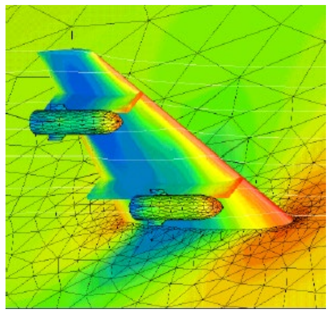
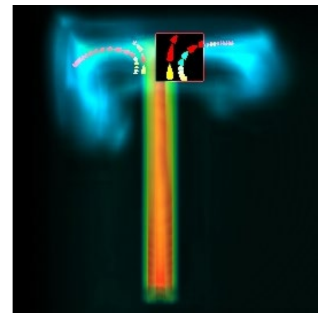
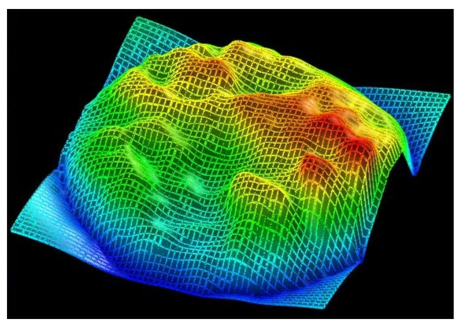
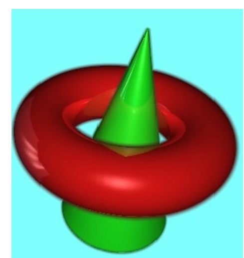
Scientific Visualization Module 6
Volumetric Algorithms



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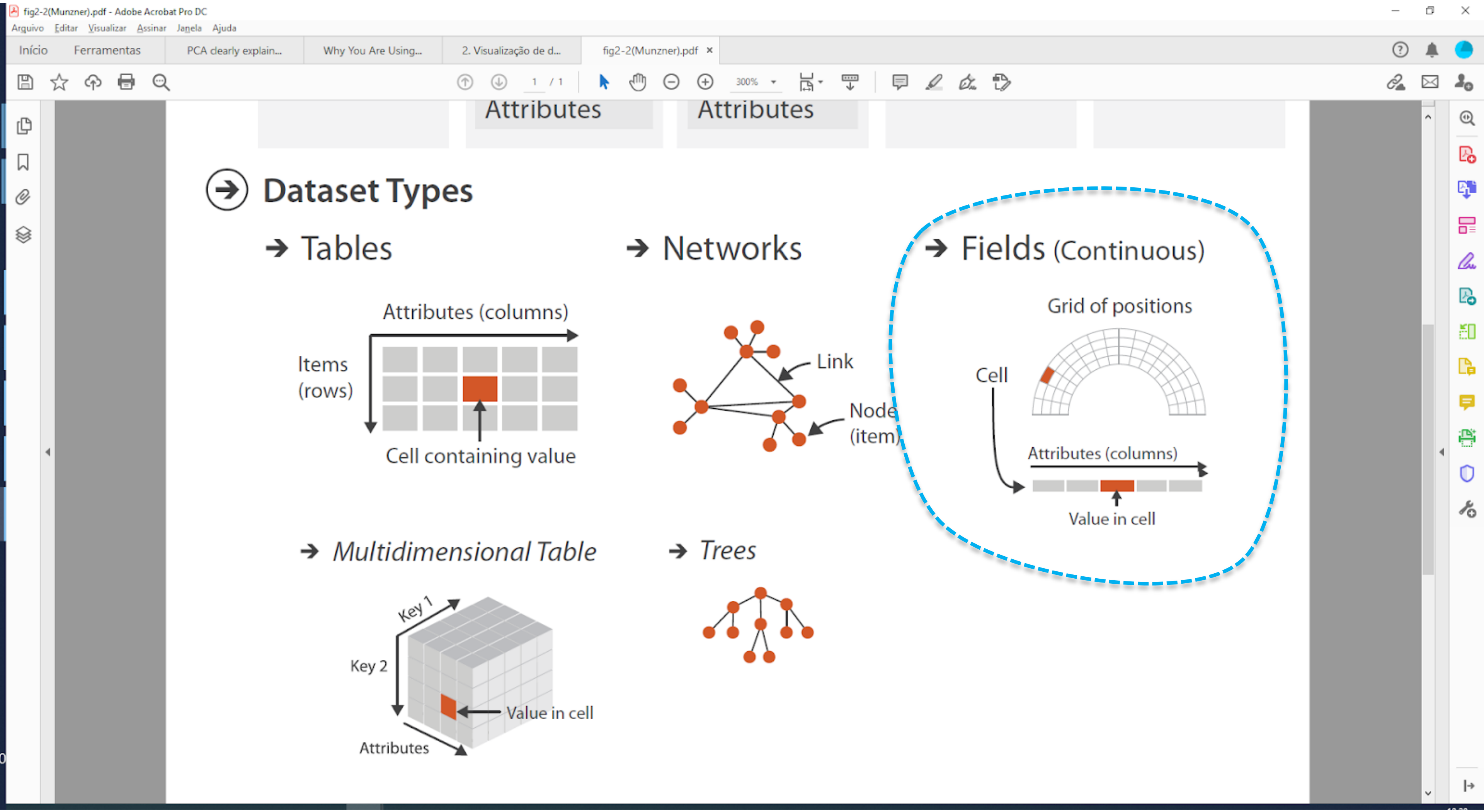


