Eletromagnetismo

$$\overrightarrow{\nabla} \cdot \overrightarrow{D} = \frac{\rho}{\epsilon_0}$$

$$\overrightarrow{\nabla} \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$$

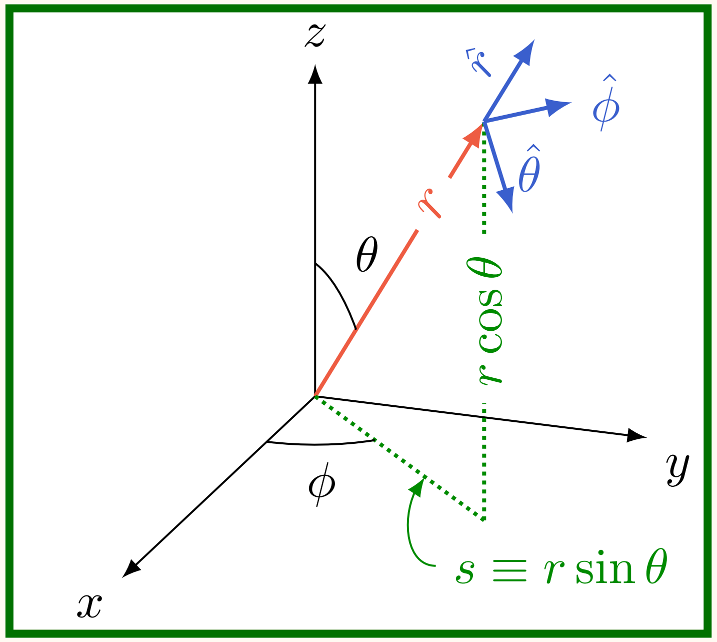
$$\overrightarrow{\nabla} \times \overrightarrow{H} = \vec{j} + \epsilon_0 \frac{\partial \overrightarrow{D}}{\partial t}$$

Aula de 19 de maio

Eletrostática

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$



$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

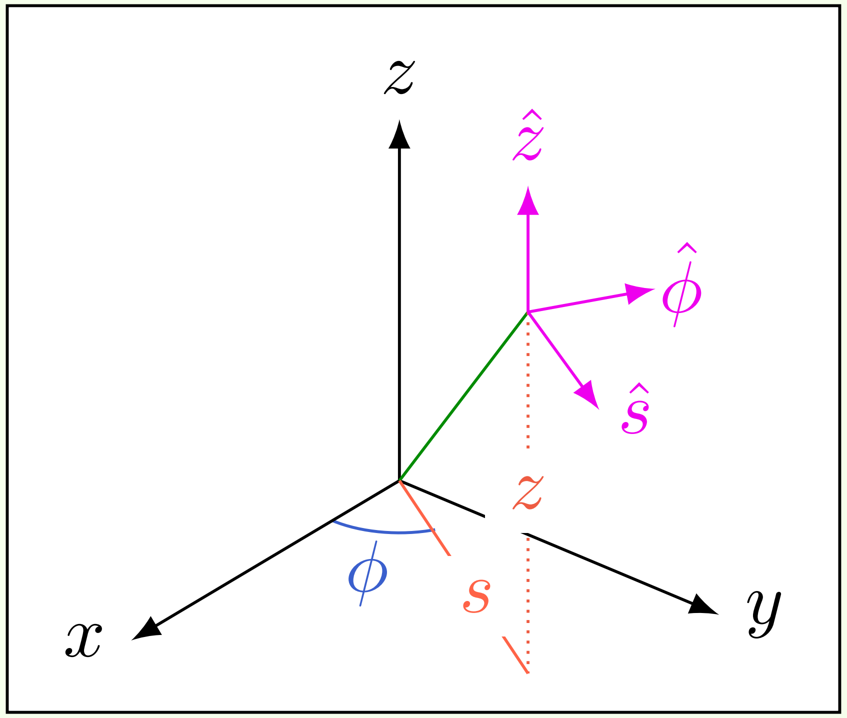
$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Coordenadas cilíndricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$



$$\vec{\boldsymbol{\nabla}} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{v} = \frac{1}{s}\frac{\partial(s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

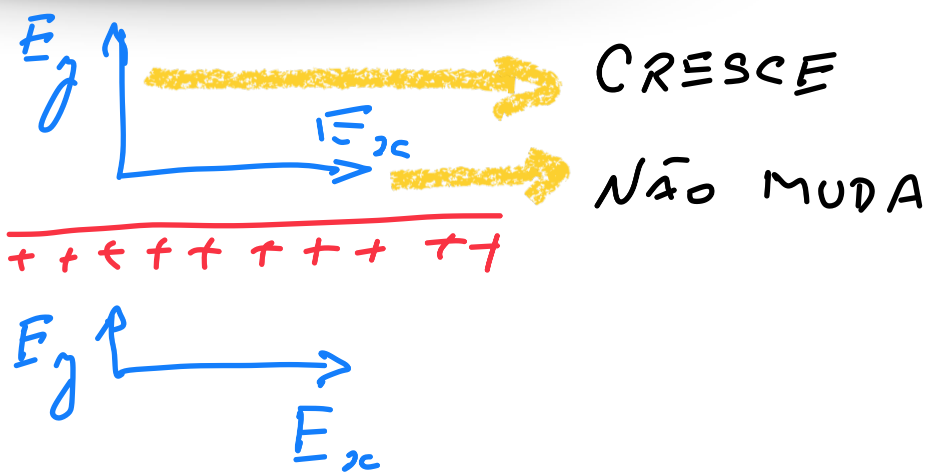
$$\vec{\boldsymbol{\nabla}} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial(s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

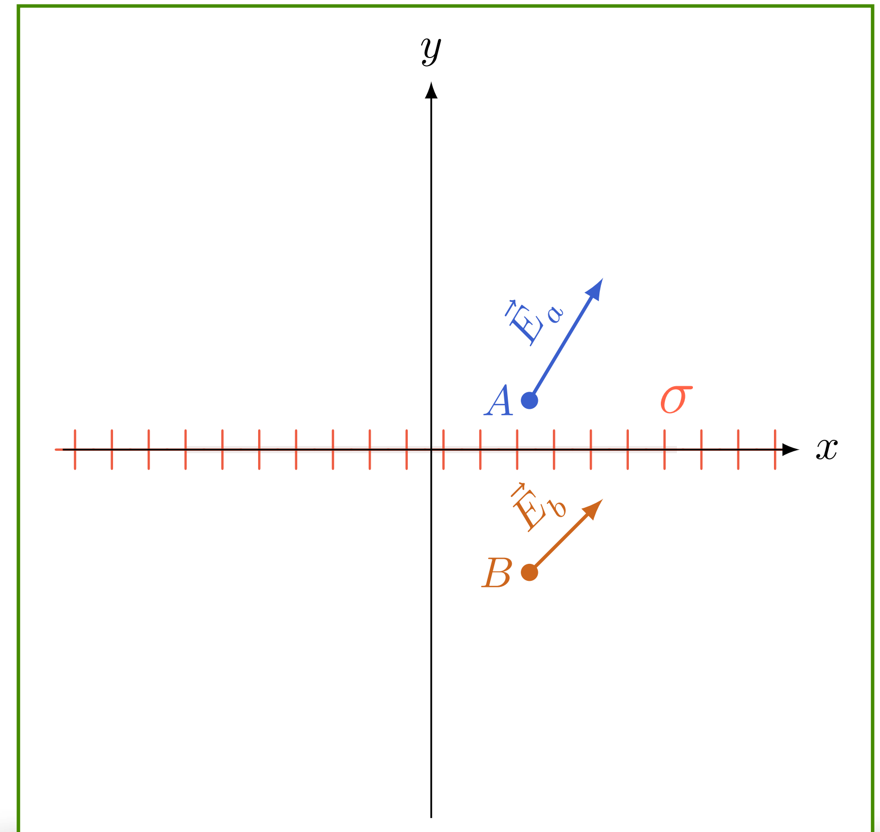
$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Condições de contorno

