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Photographs of the Center of Mass

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NOTES AND DISCUSSION

Let's Be Rational

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THE so-called "rationalized mks system" that is stylish today in electricity and magnetism is justified by its proponents as the answer to Heaviside's objections to the "eruption of the 4π's." What we now have, however, is not just an eruption of 4π's, but an eruption of $4\pi\epsilon_0$'s and of $\mu_0/4\pi$'s. If the purpose of "rationalization" was to eliminate the appearance of the factor 4π in the equations of electricity and magnetism, it has certainly failed.

The "rationalists" write Coulomb's law as

$$F=(1/4\pi\epsilon_0)(qq'/r^2). \tag{1}$$

Then Gauss' law becomes

$$q=\epsilon_0\oint\mathbf{E}\cdot d\mathbf{A}. \tag{2}$$

The "nonrationalists" write Coulomb's law as

$$F=(1/\epsilon_0)(qq'/r^2), \tag{3}$$

and Gauss' law becomes

$$q=(\epsilon_0/4\pi)\oint\mathbf{E}\cdot d\mathbf{A}. \tag{4}$$

(The nonrational ϵ_0 is 4π times the rational ϵ_0.)

It has been customary to say that one cannot eliminate the 4π from both Coulomb's law and Gauss' law, and there seems to be general agreement that it is preferable to eliminate it from Gauss' law. However, it *is* possible to eliminate the 4π from *both* Coulomb's law and Gauss' law.

If the theory of electrostatics is developed from Coulomb's law, surely the *rational* way to write this law is not in the form of either Eq. (1) or Eq. (3), but as

$$F=k(qq'/r^2), \tag{5}$$

in exact analogy with Newton's law,

$$F=G(mm'/r^2).$$

When Gauss' law is then derived from Coulomb's law, we get

$$q=(1/4\pi k)\oint\mathbf{E}\cdot d\mathbf{A}.$$

To simplify the notation, we define a new constant, which we might as well call ϵ_0, by the equation

$$\epsilon_0=(1/4\pi k),$$

and write Gauss' law as

$$q=\epsilon_0\oint\mathbf{E}\cdot d\mathbf{A}.$$

We then use the constant k in any equation in which it will eliminate the factor 4π, and use ϵ_0 in equations where its use will eliminate this factor. In problems involving spherical symmetry, the factor k appears. If the symmetry is cylindrical, the factor is $2k$. If plane, the factor is ϵ_0. Thus the magnitude of the electric intensity at points outside a charged sphere is

$$E=k(q/r^2).$$

At points outside a long charged cylinder,

$$E=2k(\lambda/r).$$

At points between two large parallel plates,

$$E=\sigma/\epsilon_0.$$

On the other hand, if the theory of electrostatics is developed from Gauss's law, the *rational* way to write this law is in the form of Eq. (2). (The net charge inside a closed surface is proportional to the surface integral of $\mathbf{E}$ over the surface.) When Coulomb's law is developed from Gauss' law, we get

$$F=(1/4\pi\epsilon_0)(qq'/r^2),$$

and to simplify the notation we define a new constant k as

$$k=(1/4\pi\epsilon_0).$$

The same procedure can be followed in magnetism. The *rational* form of the expression for the flux density set up by a current element is

$$d\mathbf{B}=k'I(d\mathbf{l}\times\mathbf{r}/r^3).$$

When Ampère's law is derived from this law, we get

$$\oint\mathbf{B}\cdot d\mathbf{s}=4\pi k'I.$$

To simplify the notation we define a new constant μ_0 by the equation

$$\mu_0=4\pi k',$$

and Ampère's law can be written

$$\oint\mathbf{B}\cdot d\mathbf{s}=\mu_0 I.$$

The disadvantage of this proposal, of course, is that one has to use *four* proportionality constants instead of *two*, and to remember that

$$\epsilon_0=(1/4\pi k),\quad \mu_0=4\pi k', \tag{6}$$

but may not this be a price worth paying? The numerical values of k and k' are easy to remember (that of k is approximately 9×10^9 and that of k' is *exactly* 10^{-7}.) It is surely as easy to remember these as to remember that the numerical value of $1/4\pi\epsilon_0$ is 9×10^9 and that of $\mu_0/4\pi$ is exactly 10^{-7}. If the numerical values of ϵ_0 and μ_0 are needed, they can readily be found from Eq. (6), which is the procedure that must be followed anyway if one simply remembers the numerical values of $1/4\pi\epsilon_0$ and of $\mu_0/4\pi$.