Munzner, Fig. 2.2

Volume visualization (Chapter 10)

1. **Motivation**
   - how to see through 3D scalar volumes?

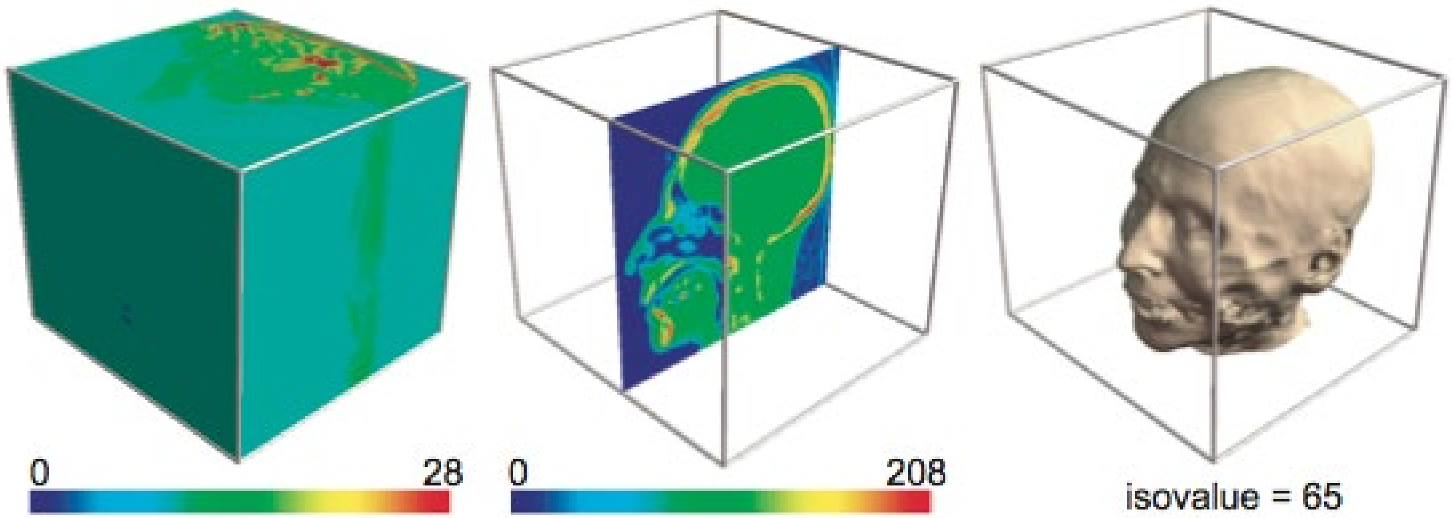
2. **Methods and techniques**
   - ray function (MIP, average intensity, distance to value, isosurface)
   - classification
   - compositing
   - volumetric shading

3. **Advanced points**
   - sampling and interpolation
   - classification and interpolation order
   - performance issues

Motivation

Scalar volume $s : \mathbf{R}^3 \rightarrow \mathbf{R}$

How to visualize this?



direct color mapping	slicing	contouring
•see only outer surface	•all details on slice •no info outside slice	•all details on contour •no info outside contour