Descontinuidade do campo elétrico

$$\boxed{\Delta\vec{E}\cdot\hat{n} = \frac{\sigma}{\epsilon_0}}$$

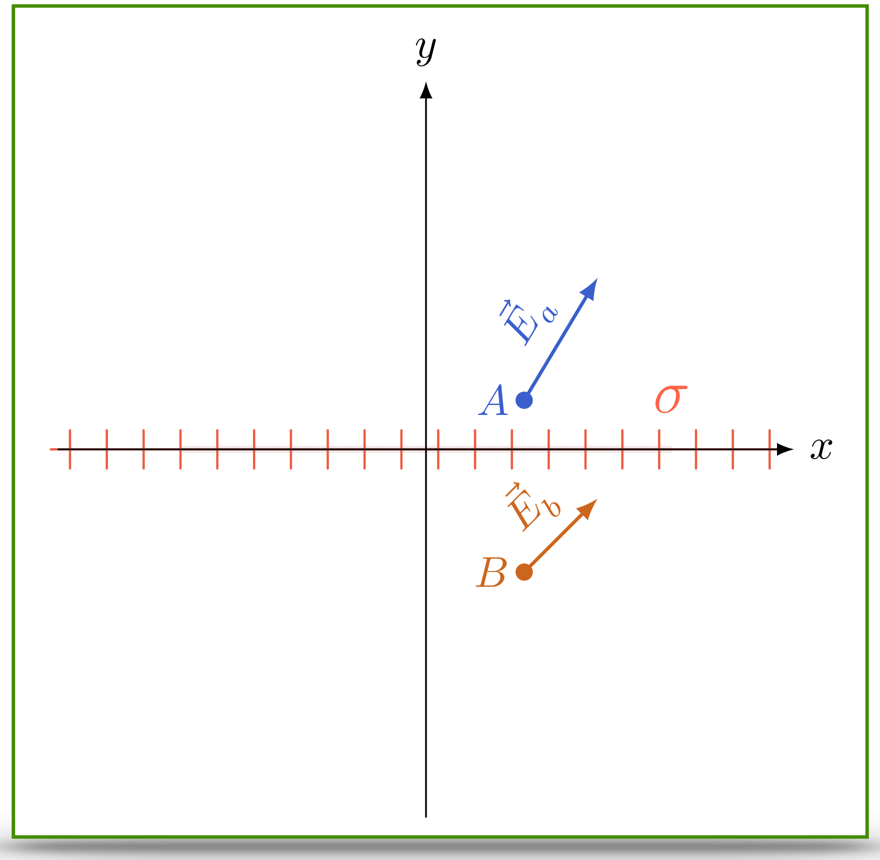




Condições de contorno

Descontinuidade do campo elétrico

$$\Delta\vec{E}\cdot\hat{n} = \frac{\sigma}{\epsilon_0}$$



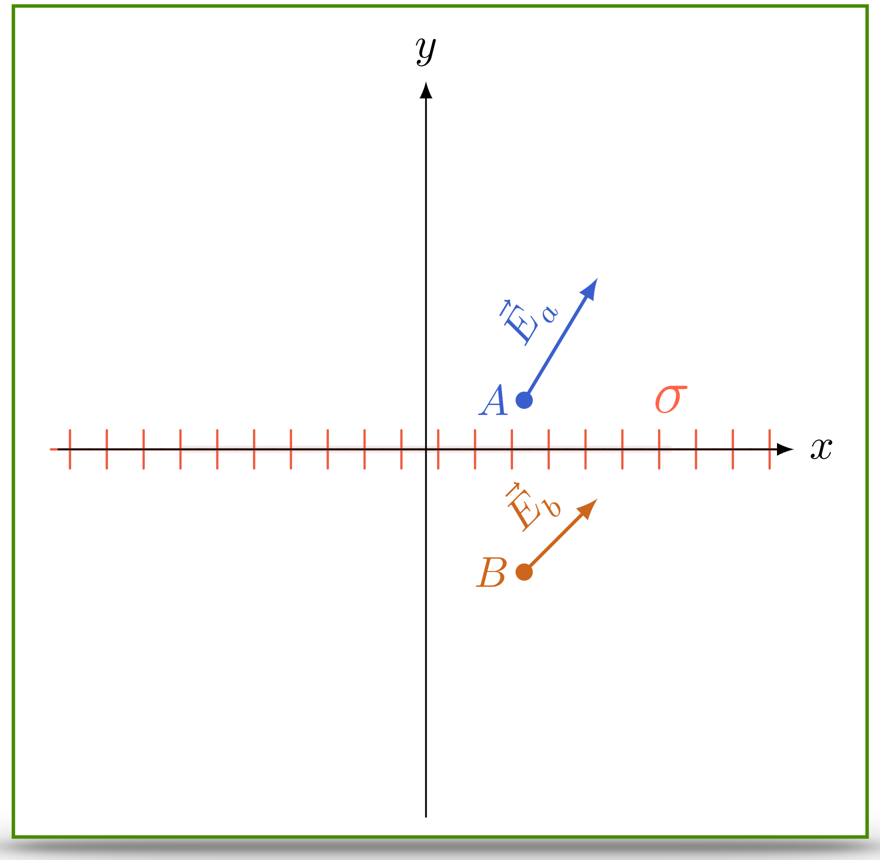
$$\Delta V = \int_A^B \vec{E}\cdot d\vec{\ell} \rightarrow \boxed{A \to B \Rightarrow \Delta V \to 0}$$

