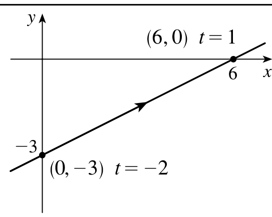
10.1 SOLUÇÕES

Revisão técnica: Ricardo Miranda Martins – IMECC – Unicamp

1. (a) $x = 2t + 4, y = t - 1$

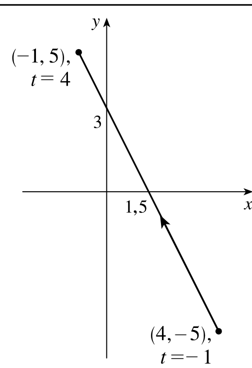
t	-3	-2	-1	0	1	2
x	-2	0	2	4	6	8
y	-4	-3	-2	-1	0	1



(b) $x = 2t + 4, y = t - 1 \quad \Rightarrow$
$x = 2(y + 1) + 4 = 2y + 6$ ou $y = \frac{1}{2}x - 3$

2. (a) $x = 3 - t, y = 2t - 3, -1 \le t \le 4$

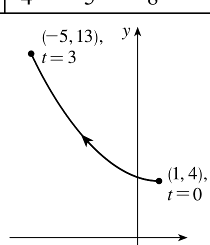
t	-1	0	1	2	3	4
x	4	3	2	1	0	-1
y	-5	-3	-1	1	3	5



(b) $x = 3 - t \quad \Rightarrow \quad t = 3 - x \quad \Rightarrow$
$y = 2t - 3 = 2(3 - x) - 3 \quad \Rightarrow \quad y = 3 - 2x$

3. (a) $x = 1 - 2t, y = t^2 + 4, 0 \le t \le 3$

t	0	1	2	3
x	1	-1	-3	-5
y	4	5	8	13



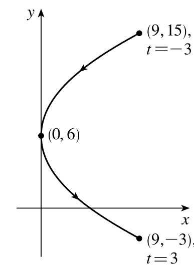
(b) $x = 1 - 2t \quad \Rightarrow \quad 2t = 1 - x \quad \Rightarrow \quad t = \dfrac{1 - x}{2} \quad \Rightarrow$

$$y = t^2 + 4 = \left(\frac{1 - x}{2}\right)^2 + 4 = \tfrac{1}{4}(x - 1)^2 + 4 \text{ ou}$$

$$y = \tfrac{1}{4}x^2 - \tfrac{1}{2}x + \tfrac{17}{4}$$

4. (a) $x = t^2, y = 6 - 3t$

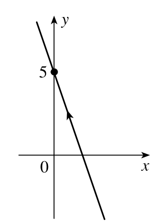
t	-3	-2	-1	0	1	2	3
x	9	4	1	0	1	4	9
y	15	12	9	6	3	0	-3



(b) $y = 6 - 3t \quad \Rightarrow \quad 3t = 6 - y \quad \Rightarrow \quad t = \dfrac{6 - y}{3} \quad \Rightarrow$

$$x = t^2 = \left(\frac{6 - y}{3}\right)^2 = \tfrac{1}{9}(y - 6)^2$$

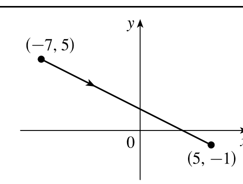
5. (a)



(b) $x = 1 - t, y = 2 + 3t \quad \Rightarrow$
$y = 2 + 3(1 - x) = 5 - 3x$, assim $3x + y = 5$

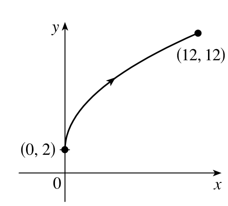
6. (a)

t	-3	-2	1	0	1	2
x	-7	-5	-3	-1	1	3
y	5	4	3	2	1	0



(b) $x = 2t - 1, y = 2 - t, -3 \le t \le 3 \quad \Rightarrow$
$x = 2(2 - y) - 1 = 3 - 2y$, logo $x + 2y = 3$,
com $-7 \le x \le 5$

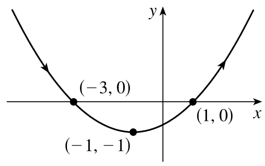
7. (a)



(b) $x = 3t^2, y = 2 + 5t, 0 \le t \le 2 \quad \Rightarrow$

$$x = 3\left(\frac{y - 2}{5}\right)^2 = \tfrac{3}{25}(y - 2)^2, 2 \le y \le 12$$

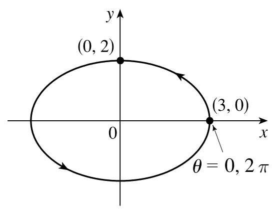
8. (a)



(b) $x = 2t - 1, y = t^2 - 1 \quad \Rightarrow \quad y = \left(\dfrac{x+1}{2}\right)^2 - 1,$

logo $y + 1 = \frac{1}{4}(x+1)^2$

9. (a)