How to visualize this so we see through the volume

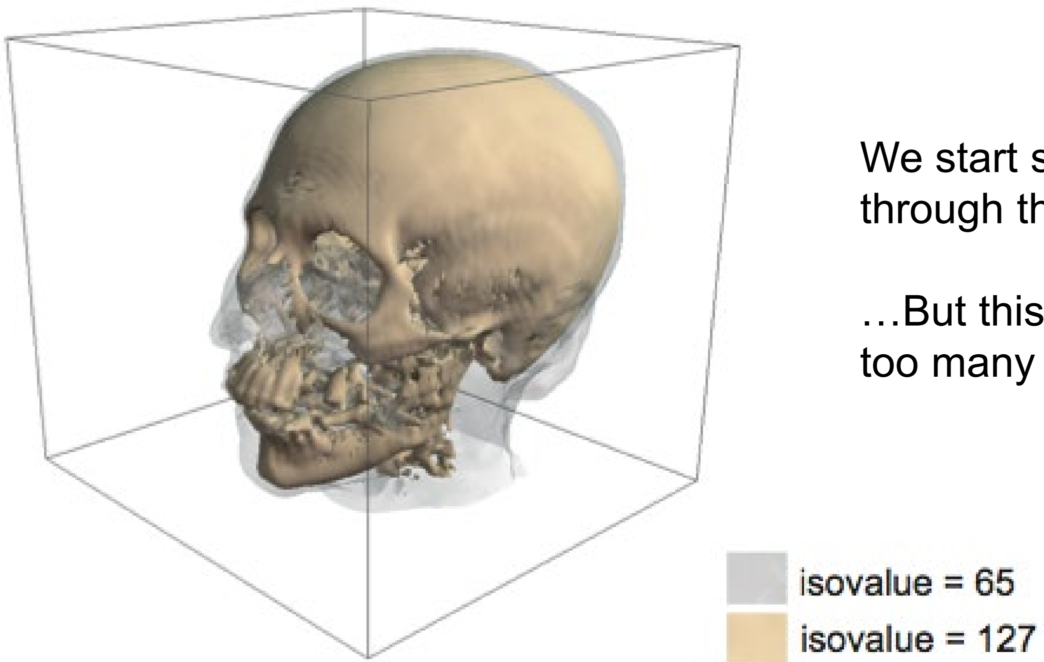
Seeing through a volume

Idea

- use known techniques (slices and contours)
- use transparency

First try

- draw several contours C_i for several values s_i
- transparency α_i proportional to scalar value s



We start seeing a little bit through the volume…

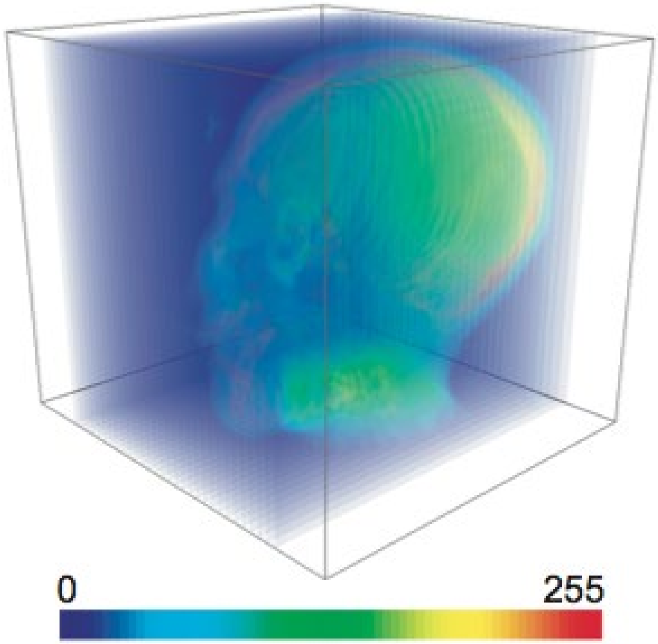
…But this won't work for too many contours (**why?**)

Seeing through a volume

Second try

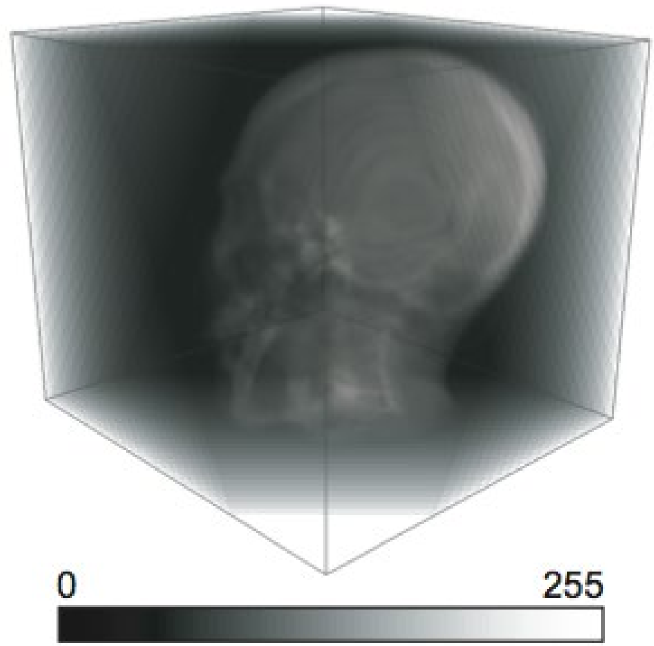
- draw several parallel slices S_i
- transparency α_i inversely proportional to number of slices