Photographs of the Center of Mass

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SINCE the center of mass is an invisible point, it is interesting to see its photograph.

The apparatus consists of two physical pendulums, 2 m in length, whose points of suspension are separated 50 cm

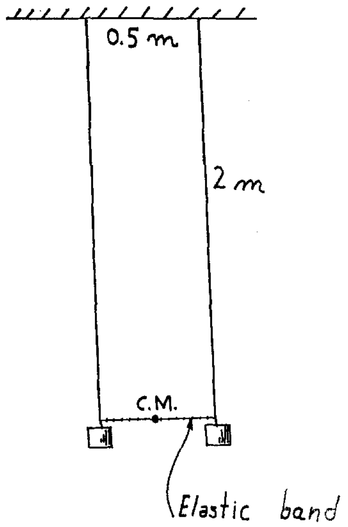

FIG. 1.

in a horizontal plane. Near the masses, the suspending threads are joined by a very compliant elastic band (which doubles in length under 35 g) as shown in Fig. 1. The elastic band has the property of maintaining the ratio of the distances to the two threads as it stretches, and this makes it possible to mark the center of mass as calculated in advance with a bright spot which will trace its trajectory in a photograph made with the lens open.

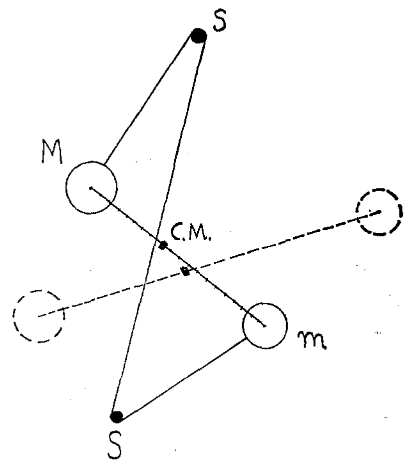

FIG. 3.

In Fig. 2 we see the center of mass fixed although the two equal bodies (each of 1 kg) are moving along individual paths. This is accomplished by first leaving the system at rest so that the center of mass remains quiet. Mark the position of equilibrium with respect to some fixed point,

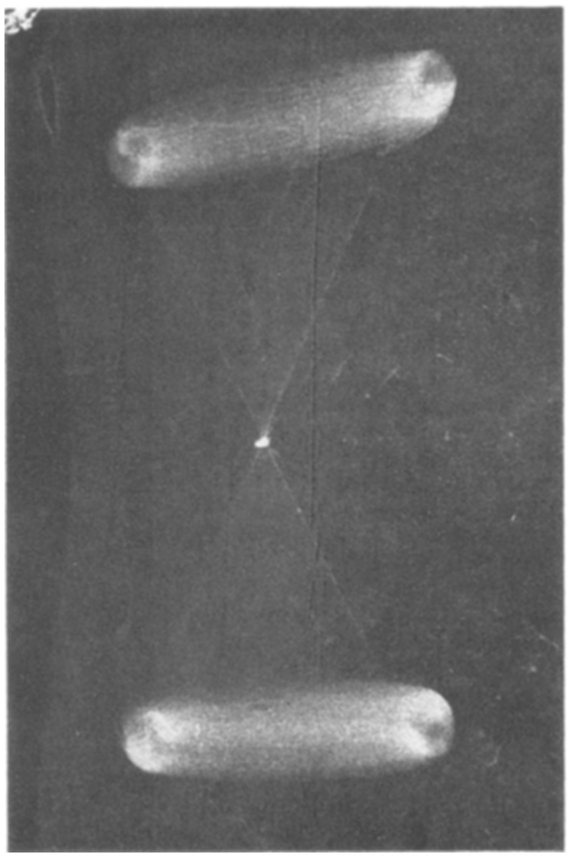

FIG. 2.