V É CONTÍNUO

Condições de contorno

Descontinuidade do campo elétrico

$$\boxed{\Delta\vec{E}\cdot\hat{n} = \frac{\sigma}{\epsilon_0}}$$

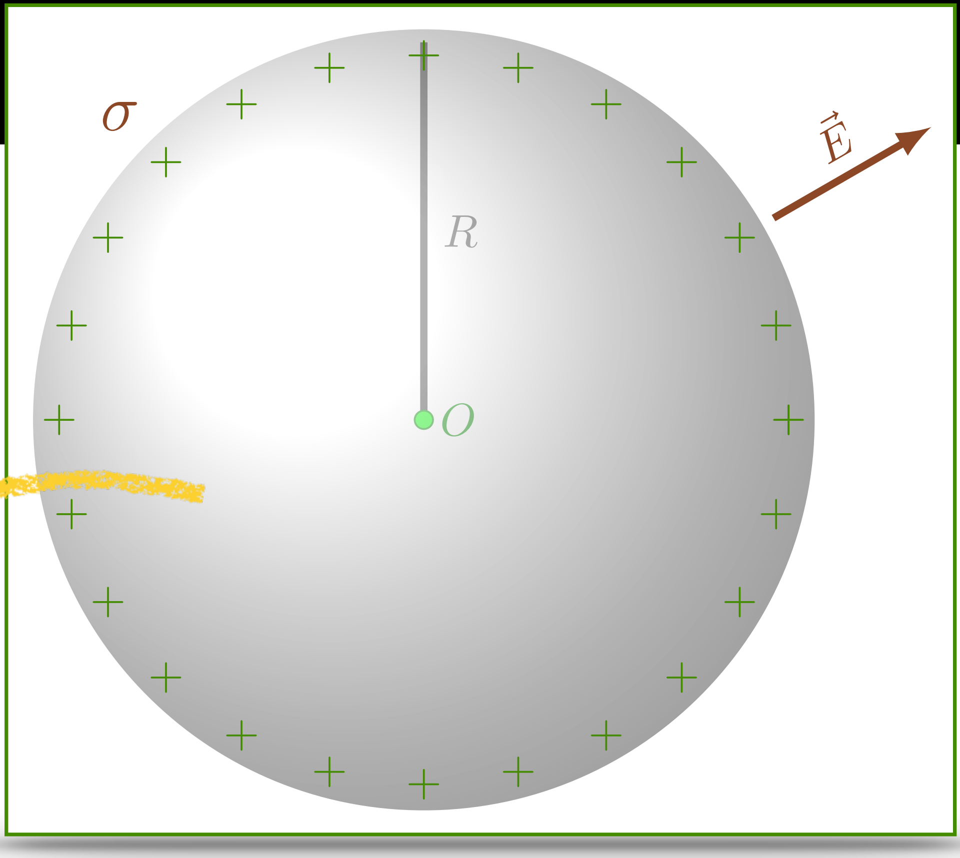


$$\Delta V = \int \vec{E}\cdot\mathrm{d}\vec{\ell}$$

$$\Delta V = \int \vec{E}\cdot\mathrm{d}\vec{\ell} \to 0$$

$$\Delta \vec{E} \cdot \hat{n} = \frac{\sigma}{\epsilon_0}$$

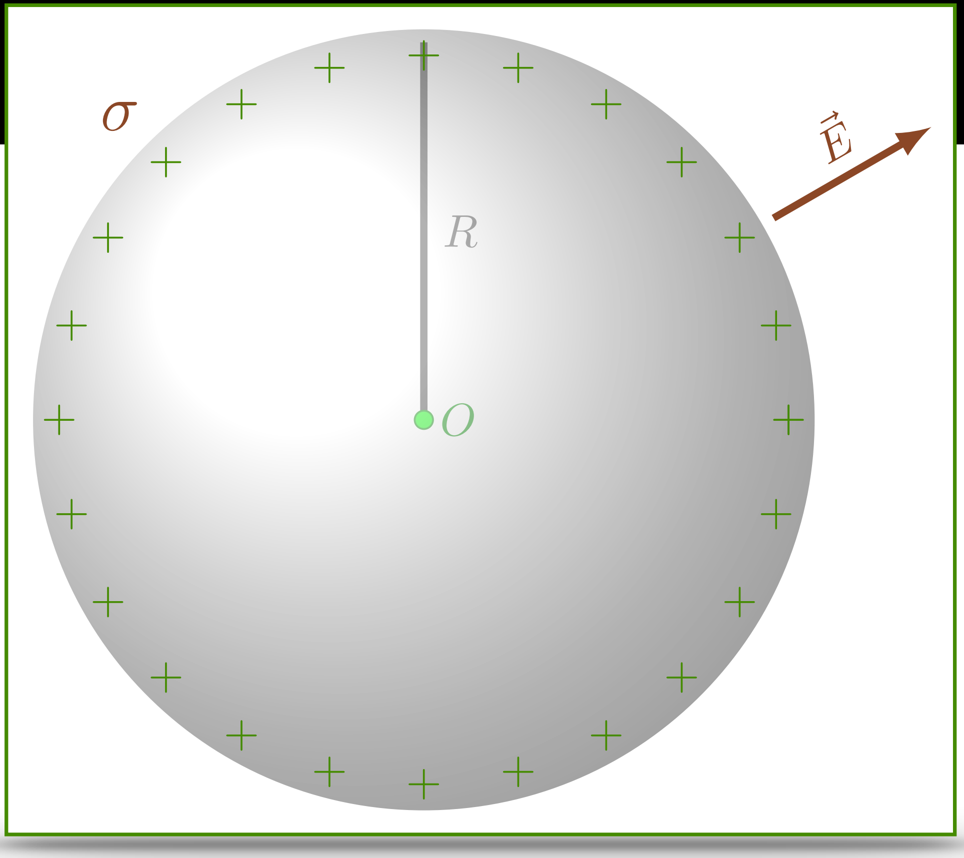
Pratique o que aprendeu



$\vec{E} = 0$
NO INTERIOR

$$\Delta \vec{E} \cdot \hat{n} = \frac{\sigma}{\epsilon_0}$$

Pratique o que aprendeu

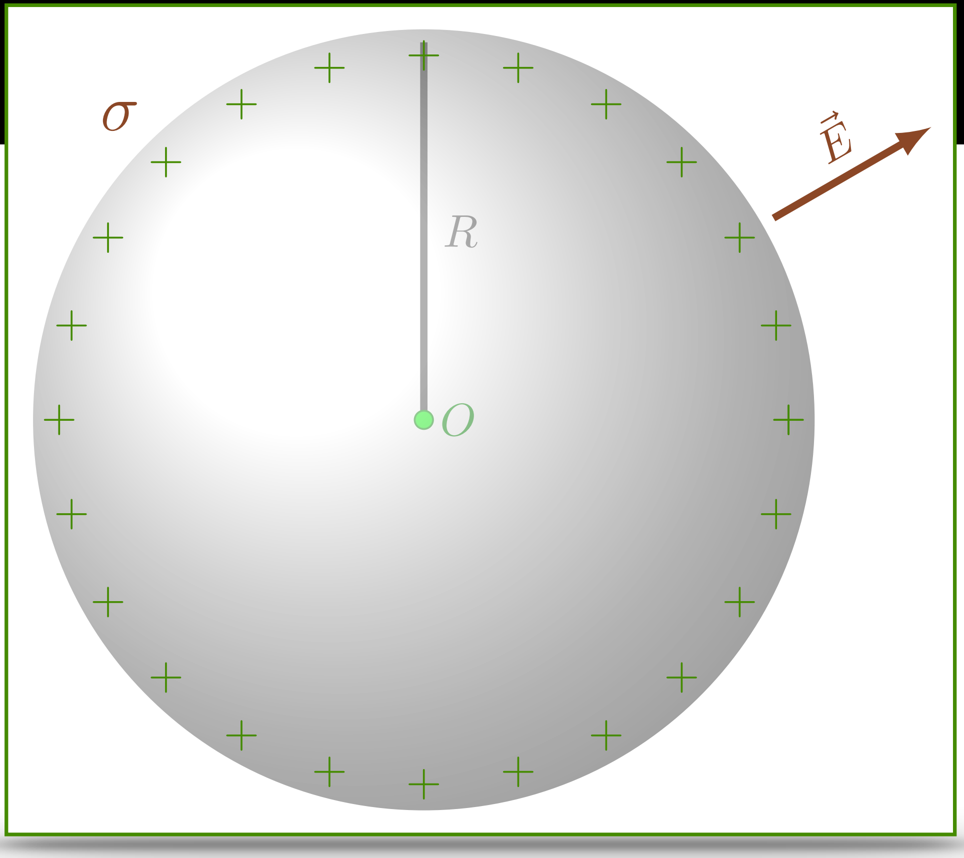


$$\vec{E} = E\hat{r}$$

↳ POR SIMETRIA

$$\Delta\vec{E}\cdot\hat{n} = \frac{\sigma}{\epsilon_0}$$

Pratique o que aprendeu



$$\vec{E} = E\hat{r}$$

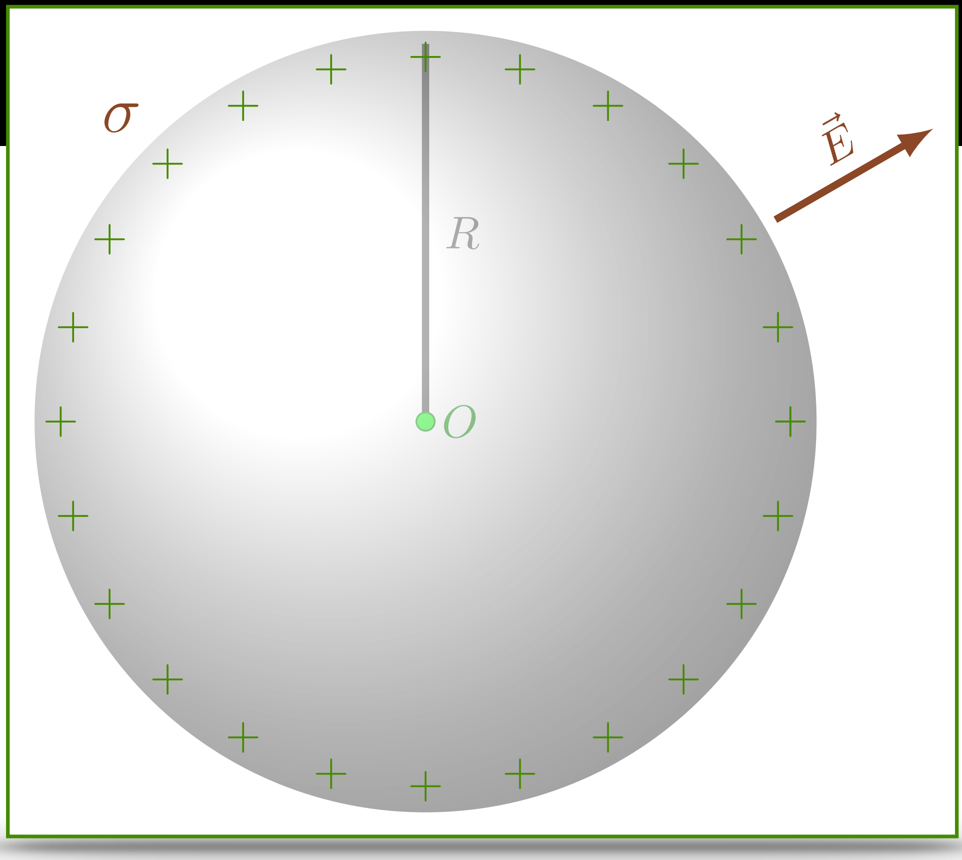
$$\Delta\vec{E}\cdot\hat{n} = E_{FORA} - E_{DENTRO}$$

$$E \quad - \quad 0$$

$$\Delta\vec{E}\cdot\hat{n} = E$$

$$\Delta\vec{E}\cdot\hat{n} = \frac{\sigma}{\epsilon_0}$$

Pratique o que aprendeu



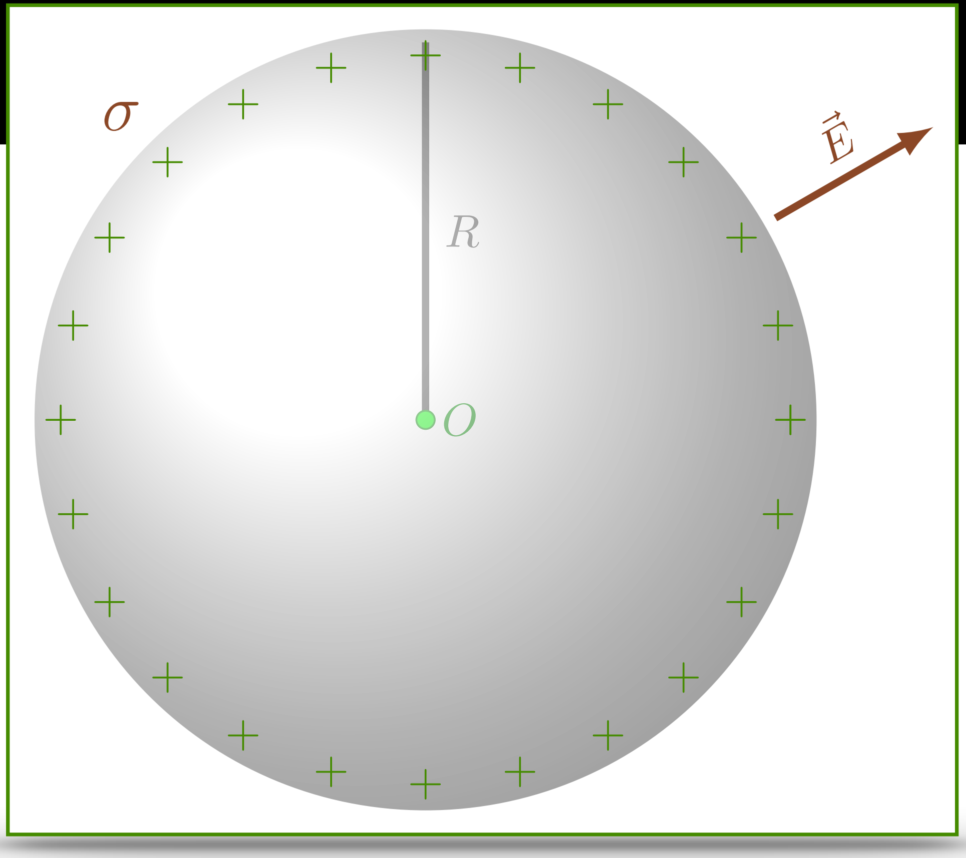
$$\vec{E} = E\hat{r}$$

$$\Delta\vec{E}\cdot\hat{n} = E$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$\Delta\vec{E}\cdot\hat{n} = \frac{\sigma}{\epsilon_0}$$

Pratique o que aprendeu



$$\vec{E} = E\hat{r}$$

$$\Delta\vec{E}\cdot\hat{n} = E$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$\sigma = \frac{q}{\text{ÁREA}} = \frac{q}{4\pi R^2}$$