$$\alpha_i = \frac{1}{\| S \|}$$



axis-aligned slices

- not OK if we view volume across slicing direction



view direction-aligned slices

- any viewing direction OK
- must reslice when changing viewpoint

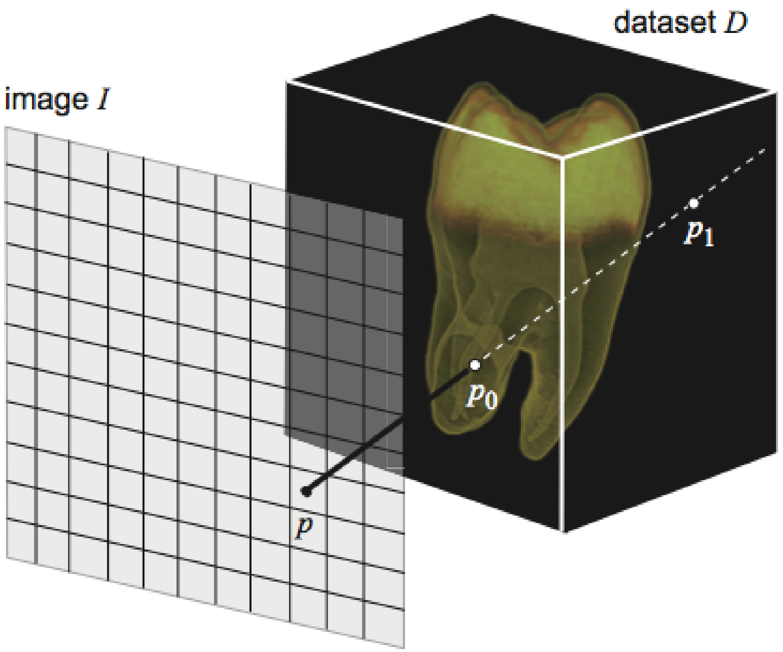
Volume rendering basics

Main idea

•consider **a scalar signal** $s : D \rightarrow \mathbf{R}$ to be drawn on the screen image I

•for each pixel $p \in I$

- construct a ray $\mathbf{r}$ orthogonal to I passing through p
- compute intersection points p_0 and p_1 of $\mathbf{r}$ with D
- express $I(p)$ as function of s along $\mathbf{r}$ between p_0 and p_1



1. Parameterize ray

$$p(t) = (1-t)p_0 + tp_1, \quad t \in [0,1]$$

1. Compute pixel color

$$I(p) = f(F(s(t)), \quad t \in [0,1]$$

ray function

transfer function

Volume rendering

Define a ray function $F:\{s(t)\,|\,t\in[0,1]\}\rightarrow \mathbf{R}$

all scalar values along ray

a single resulting scalar value

The ray function 'aggregates' all scalar values along a ray

Next, define a transfer function

$$f:\mathbf{R}\rightarrow[0,1]^4$$

a single scalar value

an RGBA color

•same concept as color mapping (see Module 2)

Idea

•ray function: says how to combine all scalar values along a ray into a single value

•transfer function: says how to map a single scalar value to a color

•The process of computing all rays for an image I is called ray casting

Maximum intensity projection (MIP)

First example of ray function

- find maximum scalar along ray, then apply transfer function to its value

$$I(p) = f(\max_{t \in [0,T]} s(t))$$

- useful to emphasize high-value points in the volume

Example
MIP of human head CT

- white = low density (air)
- black = high density (bone)

OK, but gives no depth cues

Average intensity projection

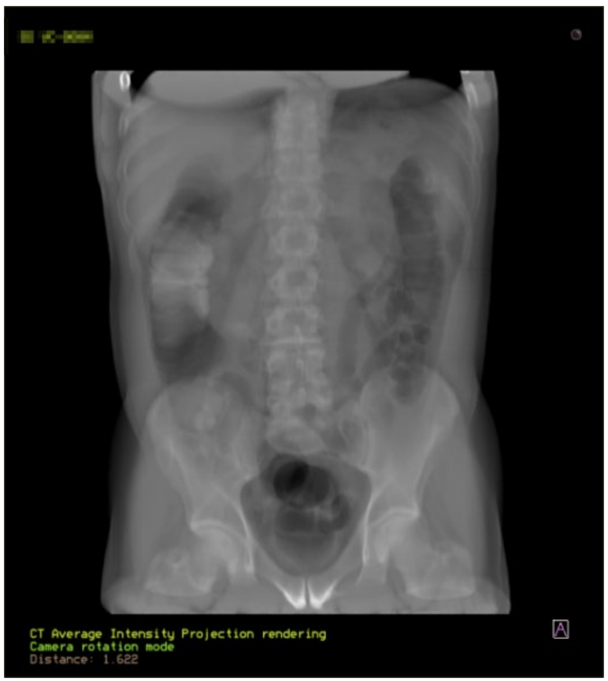
Second example of ray function

- compute average scalar along ray, then map it to color

$$I(p) = f\left(\frac{\int_{t=0}^{T} s(t)dt}{T}\right)$$

- useful to emphasize average tissue type (e.g. density in a CT scan)

maximum intensity projection



average intensity projection

Example
Human torso CT

- black = low density (air)
- white = high density (bone)

