Hoje: 26/11/20 Dúvidas e Exercícios

Relações matemáticas importantes: $\sum_{n=0}^{\infty} x^n = (1-x)^{-1}$ $e^{x} = 1+x$ para $x \ll 1$

Dados:

1 u.m.a. = 1,66x10^{-27} kg; 1ns = 10^{-9} s; 1 Å = 10^{-10} m; 1 atm ≅ 10^5 Pa; k = 1,38x10^{-23} J/K.

m(He) = 4 u.m.a., m(C)= 12 u.m.a., m(O)= 16 u.m.a., m(Ne)= 20 u.m.a., $m(O_2)$ = 32 u.m.a., $m(N_2)$ = 28 u.m.a. e m(Ar) = 39,95 u.m.a.

Formulário:

$G_n = \int_0^{\infty} x^n e^{-\alpha x^2} dx \Rightarrow G_{2i} = \frac{1\cdot 3\cdots(2i-1)}{2^{i+1}}\sqrt{\frac{\pi}{\alpha^{2i+1}}}$ e $G_{2i+1} = \frac{i!}{2\alpha^{i+1}}$; $x \ll 1 \rightarrow e^x \cong 1 + x$; $P(n) = \frac{N!}{n!(N-n)!}p^n q^{N-n}$;

$P(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(n-n_0)^2}{2\sigma_n^2}}$; $\langle n\rangle = Np$; $\langle n^2\rangle = Np(q+Np)$; $\sigma_n^2 = \langle n^2\rangle - \langle n\rangle^2$; $D = \frac{2l^2pq}{\tau}$; $D = \frac{kT}{6\pi a\eta}$; $dxdydz = 4\pi r^2 dr$;

$dxdy = 2\pi r dr$; $f(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$; $v_{qm} = \sqrt{\langle v^2\rangle}$; $\beta = (1/kT)$; $dp(\Gamma) = (1/Z)e^{-\beta E}\, d\Gamma$; $Z = z^N$;

$z = \int\ldots\int e^{-\beta E_i}\, d\Gamma_i$; $\langle E\rangle = -\frac{\partial}{\partial\beta}(lnZ)$; $F = -\left(\frac{1}{\beta}\right) lnZ$; $P = -\left(\frac{\partial F}{\partial V}\right)_{T,N}$; $S = -\left(\frac{\partial F}{\partial T}\right)_{V,N}$; $\lambda = \frac{1}{\sqrt{2}\rho_N \pi d^2}$; $P = \frac{\rho}{3}\langle v^2\rangle$;

Movimento Browniano

passo: tamanho l e duração t

$m = N - n$

$x = (n-m)l$

$x = (2n-N)l$

$\rho_N = \frac{N}{V} = \frac{P}{kT}$

$N = n + m$

$1 = p + q$

$t = N\tau$

$P(n)dn = P(x)dx \rightarrow P(x;t) \Rightarrow \langle x\rangle = Nl(p-q)$

$\hookrightarrow P(n;N)$

$\sigma_x^2 = 4Npql^2$

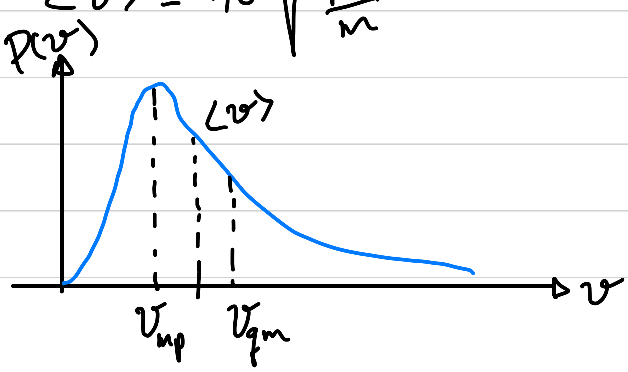
Teoria Cinética dos Gases (ok para gás ideal 3D)

$\langle v\rangle = \int_0^{\infty} v f(v)\, dv$

$f(v)$ = Distr. de Maxwell

$\langle v\rangle = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$ $v_{mp} = \sqrt{2\frac{kT}{m}}$ $v_{qm} = \sqrt{3\frac{kT}{m}}$

$\langle v\rangle = 1{,}6\sqrt{\frac{kT}{m}}$ $v_{mp} = 1{,}4\sqrt{\frac{kT}{m}}$ $v_{qm} = 1{,}7\sqrt{\frac{kT}{m}}$

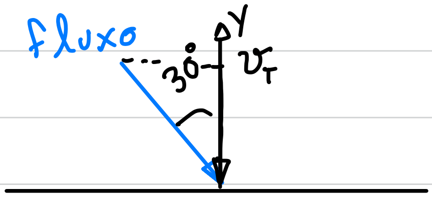


Estimativa

$P = \frac{\rho}{3}\langle v^2\rangle \cong \frac{\rho}{3}\langle v\rangle^2$

considerando a isotropia do espaço

Exercício 4 Lista T.C.G. fluxo $\neq$ isotropia



moléculas $O_2 \Rightarrow m_{O_2} = 2 \times 16 \times 1{,}66 \times 10^{-27}$ kg

$$\rho_N = \frac{N}{V} = 10^{10}\, \text{mol.}/\text{cm}^3 = \frac{10^{10}\,\text{mol.}}{(10^{-2}\text{m})^3} = 10^{16}\, \text{mol.}/\text{m}^3$$