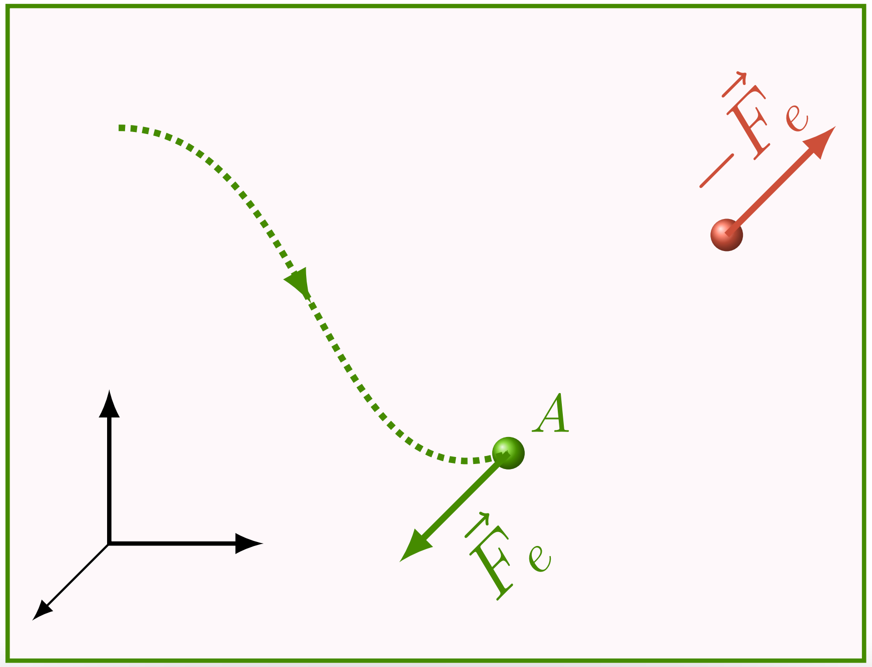
$$E = \frac{q}{4\pi\epsilon_0 R^2}$$

Trabalho e energia

$$W = \int_{\mathcal{O}}^{A} \vec{F} \cdot \mathrm{d}\vec{\ell}$$

FORÇA EXTERNA, IGUAL E CONTRÁRIA À FORÇA ELÉTRICA

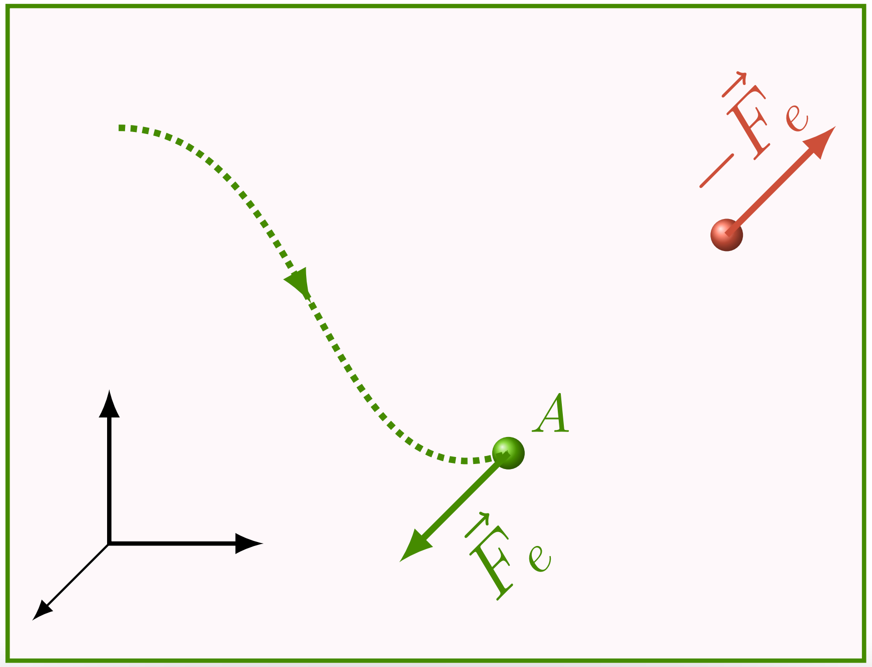
RESULTANTE = 0



Trabalho e energia

$$W = \int_{\mathcal{O}}^{A} \vec{F} \cdot \mathrm{d}\vec{\ell}$$

$$\vec{F} = -\vec{F}_e = -q\vec{E}$$

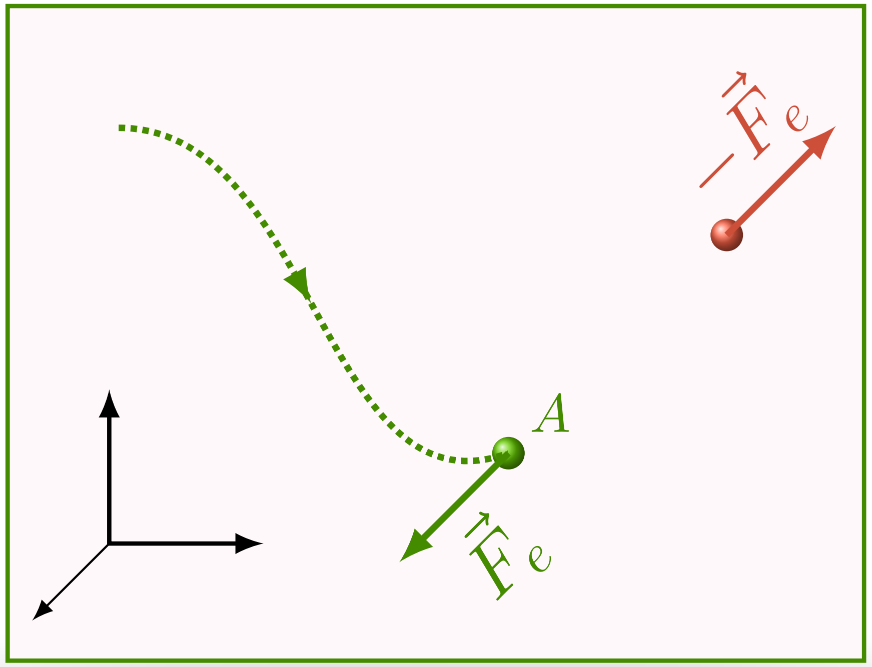


Trabalho e energia

$$W = \int_{\mathcal{O}}^{A} \vec{F} \cdot \mathrm{d}\vec{\ell}$$

$$\vec{F} = -\vec{F}_e = -q\vec{E} \Rightarrow W = -q \int_{0}^{A} \vec{E} \cdot d\vec{\ell}$$

$$W = qV(A) \leftarrow V(A) = -\int_{0}^{A} \vec{E} \cdot d\vec{\ell}$$



Trabalho e energia

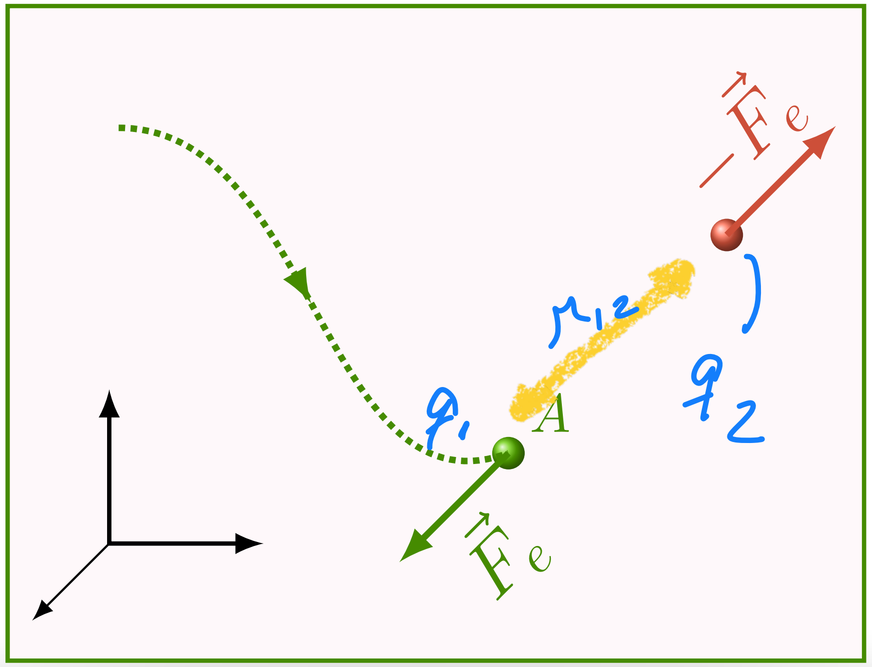
$$W = \int_{\mathcal{O}}^{A} \vec{F} \cdot \mathrm{d}\vec{\ell}$$

$$\vec{F} = -\vec{F}_e = -q\vec{E}$$

$$W = qV(A)$$

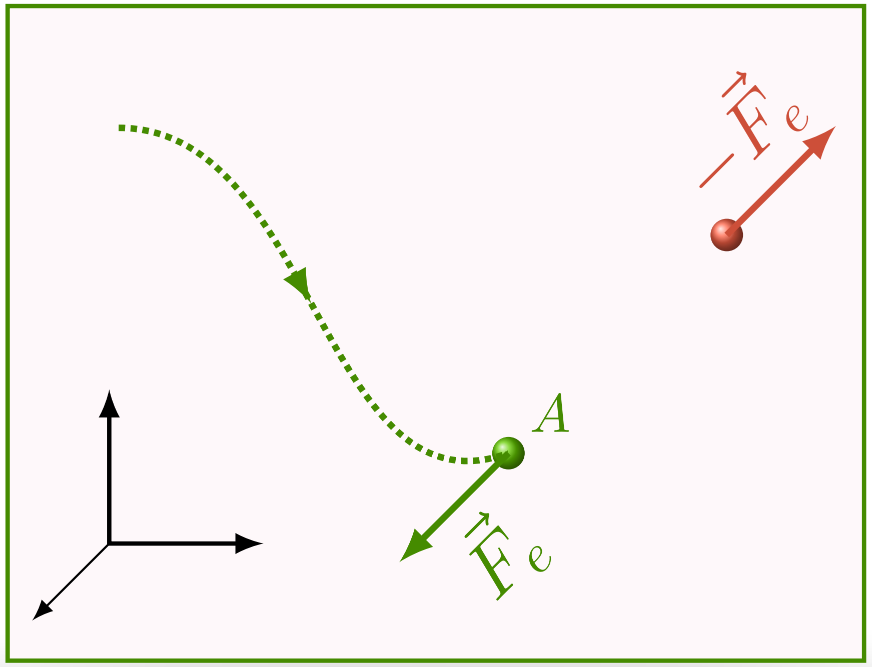
$$W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$$

