esfera. Os pontos da superfície da esfera não fazem parte do domínio.

5) Sejam $z = e^{xy}$, $x = 2u + v$ e $y = \frac{u}{v}$.

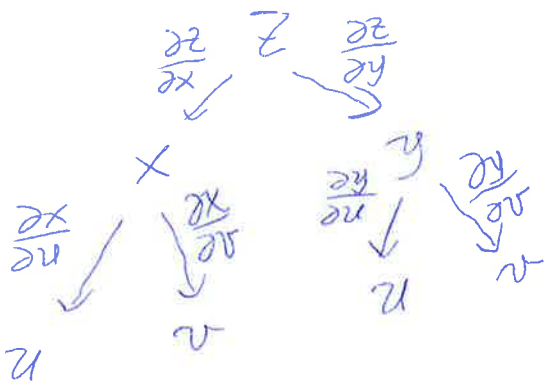
Usando a regra da cadeia encontre $\frac{\partial z}{\partial u}$ e $\frac{\partial z}{\partial v}$ em função de u e v.

Sol.:

$$z(x,y) = e^{xy}$$

$$x(u,v) = 2u + v$$

$$y(u,v) = \frac{u}{v}$$



$$\frac{\partial z}{\partial u} = \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial x}{\partial u}\right) + \left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial y}{\partial u}\right)$$

$$\frac{\partial z}{\partial u} = y e^{xy} \cdot 2 + x e^{xy} \cdot \frac{1}{v}$$

$$\frac{\partial z}{\partial u} = 2\frac{u}{v} e^{\frac{u}{v}(2u+v)} + \frac{1}{v}(2u+v) e^{\frac{u}{v}(2u+v)}$$

$$\frac{\partial z}{\partial u} = \frac{1}{v} e^{\frac{u}{v}(2u+v)} \cdot \left[2u + 2u + v\right]$$

$$\boxed{\frac{\partial z}{\partial u} = \frac{1}{v} e^{\frac{u}{v}(2u+v)} \cdot \left[4u + v\right]}$$

$$\frac{\partial z}{\partial v} = \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial x}{\partial v}\right) + \left(\frac{\partial z}{\partial y}\right)\left(\frac{\partial y}{\partial v}\right)$$

$$\frac{\partial z}{\partial v} = ye^{xy} \cdot \underline{1} + xe^{xy}\frac{(-1)u}{v^2}$$

$$= \frac{u}{v} e^{(2u+v)\frac{u}{v}} + (2u+v)\frac{u}{v^2}(-1)\, e^{(2u+v)\frac{u}{v}}$$

$$= \frac{u}{v} e^{(2u+v)\frac{u}{v}} \left[1 - \frac{(2u+v)}{v}\right]$$

$$\frac{\partial z}{\partial u} = \frac{u}{v^2} e^{(2u+v)\frac{u}{v}} \left[-2u\right] = -2\left(\frac{u}{v}\right)^2 e^{(2u+v)}$$