$\langle v \rangle = 500\, \text{m/s} \Rightarrow v_\perp = \langle v \rangle \cos\theta = 500 \cos 30$

$v_\perp = 433\, \text{m/s}$

$P = ?$

Mecânica
$$P = \frac{F}{A} = \frac{\Delta p}{\Delta t\, A} = \frac{2 m v_\perp N}{\Delta t A} = \frac{2 m v_\perp N}{\frac{\Delta y}{v_\perp} A}$$

$\Delta p = 2 m v_\perp N$

$$P = \frac{2 m v_\perp^2 N}{V} = 2 m \rho_N v_\perp^2$$

$$v_\perp = \frac{\Delta y}{\Delta t} \quad \therefore \quad \Delta t = \frac{\Delta y}{v_\perp}$$

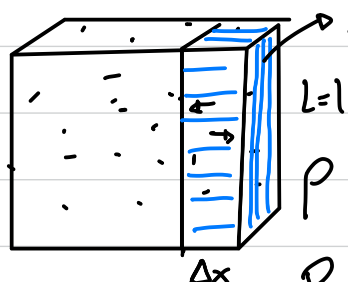
$\langle v_\perp^2 \rangle \cong \langle v_\perp \rangle^2$

$$\boxed{P = 2 m \rho_N v_\perp^2} \quad \text{ou} \quad \boxed{P = 2 \rho v_\perp^2}$$

$\hookrightarrow P = 2 \times 5{,}31 \times 10^{-26} \times 10^{16} \times 433^2 = 2 \times 10^{-4}$ Pa

$$\boxed{P = 2 \times 10^{-4}\, \text{Pa}}$$ resposta.

5)



$A = L^2$

$L = 1\,\text{cm} = 10^{-2}\,\text{m}$ $V = L^3 = 10^{-6}\,\text{m}^3$

$\rho = 1{,}2\, \text{g/m}^3$ $1{,}2 \times 10^{-3}\, \text{kg/m}^3$

$P = 1\,\text{atm} = 10^5\,\text{Pa}$; $T = 22°\text{C} = 295\,\text{K}$

$$\langle v_x \rangle = \frac{\Delta x}{\Delta t}$$

$$\frac{n_{col}}{\Delta t} = \frac{n/2}{\Delta t} = \frac{n}{2\Delta t} = \frac{\rho_N \overbrace{\Delta x}^{\langle v_x \rangle} A}{2 \Delta t} = \rho_N \frac{\langle v_x \rangle A}{2}$$

$$\rho_N = \frac{N}{V} = \frac{n}{\Delta x A} \Rightarrow n = \rho_N \Delta x A$$

$$\frac{n_{col}}{\Delta t} = \rho_N \frac{\langle v_x \rangle A}{2}$$

$$\frac{n_{col}}{\Delta t} = \rho_N \frac{\langle v_x \rangle L^2}{2}$$

$$\frac{n_{col}}{\Delta t} = \frac{P}{kT} \frac{L^2}{2} \langle v_x \rangle$$

$$\frac{n_{col}}{\Delta t} \cong \frac{P}{kT} \frac{L^2}{2} \sqrt{\frac{kT}{m}}$$

$$kT = \frac{PV}{N}$$

$$\frac{n_{col}}{\Delta t} \cong \frac{P}{kT} \frac{L^2}{2} \sqrt{\frac{PV}{mN}}$$

$$\frac{n_{col}}{\Delta t} \cong \frac{P}{kT} \frac{L^2}{2} \sqrt{\frac{P}{\rho}}$$

$$\boxed{\frac{n_{col}}{\Delta t} \cong \frac{L^2}{2kT} \sqrt{\frac{P^3}{\rho}}}$$

$$PV = NkT \therefore \frac{N}{V} = \frac{P}{kT}$$

$$\rho_N = \frac{P}{kT}$$

Gas ideal 3D isotrópico

$$\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle$$

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$$

$$\langle v^2 \rangle = 3 \langle v_x^2 \rangle$$

$$\langle v_x^2 \rangle = \frac{\langle v^2 \rangle}{3}$$

T.E.E. (3D)

$$\frac{1}{\cancel{2}} m \langle v^2 \rangle = \frac{3}{\cancel{2}} kT$$

$$\langle v^2 \rangle = 3 \frac{kT}{m}$$

T.E.E. (1D)

$$\frac{1}{2} m \langle v_x^2 \rangle = \frac{1}{2} kT \Rightarrow \langle v_x^2 \rangle = \frac{kT}{m}$$

$$\langle v_x \rangle \cong \sqrt{\langle v_x^2 \rangle}$$

$$\frac{n_{col}}{\Delta t} = \frac{10^{-4}}{2 \times 1{,}38 \times 10^{-23} \times 295} \sqrt{\frac{10^{15}}{1{,}2 \times 10^{-3}}} = 1{,}12 \times 10^{25}\ col./s$$

$$\boxed{\frac{n_{col}}{\Delta t} = 10^{25}\ \text{colisões}/s}$$ resposta.