ENERGIA POTENCIAL



Trabalho e energia

$$W = \int_{\mathcal{O}}^{A} \vec{F} \cdot \mathrm{d}\vec{\ell}$$



$$\vec{F} = -\vec{F}_e = -q\vec{E}$$

$$W = qV(A)$$

$$W = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$$

N cargas

$$W = \frac{1}{4\pi\epsilon_0}\sum_i \sum_{j>i} \frac{q_i q_j}{r_{ij}}$$

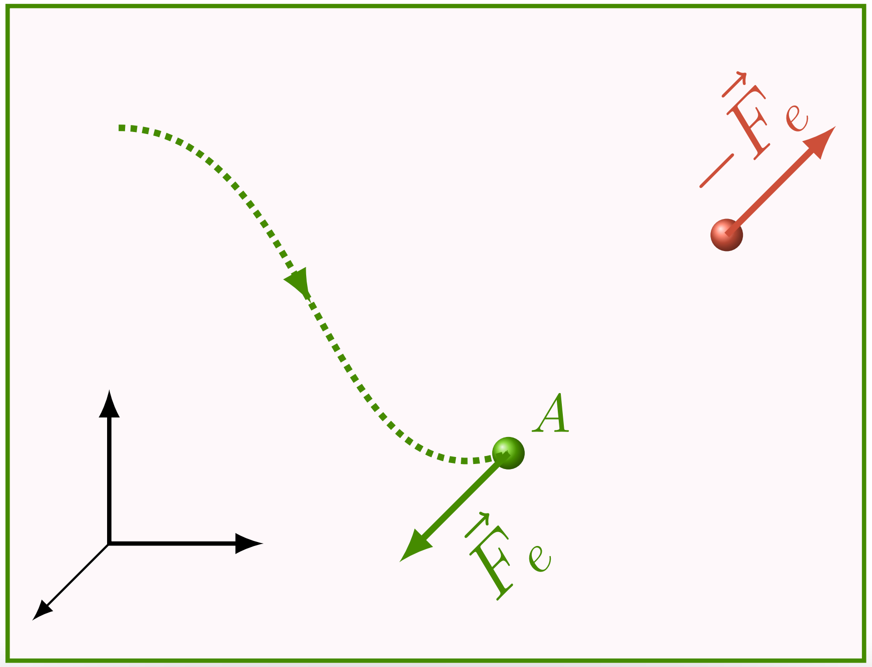
$j > i$

PARA NÃO CONTAR DUAS VEZES CADA PAR

Trabalho e energia

$$W = \int_{\mathcal{O}}^{A} \vec{F} \cdot \mathrm{d}\vec{\ell}$$

$$\vec{F} = -\vec{F}_e = -q\vec{E}$$



$$W = qV(A)$$

$$W = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\mathscr{r}_{12}}$$

N cargas

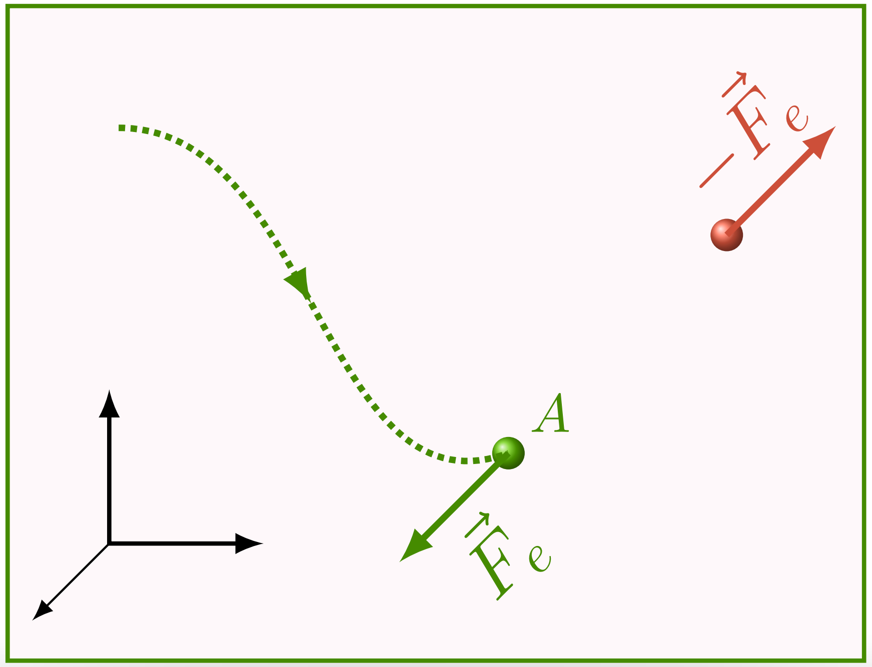
$$W = \frac{1}{4\pi\epsilon_0}\sum_i \sum_{j>i} \frac{q_i q_j}{\mathscr{r}_{ij}}$$

MAIS FÁCIL

CONTAR 2 VEZES
E DIVIDIR POR 2

$$W = \frac{1}{8\pi\epsilon_0}\sum_i \sum_j \frac{q_i q_j}{\mathscr{r}_{ij}}$$

Trabalho e energia

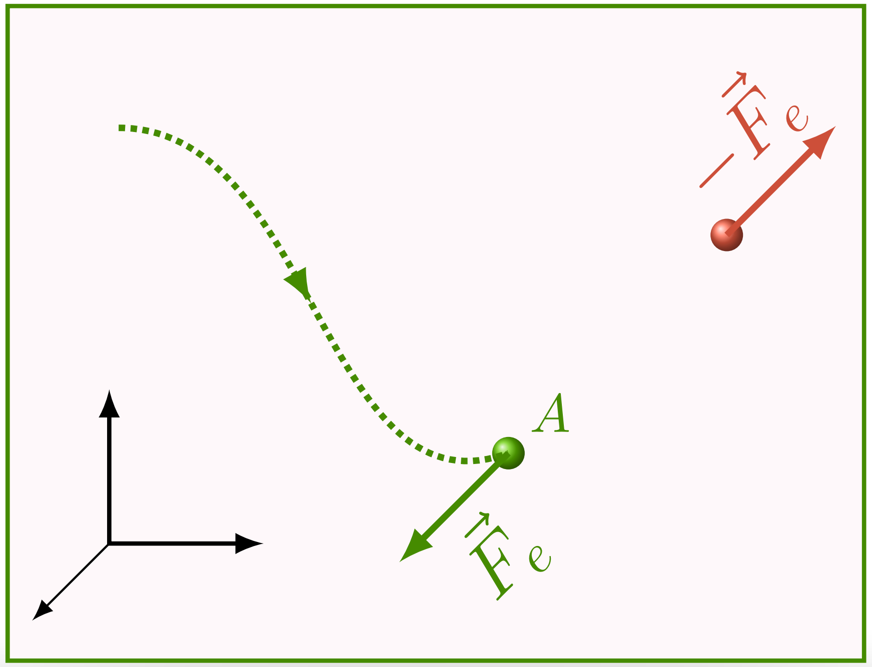


N cargas

$$W = \frac{1}{8\pi\epsilon_0}\sum_i \sum_j \frac{q_i q_j}{\mathscr{r}_{ij}}$$

Trabalho e energia

$$W = \frac{1}{8\pi\epsilon_0}\int\int \frac{\rho(\vec{r})\rho(\vec{r'})}{\mathscr{r}}\,\mathrm{d}\vec{r}\,\mathrm{d}\vec{r'}$$



N cargas

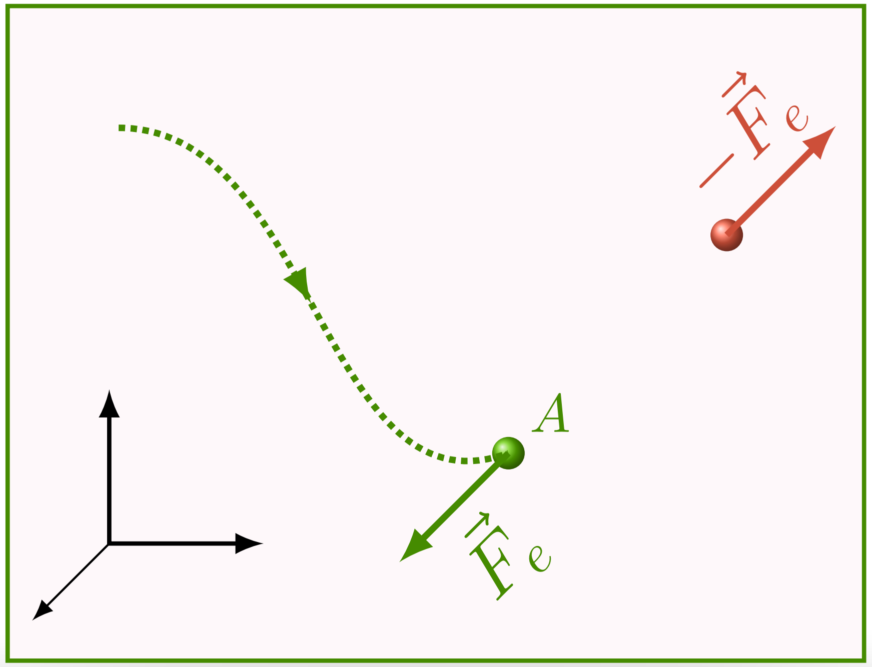
$$W = \frac{1}{8\pi\epsilon_0}\sum_i\sum_j \frac{q_i q_j}{\mathscr{r}_{ij}}$$

Trabalho e energia

$$W = \frac{1}{8\pi\epsilon_0}\int\int\frac{\rho(\vec{r})\rho(\vec{r'})}{r}\,\mathrm{d}\vec{r}\,\mathrm{d}\vec{r'}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r}')}{r}\,d\vec{r}'$$

$$W = \frac{1}{2}\int\rho(\vec{r})V(\vec{r})\,\mathrm{d}\vec{r}$$



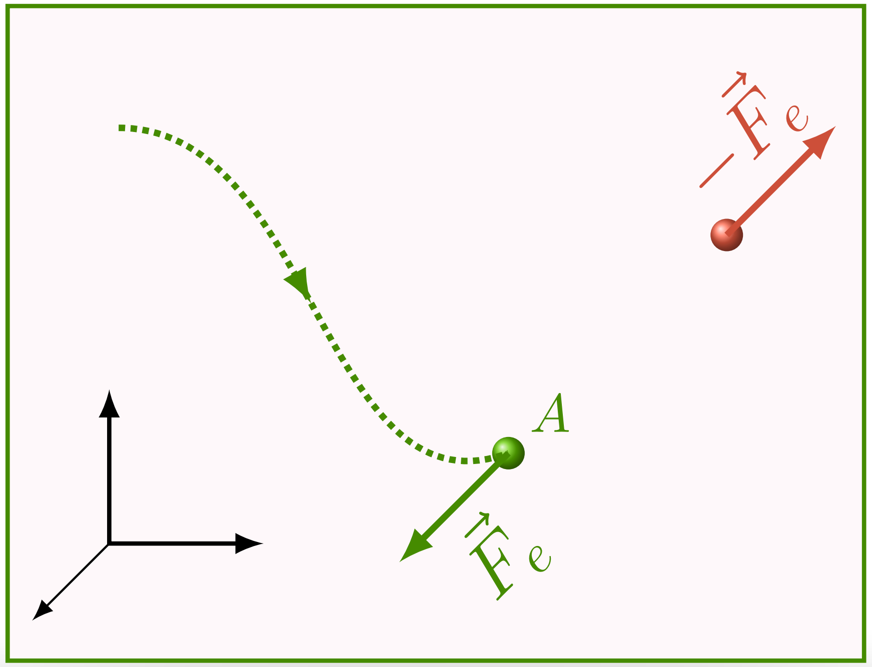
Trabalho e energia

$$W = \frac{1}{8\pi\epsilon_0}\int\int \frac{\rho(\vec{r})\rho(\vec{r'})}{r}\,\mathrm{d}\vec{r}\,\mathrm{d}\vec{r'}$$

$$W = \frac{1}{2}\int \rho(\vec{r})V(\vec{r})\,\mathrm{d}\vec{r}$$

$$\frac{\rho}{\epsilon_0} = \vec{\nabla}\cdot\vec{E}$$

$$W = \frac{\epsilon_0}{2}\int\left(\vec{\boldsymbol{\nabla}}\cdot\vec{E}\right)V\,\mathrm{d}\vec{r}$$

