Hoje: Distribuição de velocidade de Maxwell
(continuação)

Maxwell: 3 hipoteses:

1) Espaço isotrópico: $f(v_x) = f(-v_x) = f(v_y) = f(-v_y) = f(v_z) = f(-v_z)$
$\hookrightarrow f(v_i^2)$

2) Componentes da velocidade são independentes:
$$f(v_x, v_y, v_z)\,dv_x\,dv_y\,dv_z = f(v_x)f(v_y)f(v_z)\,dv_x\,dv_y\,dv_z$$

3) Validada T.E.E. $\Rightarrow$ gás ideal monoatômico
$$\langle E \rangle = \frac{3}{2}kT$$ (energia média por molécula/partícula)

Proposta: $f(v_i) = e^{-\alpha v_i^2}$

transformação para coordenadas esfericas

$$\begin{cases} v^2 = v_x^2 + v_y^2 + v_z^2 \\ dv_x\,dv_y\,dv_z = 4\pi v^2\,dv \end{cases}$$

$$dx\,dy\,dz = r^2 \operatorname{sen}\theta\,dr\,d\theta\,d\varphi$$
$$= 4\pi r^2 dr$$ (dependencia radial apenas)

$$dp = f(v_x, v_y, v_z)\,dv_x\,dv_y\,dv_z = f(v)\,dv$$

$$\boxed{dp = 4\pi A v^2 e^{-\alpha v^2} dv}$$

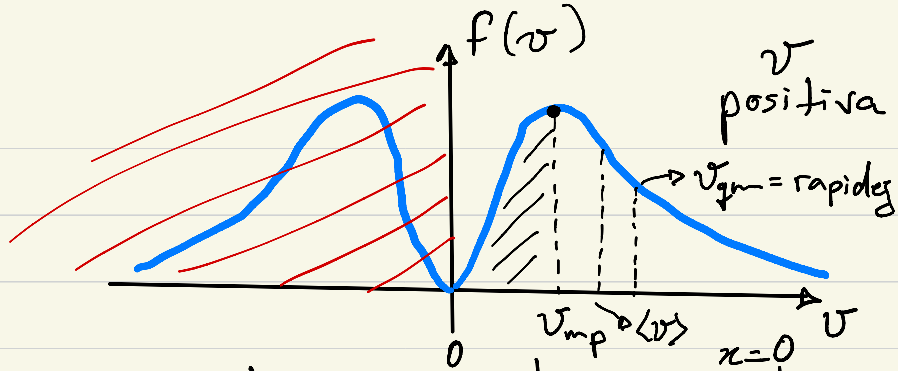
$$\int_0^\infty dp(v) = 1$$ } condição normalização

$$\underbrace{\langle E \rangle = \int_0^\infty \frac{1}{2}mv^2\,dp(v) = \frac{3}{2}kT}_{\text{Validade T.E.E.}}$$

$$A = \left(\frac{\alpha}{\pi}\right)^{3/2} \quad e \quad \alpha = \frac{m}{2kT}$$

$$\boxed{dp(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{1}{2}mv^2/kT}\,dv}$$

$$dp = \underbrace{C v^2 e^{-\alpha v^2}}_{f(v)} dv$$



$-\infty \leq f(v) \leq \infty \neq$ matematicamente $\Rightarrow$ simétrica

$0 \leq f(v) \leq \infty \neq$ fisicamente $\Rightarrow$ assimétrica

$$\langle E \rangle = \frac{1}{2} m \langle v^2 \rangle \quad \Rightarrow \quad \boxed{v_{qm} = \sqrt{\langle v^2 \rangle} = \text{rapidez}}$$

Qual o valor de v_{mp}?

$$\left.\frac{dp(v)}{dv}\right|_{v=v_{mp}} = 0 \qquad \frac{dp}{dv} = C\left[v^2(-2\alpha v)e^{-\alpha v^2} + 2v e^{-\alpha v^2}\right] = 0$$

$$C 2v e^{-\alpha v^2}\left[-\alpha v^2 + 1\right] = 0 \qquad -\alpha v^2 + 1 = 0 \therefore v_{mp}^2 = \frac{1}{\alpha}$$

$$v_{mp} = \sqrt{\frac{1}{\alpha}} \quad \Rightarrow \quad \boxed{v_{mp} = \sqrt{\frac{2kT}{m}}}$$