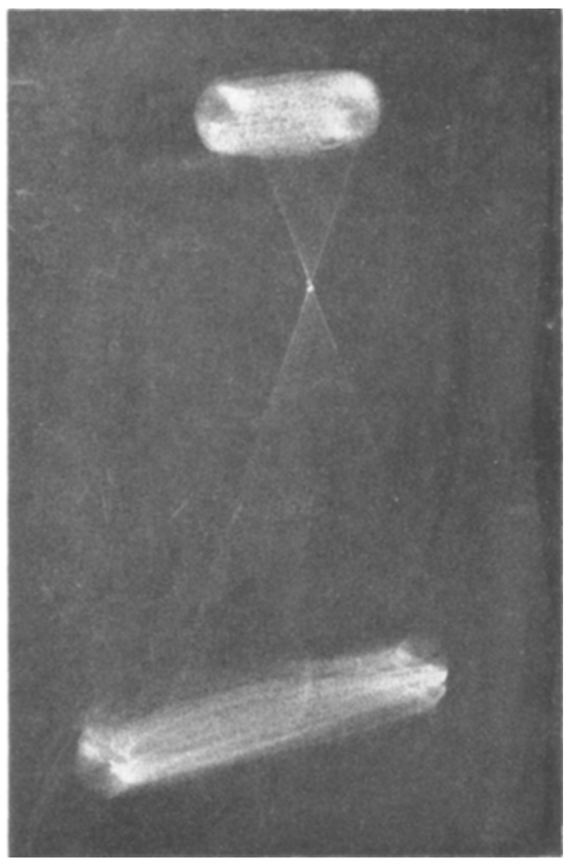

FIG. 4.

FIG. 5.

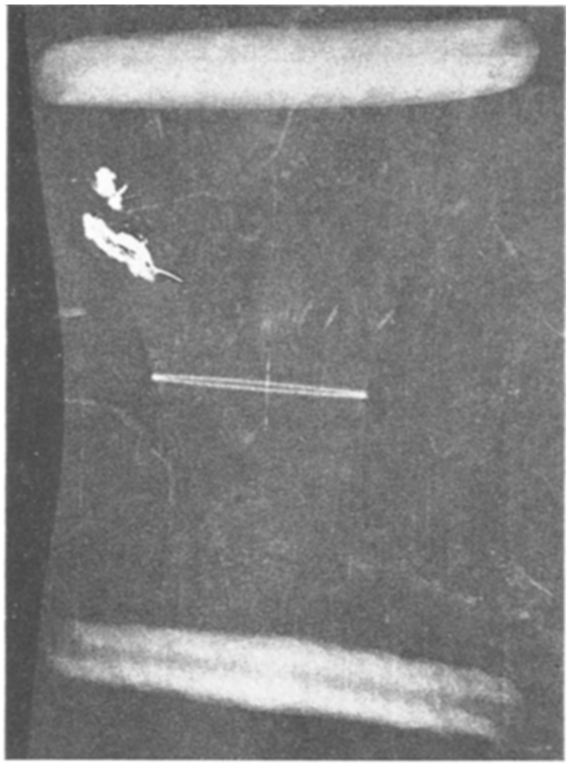

FIG. 6.

FIG. 7.

then take one body in each hand and bring it to such a place that the center of mass remains in the same position as when it was in equilibrium, and release the bodies simultaneously. If there is any difficulty in releasing them simultaneously, fasten the two bodies with a single thread which passes around two supports S. See Fig. 3 which is a plan view of the system ready to be launched. When the thread is burned with a match, the two bodies will begin to move in accordance with the conditions of the system keeping the center of mass in a fixed position. The supports S may be a pair of heavy bodies which can be moved until the system has been located as desired.

Figure 4 differs from Fig. 2 only in having the two bodies differ in mass (1 and $\frac{1}{2}$ kg). By hypothesis, the center of mass is located by the equation $l = mL/(m+M)$, where l is the distance of the center of mass from the body of mass M, L is the total length of the elastic band, and m is the mass of the other body.

Figure 5 shows the ellipse which the center of mass travels as though it were a suspended body actuated as a pendulum in spite of the irregular trajectories of the two bodies which compose the system. Obviously this elliptical trajectory is the easiest to achieve since it is enough to displace one of the bodies and release it to make the center of mass of the system describe an ellipse. If it is too nearly closed, give one of the two bodies small impulses normal to the major axis of the ellipse.

Figures 6 and 7 show closed ellipses, practically straight lines, described by the center of mass. In Fig. 6 the two bodies also move in approximately straight lines, while in Fig. 7 the bodies have irregular paths. To obtain them, it is sufficient to draw one body aside, out of its position of equilibrium, by means of a horizontal thread tied to it. Fasten the thread, quiet the bodies with the band in order to bring them to rest, and burn the thread, so that the center of mass oscillates like a simple pendulum while the bodies follow the required trajectory. The two bodies may also be tied with a single thread as shown in Fig. 3, where the dashed line indicates the equilibrium position.