Average intensity projection is equivalent to an X-ray

Distance to value function

Third example of ray function

- compute distance along ray until a specific scalar value σ

$$I(p) = f\left(\min_{t\in[0,T],s(t)\geq\sigma} t\right)$$

- useful to emphasize depth where some specific tissue is located

distance to value 20

distance to value 50

Example
Human head CT

- black = low distance
- white = high distance

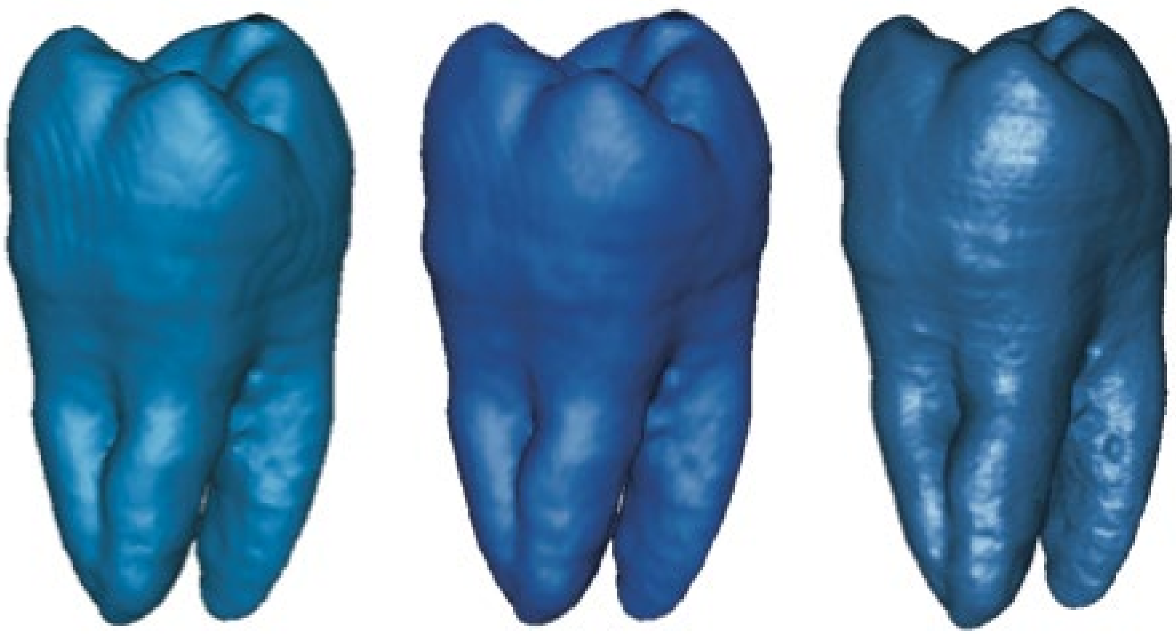
Isosurface function

Fourth example of ray function

- compute whether a given isovalue σ exists along ray

$$I(p) = \begin{cases} f(\sigma), & \exists t \in [0,T], s(t) = \sigma \\ I_0, & \text{otherwise.} \end{cases}$$

- produces same result as marching cubes, but with a higher accuracy



isosurface (marching cubes) | isosurface (software ray casting) | isosurface (hardware ray casting)

Volumetric shading

Shading

- is required if we compute e.g. isosurfaces
- but can also be useful for composite ray function

Method

- instead of simply using the colors $I(t) = f(s(t))$
- composite the shaded colors (see Chapter 2, Phong lighting)

$$I(t) = c_{\text{amb}} + c_{\text{diff}}(t)\max(-\mathbf{L}\cdot\mathbf{n}(t), 0) + c_{\text{spec}}(t)\max(-\mathbf{r}\cdot\mathbf{v}, 0)^{\alpha}$$