Qual o valor de $\langle v \rangle$?

$$\langle v \rangle = \int_0^\infty v\, dp(v) = \int_0^\infty v C v^2 e^{-\alpha v^2} dv = C \underbrace{\int_0^\infty v^3 e^{-\alpha v^2} dv}_{= G_3}$$

$$\langle v \rangle = C G_3 = \frac{C}{2\alpha^2} = \cancel{4}\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1}{\cancel{2}} \left(\frac{2kT}{m}\right)^2 \qquad \Bigg| \; G_3 = 1/2\alpha^2, \quad i=1 \Rightarrow G_{2i+1}$$

$$\langle v \rangle = (2\pi)^{2/2} \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{2kT}{m}\right)^{4/2} = \left[\frac{2^2\pi^2 m^3}{2^3\pi^3 k^3 T^3} \frac{2^4 k^4 T^4}{m^4}\right]^{1/2} = \left[\frac{2^3 kT}{\pi m}\right]^{1/2}$$

$$\boxed{\langle v\rangle = \sqrt{\frac{8}{\pi}\frac{kT}{m}}}$$

$$\frac{8}{\pi} = 2,54$$

$$\langle v\rangle = \sqrt{2,54\,\frac{kT}{m}} > v_{mp} = \sqrt{2\frac{kT}{m}}$$

Qual é o valor $\langle v^2\rangle$?

$$\langle v^2\rangle = \int_0^\infty v^2\, dp(v) \quad \therefore \quad \langle v^2\rangle = \int_0^\infty v^2 C v^2 e^{-\alpha v^2} dv = C\underbrace{\int_0^\infty v^4 e^{-\alpha v^2} dv}_{=G_4}$$

$$G_4 = \frac{3}{2^3}\sqrt{\frac{\pi}{\alpha^5}}$$

$i=2 \quad G_{2i}$

$$\langle v^2\rangle = C\,G_4 = \cancel{4}\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\frac{3}{\cancel{8}}\left(\frac{\pi\, 2^5 k^5 T^5}{m^5}\right)^{1/2}$$

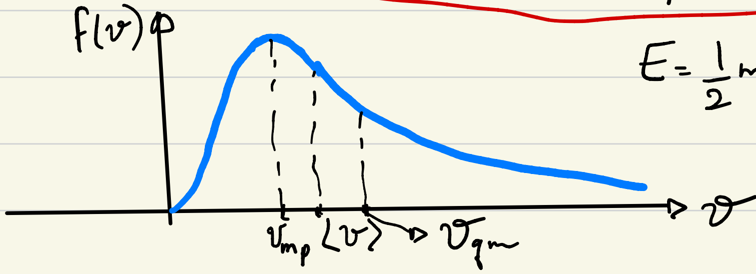
$$\langle v^2\rangle = \frac{3\pi}{2}\left(\frac{m^3}{2^3\pi^3k^3T^3}\,\frac{\pi\, 2^5k^5T^5}{m^5}\right)^{1/2} = \left(3^2\,\frac{k^2T^2}{m^2}\right)^{1/2} = 3\,\frac{kT}{m}$$

$$v_{qm} = \sqrt{\langle v^2\rangle} = \sqrt{3\,\frac{kT}{m}} \Rightarrow \boxed{v_{qm} = \sqrt{3\,\frac{kT}{m}}}$$

$$v_{qm}^2 = \langle v^2\rangle$$

Sumário: $\langle v\rangle^2 \neq \langle v^2\rangle$

$$\boxed{v_{mp} = \sqrt{2\,\frac{kT}{m}} < \langle v\rangle = \sqrt{\underset{\sim 2,5}{\underset{\downarrow}{\frac{8}{\pi}}}\,\frac{kT}{m}} < v_{qm} = \sqrt{3\,\frac{kT}{m}}}$$



$$E = \frac{1}{2} m v^2 \qquad \langle E\rangle = \frac{1}{2} m \langle v^2\rangle = \frac{1}{2} m v_{qm}^2$$

$$\langle E\rangle = \frac{3}{2} kT \quad \text{T.E.C.}$$

$$E_{mp} \underset{?}{\neq} \frac{1}{2} m v_{mp}^2$$

Se for verdadeiro que $E_{mp} = \frac{1}{2} m v_{mp}^2 = \underline{kT}$

Para conhecer informações de energia temos que fazer uma mudança de variável e encontrar a distribuição de energia.