For class demonstrations when no photographs are desired, it is sufficient to place a reference mark below the center of mass: that is to say to observe the center of mass at rest, set up a vertical wire which will not interfere with the two bodies in motion; for the straight-line trajectory of the center of mass, use as a reference an elastic band stretched between two nails in a board; to observe the ellipses, set down a disc like a coin to give a basis for comparison.

Apparatus for Determining Brewster's Angle

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The determination of Brewster's angle for a transparent solid or a liquid affords a good experimental introduction to polarized light.[1] However, the conventional experimental setup usually involves troublesome adjustments of source and viewing positions to maintain equality of angles of incidence and reflection with consequent waste of the student's time.[2] The apparatus to be described automatically maintains this equality and also simplifies the measurement of the Brewster angle and refractive index of the material used.

A parallelogram frame ABCD of wooden strips approximately $1\frac{1}{2}\times\frac{3}{4}\times23$ in. is made by drilling the strips with holes exactly 50 cm apart and assembling them with 8–32 bolts as shown in Fig. 1. Corners B and D are fastened to collars of square-section brass which are clamped by wing bolts to a 36-in. ring stand. On the lower left member AD of the frame is mounted a small auto lamp (6 V) with the filament G at the principal focus of a short-focus convex lens *H* carried by a brass tube. This provides a parallel beam of light of considerable intensity. On the lower right member CD is mounted a wooden block J which carries a

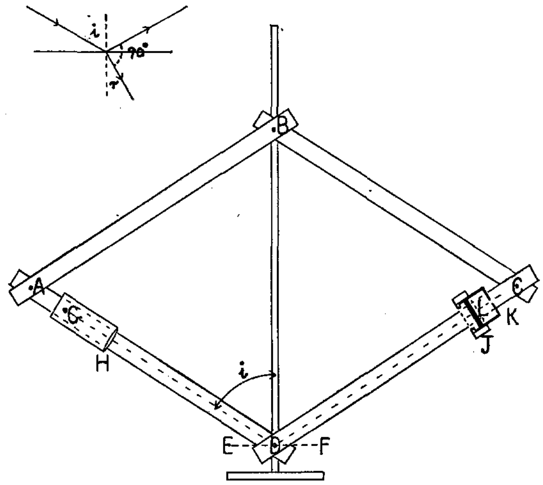

Fig. 1. Apparatus for Brewster's angle.

brass cup K containing a Polaroid disk L. The top of the cup is drilled with a $\frac{1}{4}$-in. sighting hole. Rotation of the cup in the block permits adjustment of the Polaroid to the position of minimum transmitted light.

The sample to be used (e.g., glass slab or liquid in a small beaker) is placed on a piece of black velvet resting on the base of the ringstand and the lower collar is clamped so that the intersection D of the lower frame members is level with the upper reflecting surface EF of the sample. The upper collar is then raised or lowered until, with suitable orientation of the Polaroid, the minimum condition is obtained. The high intensity source permits easy observation of this condition in a normally lighted room.

To obtain the greatest accuracy the positions on either side of the minimum for which transmission is *just* perceptible are found three or four times each and the values for AC are averaged. This procedure usually gives the angle to $\frac{1}{10}$ of a degree and the refractive index to three significant figures. Since the sine of Brewster's angle is $\frac{1}{2}AC/50$ or $AC/100$ the value is readily obtained. The refractive index is then the tangent of Brewster's angle since the reflected ray and refracted ray are at right angles under the Brewster condition.

[1] H. E. White, *Modern College Physics* (D. Van Nostrand Company, New York, 1962), p. 372.
[2] W. A. Schneider and L. B. Ham, *Experimental Physics for Colleges* (The Macmillan Company, New York, 1960), p. 396.

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LETTERS TO THE EDITOR

Behavior of a Helium Balloon in a Car

A HELIUM-FILLED balloon inside an automobile will behave in a manner which is quite unexpected and startling at first sight. If the car accelerates forward, the balloon will move *forward* inside the car. If the car makes a right turn, the balloon will move to the right inside the car, *toward* the center of curvature rather than away from it.

Two students in the advanced mechanics laboratory worked with this effect for three afternoons. They were given no specific instructions at all on what to measure,