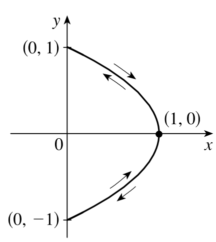


(b) $x = 3\cos\theta, y = 2 \operatorname{sen}\theta, 0 \leq \theta \leq 2\pi \quad \Rightarrow$

$\left(\dfrac{x}{3}\right)^2 + \left(\dfrac{y}{2}\right)^2 = \cos^2\theta + \operatorname{sen}^2\theta = 1$, ou

$\frac{1}{9}x^2 + \frac{1}{4}y^2 = 1$

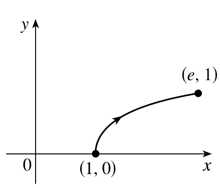
10. (a)



(b) $x = \cos^2\theta, y = \operatorname{sen}\theta \quad \Rightarrow$

$x + y^2 = \cos^2\theta + \operatorname{sen}^2\theta = 1, -1 \leq y \leq 1$

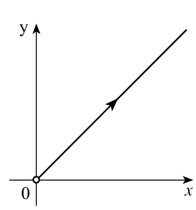
11. (a)



(b) $x = e^t, y = \sqrt{t} \quad \Rightarrow \quad x = e^{y^2}, 0 \leq y \leq 1$

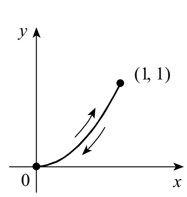
Ou: $y = \sqrt{\ln x}, 1 \leq x \leq e$

12. (a)



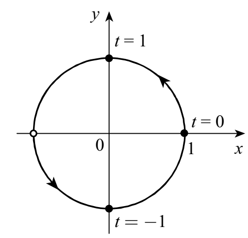
(b) $x = e^t, y = e^t \quad \Rightarrow \quad y = x, x > 0$

13. (a)



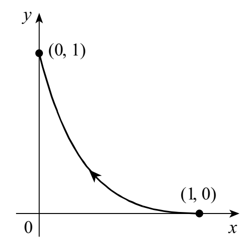
(b) $x = \cos^2 t, y = \cos^4 t \quad \Rightarrow \quad y = x^2, 0 \leq x \leq 1$

14. (a)



(b) $x = \dfrac{1-t^2}{1+t^2}, y = \dfrac{2t}{1+t^2} \quad \Rightarrow \quad x^2 + y^2 = 1, x \neq -1$

15. (a)



(b) $x = \dfrac{1-t}{1+t}, y = t^2, 0 \leq t \leq 1 \quad \Rightarrow \quad x = \dfrac{1-\sqrt{y}}{1+\sqrt{y}},$

$0 \leq y \leq 1$

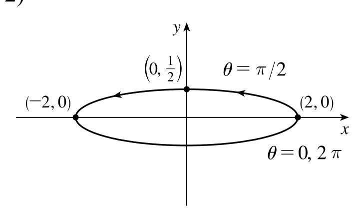
Ou: $y = \left(\dfrac{1-x}{1+x}\right)^2, 0 \leq x \leq 1$

16. (a) $x = 2\cos\theta, y = \frac{1}{2}\operatorname{sen}\theta, 0 \leq \theta \leq 2\pi.$

$1 = \cos^2\theta + \operatorname{sen}^2\theta = \left(\dfrac{x}{2}\right)^2 + \left(\dfrac{y}{1/2}\right)^2$, logo

$\dfrac{x^2}{2^2} + \dfrac{y^2}{(1/2)^2} = 1.$

(b)



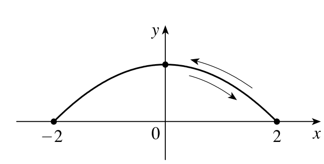
17. (a) $x = 2\cos\theta, y = \operatorname{sen}^2\theta.$

$1 = \cos^2\theta + \operatorname{sen}^2\theta = \left(\dfrac{x}{2}\right)^2 + y$, assim $y = 1 - \dfrac{x^2}{4}$,

$-2 \leq x \leq 2$. A curva está em $(2, 0)$ sempre que

$\theta = 2\pi n$.

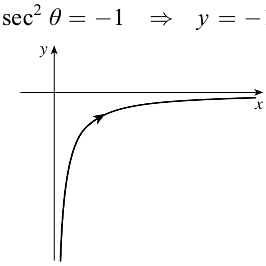
(b)



18. (a) $x = \operatorname{tg}\theta + \sec\theta, y = \operatorname{tg}\theta - \sec\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$

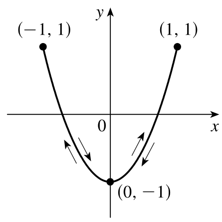
$xy = \operatorname{tg}^2\theta - \sec^2\theta = -1 \quad \Rightarrow \quad y = -1/x, x > 0.$

(b)



19. (a) $x = \cos t$, $y = \cos 2t$.
$y = \cos 2t = 2\cos^2 t - 1 = 2x^2 - 1$, logo $y + 1 = 2x^2$, $-1 \leq x \leq 1$.