How to implement

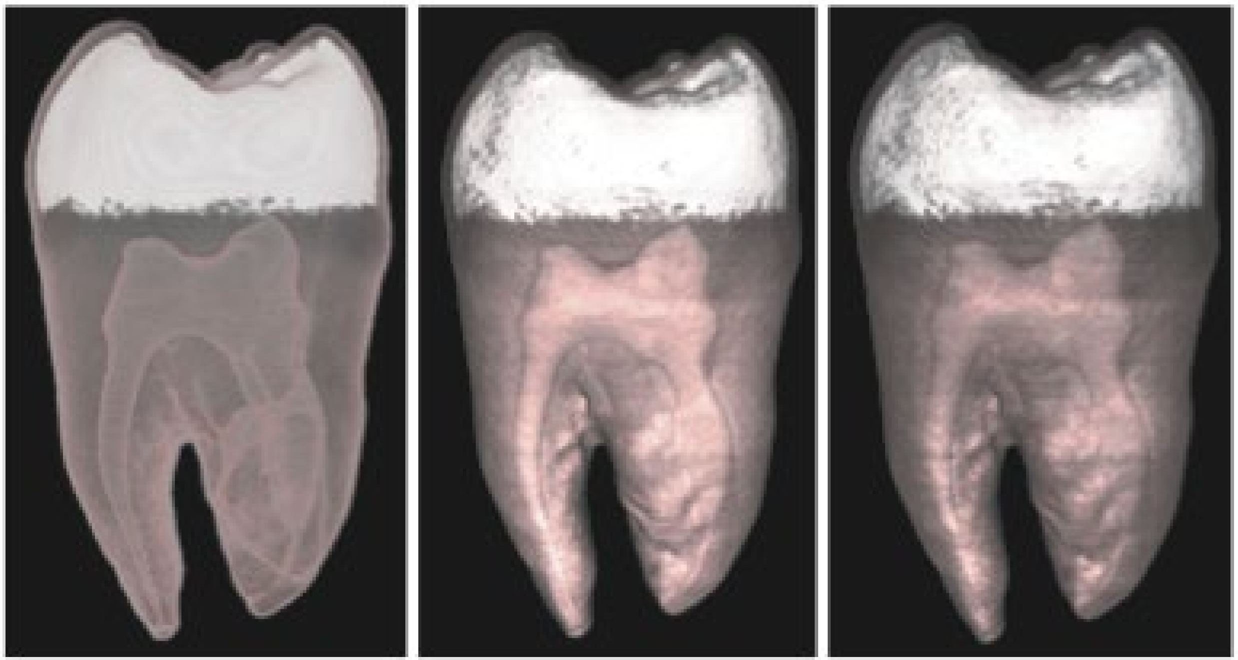
- lighting coefficients c and light vector $\mathbf{L}$: user sets them as desired
- surface normal $\mathbf{n}$: compute from gradient of scalar value

$$\nabla s(t) = \frac{\partial s(t)}{\partial x} + \frac{\partial s(t)}{\partial y} + \frac{\partial s(t)}{\partial z}$$

(we did the same for isosurfaces, see Module 3)