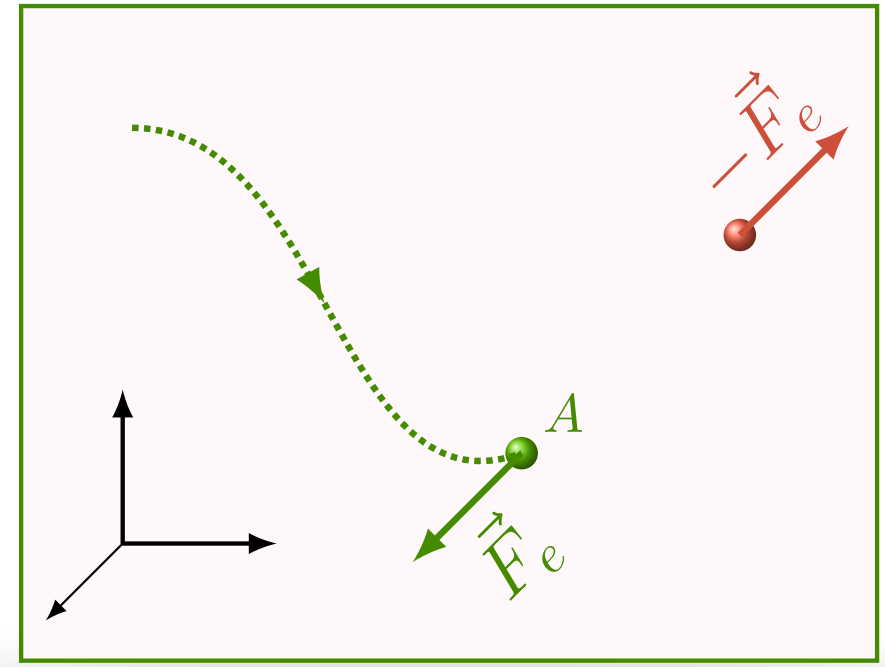


Trabalho e energia

$$W = \frac{1}{8\pi\epsilon_0}\int\int\frac{\rho(\vec{r})\rho(\vec{r'})}{\mathscr{r}}\,\mathrm{d}\tau'\,\mathrm{d}\tau$$

$$W = \frac{1}{2}\int\rho(\vec{r})V(\vec{r})\,\mathrm{d}\tau$$

$$W = \frac{\epsilon_0}{2}\int\left(\vec{\boldsymbol{\nabla}}\cdot\vec{E}\right)V\,\mathrm{d}\tau$$

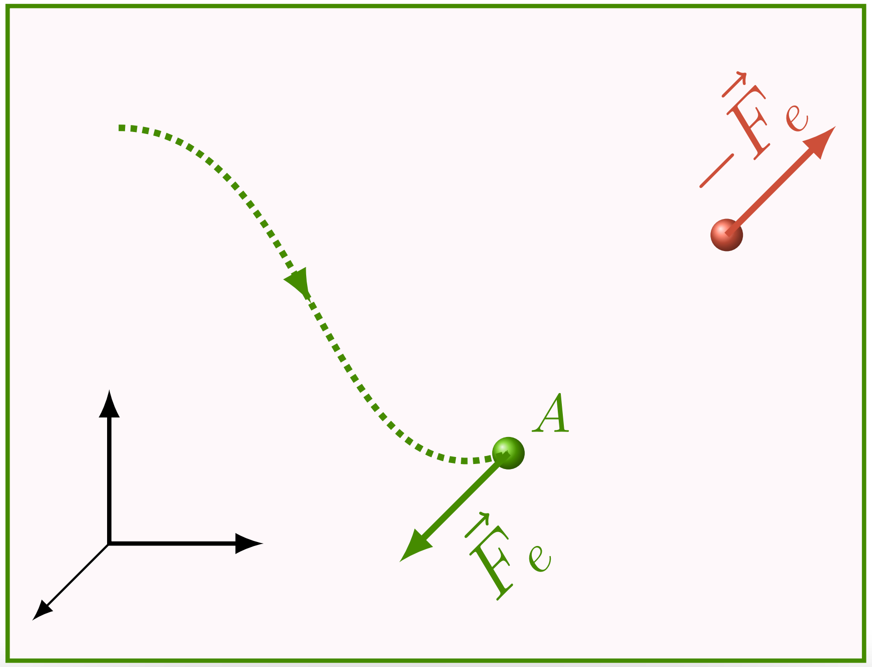


Trabalho e energia

$$W = \frac{1}{8\pi\epsilon_0}\int\int\frac{\rho(\vec{r})\rho(\vec{r'})}{\mathscr{r}}\,\mathrm{d}\tau'\,\mathrm{d}\tau$$

$$W = \frac{1}{2}\int\rho(\vec{r})V(\vec{r})\,\mathrm{d}\tau$$

$$W = \frac{\epsilon_0}{2}\int\left(\vec{\nabla}\cdot\vec{E}\right)V\,\mathrm{d}\tau$$



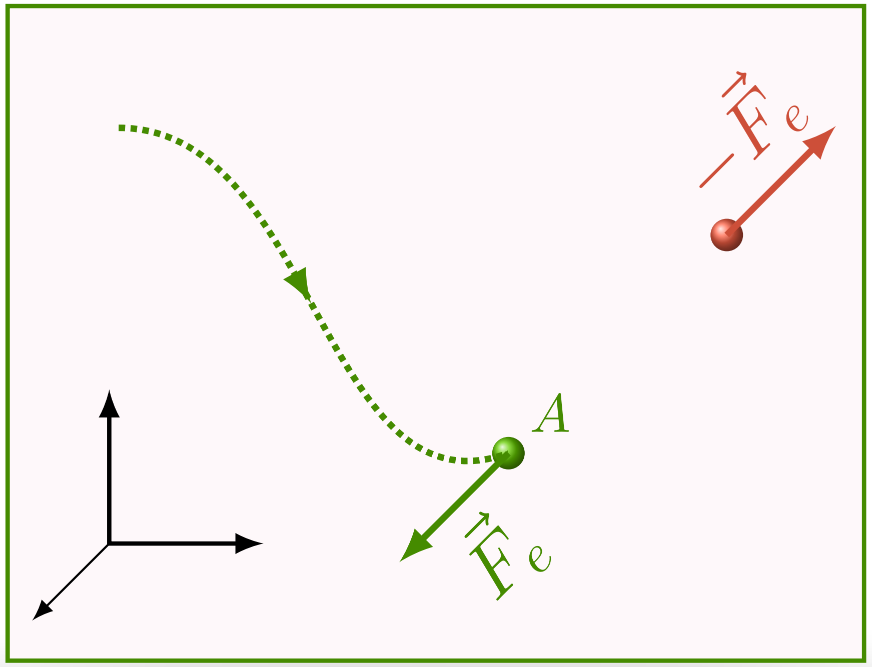
$$\vec{\nabla}\cdot(V\vec{E}) = \vec{\nabla}V\cdot\vec{E} + V\vec{\nabla}\cdot\vec{E}$$

Trabalho e energia

$$W = \frac{1}{8\pi\epsilon_0}\int\int \frac{\rho(\vec{r})\rho(\vec{r'})}{\mathscr{r}}\,\mathrm{d}\tau'\,\mathrm{d}\tau$$

$$W = \frac{1}{2}\int \rho(\vec{r})V(\vec{r})\,\mathrm{d}\tau$$

$$W = \frac{\epsilon_0}{2}\int \left(\vec{\boldsymbol{\nabla}}\cdot\vec{E}\right)V\,\mathrm{d}\tau$$



$$\vec{\boldsymbol{\nabla}}\cdot(V\vec{E}) = \vec{\boldsymbol{\nabla}}V\cdot\vec{E} + V\vec{\boldsymbol{\nabla}}\cdot\vec{E}$$