$$f(v)\,dv = f(E)\,dE \qquad f(v) = C v^2 e^{-\alpha v^2}$$

$$E = \frac{1}{2} m v^2 \Rightarrow v^2 = \frac{2E}{m} \therefore v = \sqrt{\frac{2E}{m}}$$

$$dE = m v\, dv \Rightarrow dv = \frac{1}{mv} dE = \frac{1}{m}\sqrt{\frac{m}{2E}}\, dE = \sqrt{\frac{1}{2mE}}\, dE$$

$$dv = (2mE)^{-1/2}\, dE$$

$$f(v)\,dv = C v^2 e^{-\alpha v^2} dv = C\left(\frac{2E}{m}\right) e^{-\alpha 2E/m} (2mE)^{-1/2}\, dE$$

$$\Downarrow$$

$$f(E)\,dE = C\left(\frac{\cancel{2^2} E^{\cancel{2}}}{m^2} \frac{1}{\cancel{2} m \cancel{E}}\right)^{1/2} e^{-2\alpha E/m}\, dE$$

$$f(E) = C \left(\frac{2E}{m^3}\right)^{1/2} e^{-2\alpha E/m}\, dE \qquad \Bigg| \quad \alpha = m/2kT \qquad C = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}$$

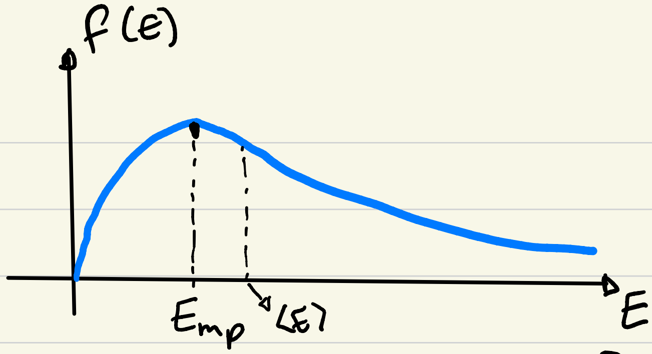
$$f(E) = 4\pi\left(\frac{\cancel{m^3}}{2^3 \pi^3 k^3 T^3} \frac{\cancel{2}E}{\cancel{m^3}}\right)^{1/2} e^{-E/kT} = \left(\frac{2^4}{\cancel{2^2}} \frac{\pi^{\cancel{2}}}{\pi^{\cancel{3}}} \frac{E}{k^3 T^3}\right)^{1/2} e^{-E/kT}$$

$$\boxed{dp(E) = f(E)\,dE = \frac{2}{\sqrt{\pi}}\left(\frac{1}{kT}\right)^{3/2} \underbrace{E^{1/2}} \underbrace{e^{-E/kT}}\, dE} \qquad \frac{1}{kT} = \beta$$

distribuição de energia de Maxwell-Boltzmann

$g(E) \Leftarrow$ densidade de estado (energia)

fator de Boltzmann



Qual o valor E_{mp}?

$$dp(E) = C\, E^{1/2} e^{-\beta E} dE$$

onde $C = \frac{2}{\sqrt{\pi}} \left(\frac{1}{kT}\right)^{3/2}$

$$\left.\frac{dp(E)}{dE}\right|_{E=E_{mp}} = 0$$

$$\frac{dp(E)}{dE} = C\left[E^{1/2}(-\beta)\, e^{-\beta E} + \frac{1}{2} E^{-1/2}\, e^{-\beta E}\right] = 0$$

$$C e^{-\beta E}\left[-\beta E^{1/2} + \frac{1}{2E^{1/2}}\right] = 0 \qquad -\beta E^{1/2} + \frac{1}{2E^{1/2}} = 0$$

$$\beta E^{1/2} = \frac{1}{2E^{1/2}} \therefore 2\beta E = 1 \therefore E_{mp} = \frac{1}{2\beta} \Rightarrow \boxed{E_{mp} = \frac{1}{2} kT}$$

Qual é o $\langle E \rangle = ?$

Resposta:
$\langle E \rangle = \frac{3}{2} kT$