Volumetric shading

Results



no shading | diffuse lighting | diffuse and specular lighting

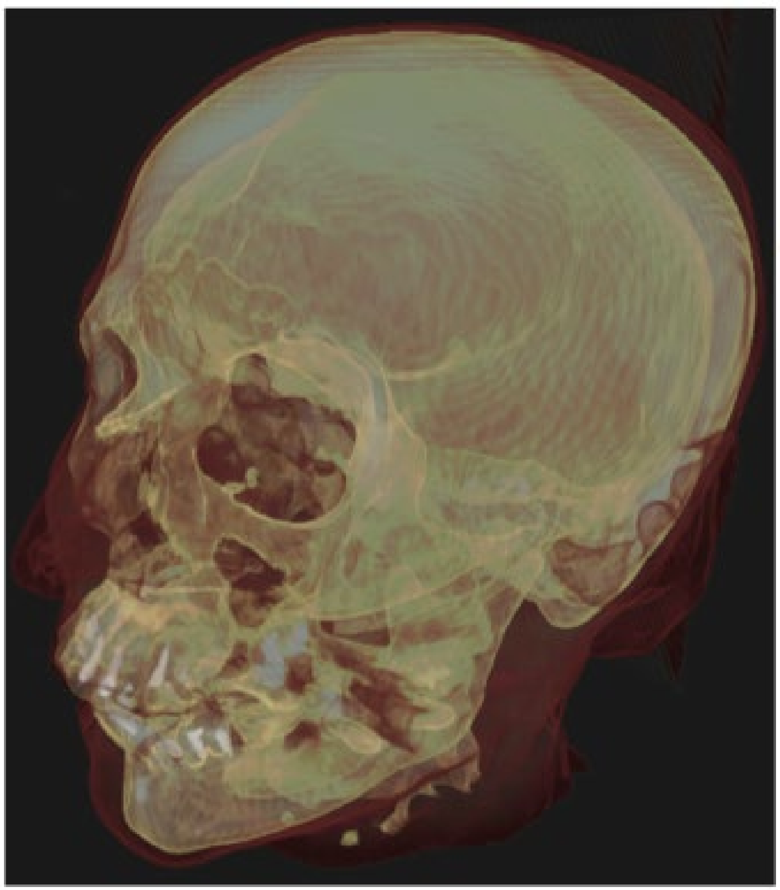
Shading

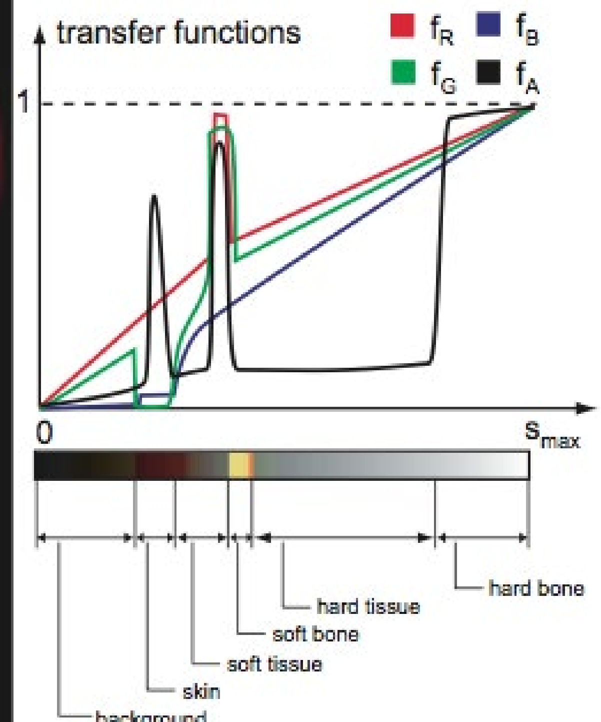
- gives very good cues of depth and shape structure
- is quite cheap and simple to compute

Transfer functions

Extremely powerful modeling tool (mainly when using composite ray function)

- design four functions f_R, f_G, f_B, f_A
- use color and transparency to emphasize desired material properties (e.g. tissue type)
- use any ray function described so far