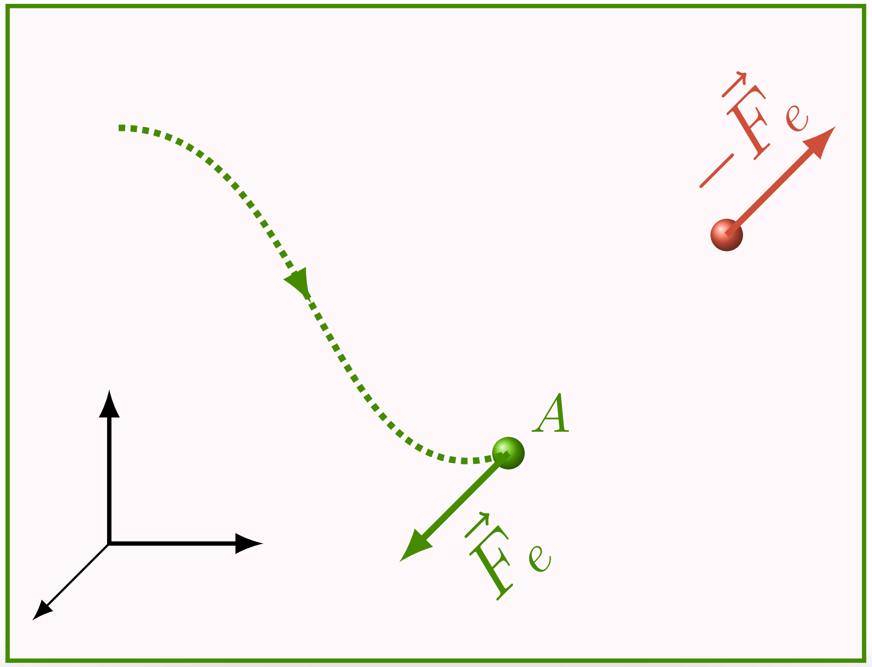
$$\int \vec{\boldsymbol{\nabla}}\cdot(V\vec{E})\,\mathrm{d}\tau = \int\left(\vec{\boldsymbol{\nabla}}V\cdot\vec{E}\right)\mathrm{d}\tau + \int\left(V\vec{\boldsymbol{\nabla}}\cdot\vec{E}\right)\mathrm{d}\tau$$

Trabalho e energia

$$W = \frac{1}{8\pi\epsilon_0}\int\int\frac{\rho(\vec{r})\rho(\vec{r'})}{\mathscr{r}}\,\mathrm{d}\tau'\,\mathrm{d}\tau$$

$$W = \frac{1}{2}\int\rho(\vec{r})V(\vec{r})\,\mathrm{d}\tau$$

$$W = \frac{\epsilon_0}{2}\int\left(\vec{\boldsymbol{\nabla}}\cdot\vec{E}\right)V\,\mathrm{d}\tau$$



$$\vec{\boldsymbol{\nabla}}\cdot(V\vec{E}) = \vec{\boldsymbol{\nabla}}V\cdot\vec{E} + V\vec{\boldsymbol{\nabla}}\cdot\vec{E}$$

$$\int\vec{\boldsymbol{\nabla}}\cdot(V\vec{E})\,\mathrm{d}\tau = \int\left(\vec{\boldsymbol{\nabla}}V\cdot\vec{E}\right)\mathrm{d}\tau + \int\left(V\vec{\boldsymbol{\nabla}}\cdot\vec{E}\right)\mathrm{d}\tau$$

$\vec{E} = -\vec{\nabla}V$

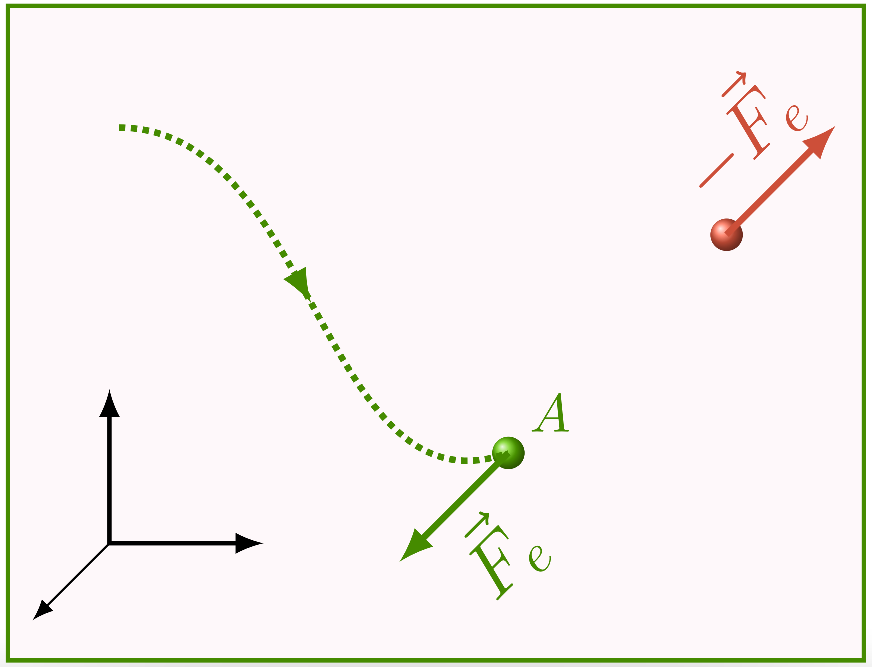
$$\int V\vec{E}\cdot\hat{n}\,\mathrm{d}A = -\int\left(\vec{E}\cdot\vec{E}\right)\mathrm{d}\tau + \int\left(V\vec{\boldsymbol{\nabla}}\cdot\vec{E}\right)\mathrm{d}\tau$$

Trabalho e energia

$$W = \frac{1}{8\pi\epsilon_0}\int\int\frac{\rho(\vec{r})\rho(\vec{r'})}{\mathscr{r}}\,\mathrm{d}\tau'\,\mathrm{d}\tau$$

$$W = \frac{1}{2}\int\rho(\vec{r})V(\vec{r})\,\mathrm{d}\tau$$

$$W = \frac{\epsilon_0}{2}\int\left(\vec{\nabla}\cdot\vec{E}\right)V\,\mathrm{d}\tau$$



$$W = \frac{\epsilon_0}{2}\int V\vec{E}\cdot\hat{n}\,\mathrm{d}A + \frac{\epsilon_0}{2}\int E^2\,\mathrm{d}\tau$$

$$\vec{\nabla}\cdot(V\vec{E}) = \vec{\nabla}V\cdot\vec{E} + V\vec{\nabla}\cdot\vec{E}$$

$$\int\vec{\nabla}\cdot(V\vec{E})\,\mathrm{d}\tau = \int\left(\vec{\nabla}V\cdot\vec{E}\right)\mathrm{d}\tau + \int\left(V\vec{\nabla}\cdot\vec{E}\right)\mathrm{d}\tau$$

GAUSS

$$\int V\vec{E}\cdot\hat{n}\,\mathrm{d}A = -\int\left(\vec{E}\cdot\vec{E}\right)\mathrm{d}\tau + \int\left(V\vec{\nabla}\cdot\vec{E}\right)\mathrm{d}\tau$$

SUPERFÍCIE
E
VOLUME $\rightarrow \infty$

$\int V\vec{E}\cdot\hat{n}\,dA \rightarrow 0$

$\hookrightarrow 1/R$ $\quad 1/R^2$

$\int 1/R^3\,dA \rightarrow 0$

Trabalho e energia

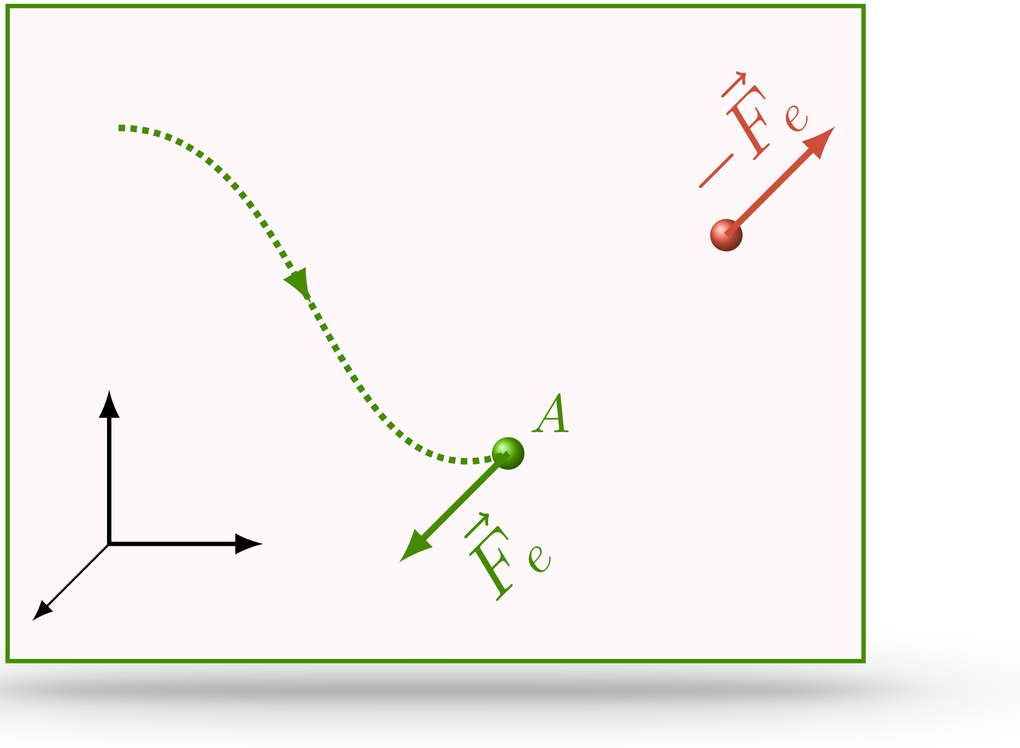
$$W = \frac{1}{8\pi\epsilon_0} \int\int \frac{\rho(\vec{r})\rho(\vec{r'})}{\mathscr{r}}\, \mathrm{d}\tau'\, \mathrm{d}\tau$$

$$W = \frac{1}{2} \int \rho(\vec{r}) V(\vec{r})\, \mathrm{d}\tau$$

$$W = \frac{\epsilon_0}{2} \int \left(\vec{\nabla} \cdot \vec{E}\right) V\, \mathrm{d}\tau$$