(b)



20. $x = 4 - 4t$, $y = 2t + 5$, $0 \leq t \leq 2$.
$x = 4 - 2(2t) = 4 - 2(y - 5) = -2y + 14$, então a partícula move-se sobre a curva $y = -\frac{1}{2}x + 7$ de $(4, 5)$ para $(-4, 9)$.

21. $x = \operatorname{tg} t$, $y = \operatorname{cotg} t$, $\frac{\pi}{6} \leq t \leq \frac{\pi}{3}$. $y = 1/x$ para $\frac{1}{\sqrt{3}} \leq x \leq \sqrt{3}$. A partícula move-se pela ramificação do primeiro quadrante da hipérbole $y = 1/x$ de $\left(\frac{1}{\sqrt{3}}, \sqrt{3}\right)$ para $\left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$.

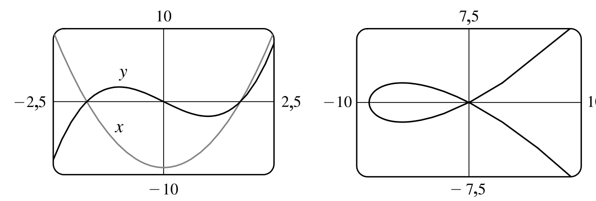
22. $x = 8t - 3$, $y = 2 - t$, $0 \leq t \leq 1 \quad \Rightarrow$
$x = 8(2 - y) - 3 = 13 - 8y$, então a partícula move-se pela reta $x + 8y = 13$ de $(-3, 2)$ para $(5, 1)$.

23. $y = \operatorname{cossec} t = 1/\operatorname{sen} t = 1/x$. A partícula desliza para baixo da ramificação do primeiro quadrante da hipérbole $xy = 1$ de $\left(\frac{1}{2}, 2\right)$ para $(\operatorname{sen} 1, \operatorname{cossec} 1) \approx (0{,}84147, 1{,}1884)$ conforme t vai de $\frac{\pi}{6}$ para 1.

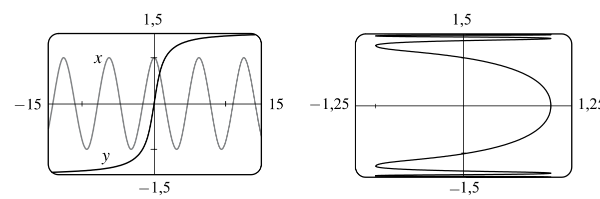
24. A partir dos gráficos, parece que $t \to -\infty$, $x \to \infty$ e $y \to -\infty$. Então, o ponto $(x(t), y(t))$ se moverá para longe do quarto quadrante conforme t aumenta. Em $t = -\sqrt{3}$, tanto x quanto y são 0, logo o gráfico passa pela origem. Depois de o gráfico passar pelo segundo quadrante (x é negativo, y é positivo), então intersecta o eixo x em $x = -9$ quando $t = 0$. Depois disto, o gráfico passa pelo terceiro quadrante, indo para a origem novamente em $t = \sqrt{3}$ e então quando $t \to \infty$, e $x \to \infty$ e $y \to \infty$. Observe que para todos os pontos $(x(t), y(t)) = (3(t^2 - 3), t^3 - 3t)$, podemos substituir $-t$ para chegar ao ponto correspondente

$$\begin{aligned}(x(-t), y(-t)) &= \left(3\left[(-t)^2 - 3\right], (-t)^3 - 3(-t)\right) \\ &= (x(t), -y(t))\end{aligned}$$

e então o gráfico é simétrico com respeito ao eixo x. A primeira figura é obtida usando $x_1 = t$, $y_1 = 3(t^2 - 3)$; $x_2 = t$, $y_2 = t^3 - 3t$; e $-2\pi \leq t \leq 2\pi$.



25. Conforme $t \to -\infty$, $y \to -\frac{\pi}{2}$ e x oscila entre 1 e -1. Então, conforme t aumenta através de 0, y aumenta enquanto x continua a oscilar e o gráfico passa através da origem. Então, quando $t \to \infty$, $y \to \frac{\pi}{2}$ conforme x oscila.



26. Como $t \to -\infty$, $x \to \infty$ e $y \to -\infty$. O gráfico passa através da origem em $t = -1$ e então passa para o segundo quadrante (x negativo, y positivo), passando pelo ponto $(-1, 1)$ em $t = 0$. Conforme t aumenta, o gráfico passa pelo ponto $(0, 2)$ em $t = 1$ e então quando $t \to \infty$, tanto x quanto y se aproximam de ∞. A primeira figura é obtida usando $x_1 = t$, $y_1 = t^4 - 1$; $x_2 = t$, $y_2 = t^3 + 1$; e $-2\pi \leq t \leq 2\pi$.