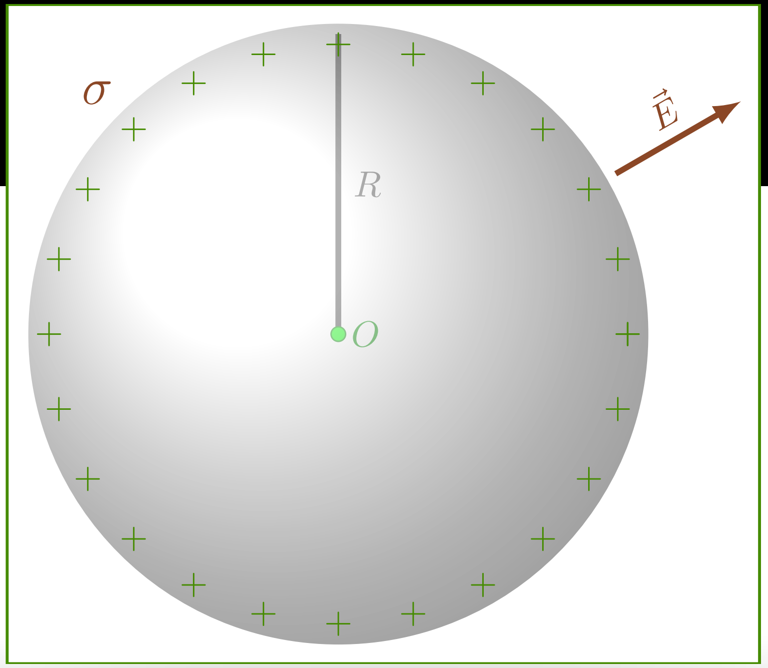
$$W = \frac{\epsilon_0}{2} \int V \vec{E} \cdot \hat{n}\, \mathrm{d}A + \frac{\epsilon_0}{2} \int E^2\, \mathrm{d}\tau$$

$$\boxed{W = \frac{\epsilon_0}{2} \int E^2\, \mathrm{d}\tau}$$



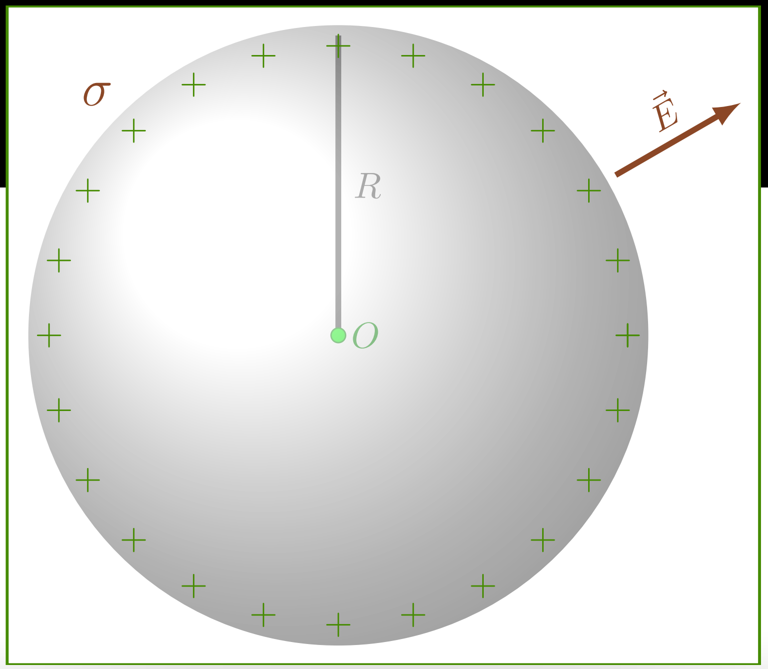
Pratique o que aprendeu

$$W = \frac{\epsilon_0}{2} \int E^2 \, \mathrm{d}\tau$$



Pratique o que aprendeu

$$W = \frac{\epsilon_0}{2} \int E^2 \, \mathrm{d}\tau$$



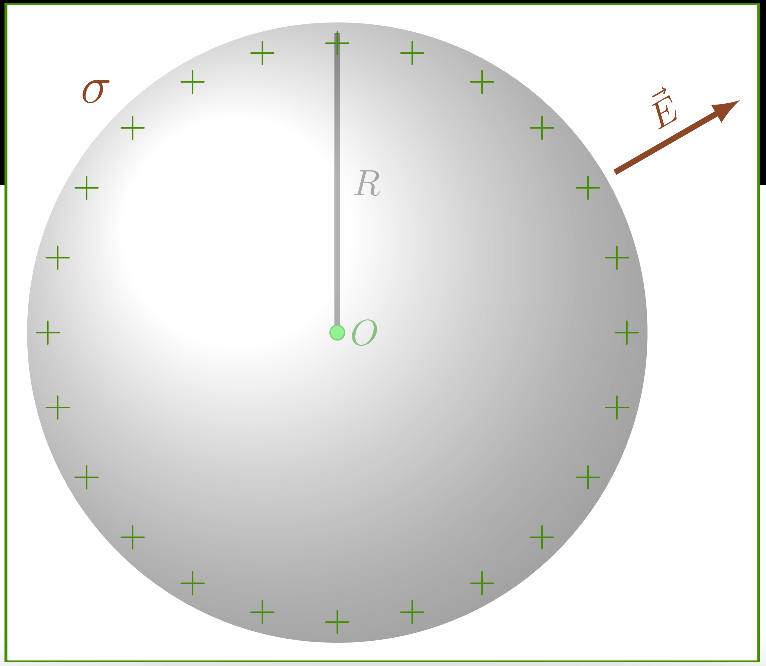
$$W = \frac{\epsilon_0}{2} \int \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \right)^2 \mathrm{d}\tau$$

FORA DA ESFERA

(DENTRO O CAMPO E' ZERO)

Pratique o que aprendeu

$$W = \frac{\epsilon_0}{2} \int E^2 \, \mathrm{d}\tau$$



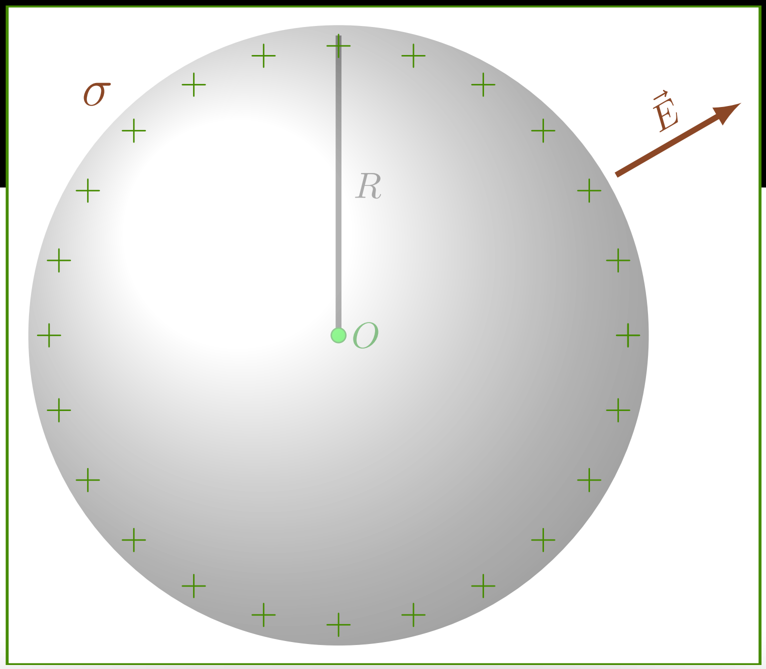
$$W = \frac{\epsilon_0}{2} \int \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \right)^2 \mathrm{d}\tau$$

$$W = \frac{\epsilon_0}{2} \frac{q^2}{(4\pi\epsilon_0)^2} \int_0^{2\pi} \int_{-1}^{1} \int_R^{\infty} \frac{1}{r^4} r^2 \, \mathrm{d}r \, \mathrm{d}u \, \mathrm{d}\phi$$

COORDENADAS ESFÉRICAS

Pratique o que aprendeu

$$W = \frac{\epsilon_0}{2} \int E^2 \, \mathrm{d}\tau$$



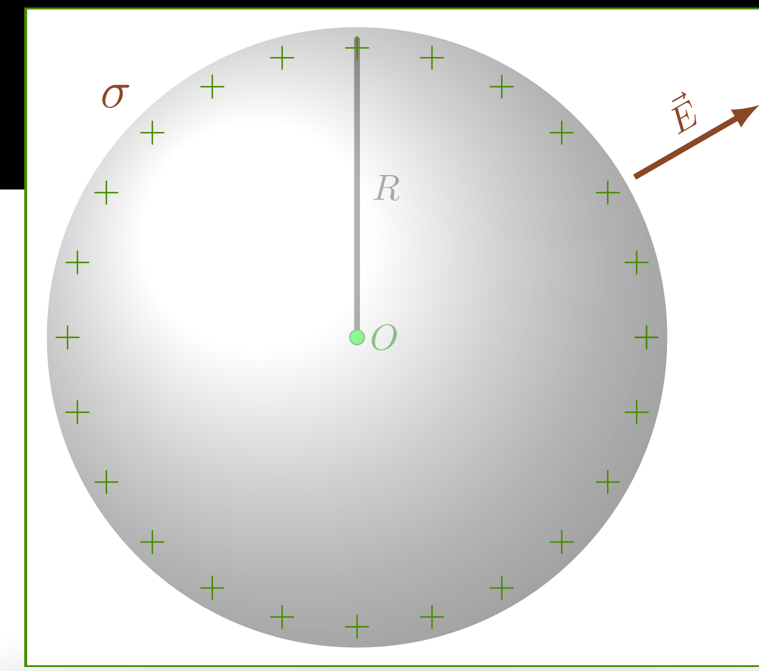
$$W = \frac{\epsilon_0}{2} \int \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \right)^2 \, \mathrm{d}\tau$$

$$W = \frac{\epsilon_0}{2} \frac{q^2}{(4\pi\epsilon_0)^2} \int_0^{2\pi} \int_{-1}^{1} \int_R^{\infty} \frac{1}{r^4} r^2 \, \mathrm{d}r \, \mathrm{d}u \, \mathrm{d}\phi$$

$$W = \frac{1}{2} \frac{q^2}{4\pi\epsilon_0} \frac{1}{R}$$

Pratique o que aprendeu

$$\boxed{W = \frac{\epsilon_0}{2} \int E^2 \, \mathrm{d}\tau}$$



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$$W = \frac{1}{2} \frac{q^2}{4\pi\epsilon_0} \frac{1}{R} = \frac{1}{8\pi\epsilon_0} \frac{q^2}{R}$$

→ ENERGIA ARMAZENADA NO CAMPO ELÉRICO

DENSIDADE DE ENERGIA É $\frac{\epsilon_0}{2} E^2$