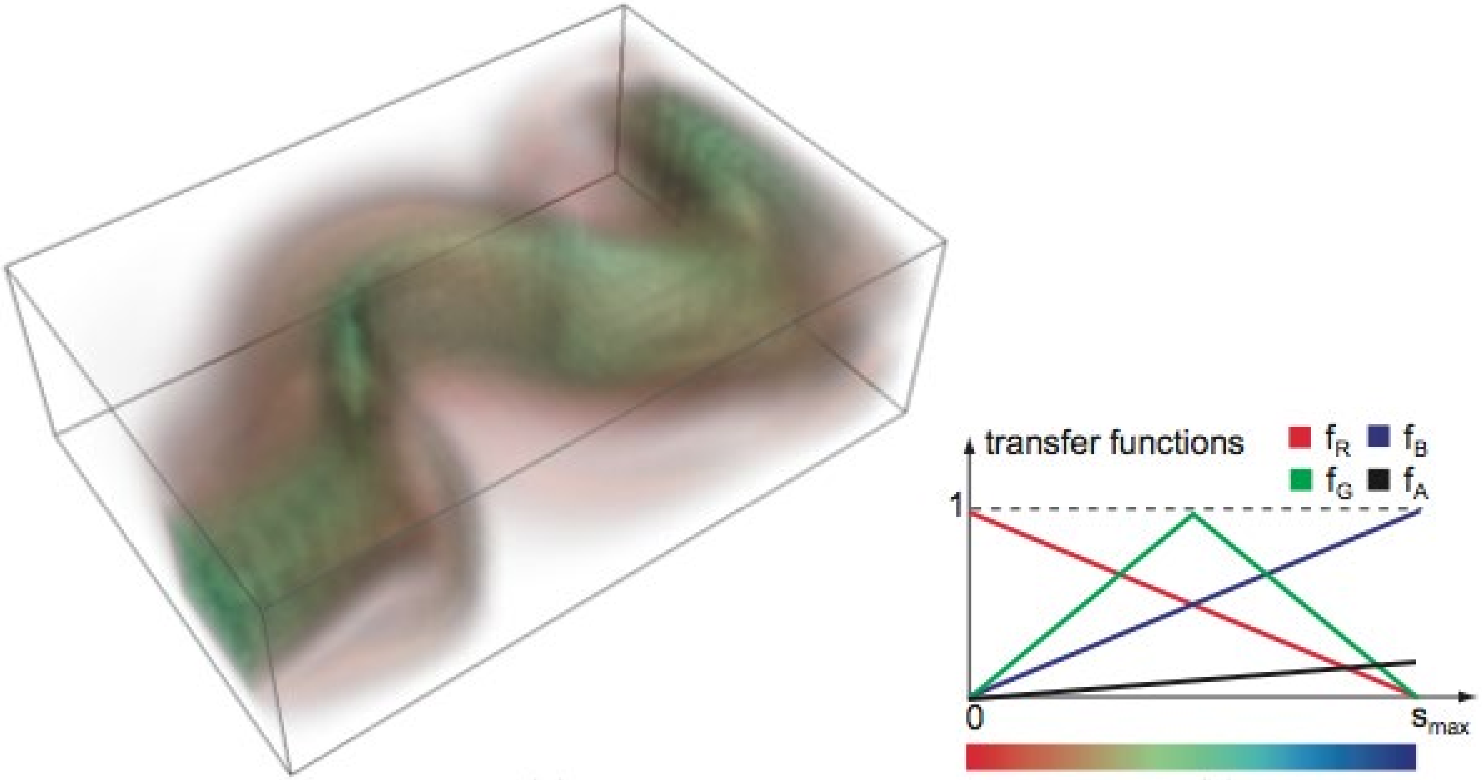


Example
Human head CT

- emphasize bone
- show also muscles

Transfer functions

Volume rendering can be used to visualize also **vector datasets**



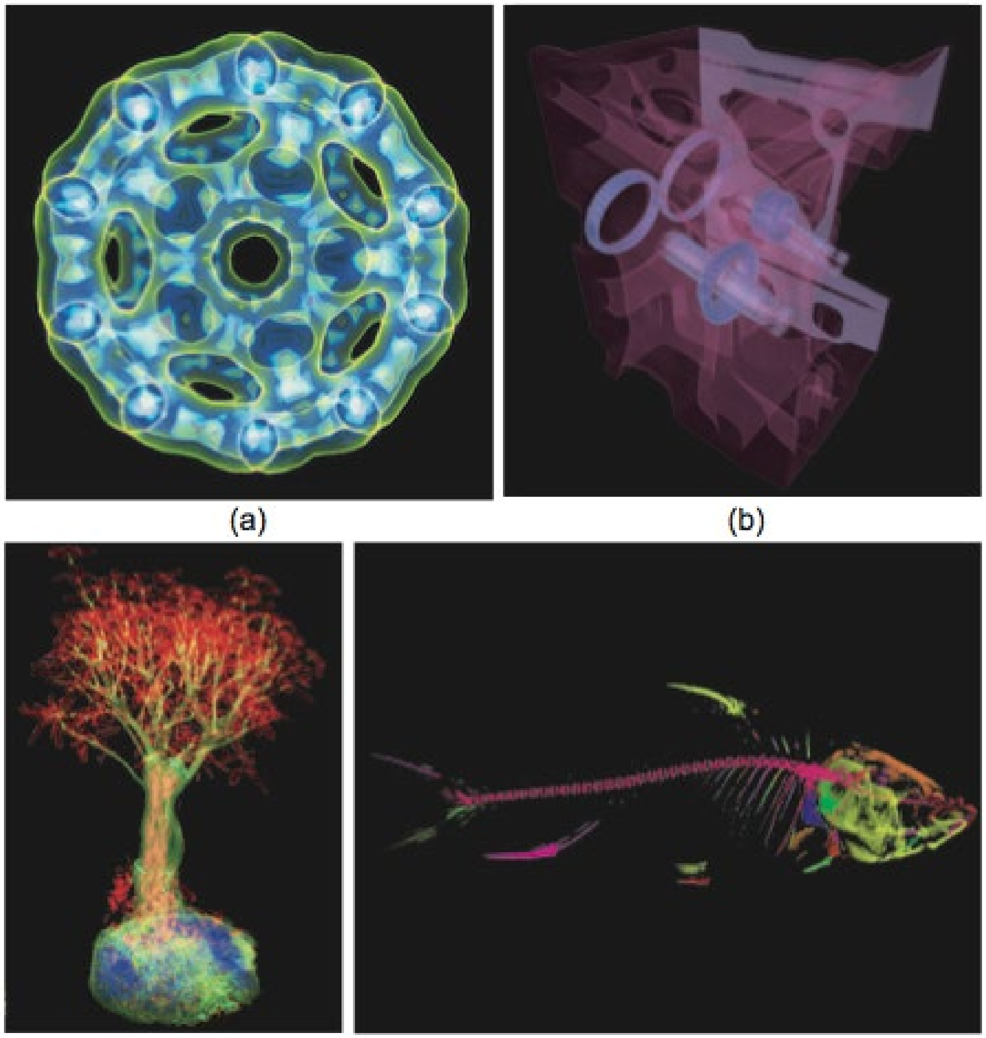
Volume rendering of fluid flow vector field magnitude

•red = slow flow
•green = more rapid flow
•blue = fastest flow

Question: why is the blue hard to see?

Transfer functions

Further examples of (artistic) volume rendering



a) electron density
b) car engine part
c) bonsai tree (scanned)
d) fish

Transfer functions

…and some extreme examples of volume rendering

Volume rendering of human MRI dataset

•shading: mimics natural lighting
•backdrop added for extra effect

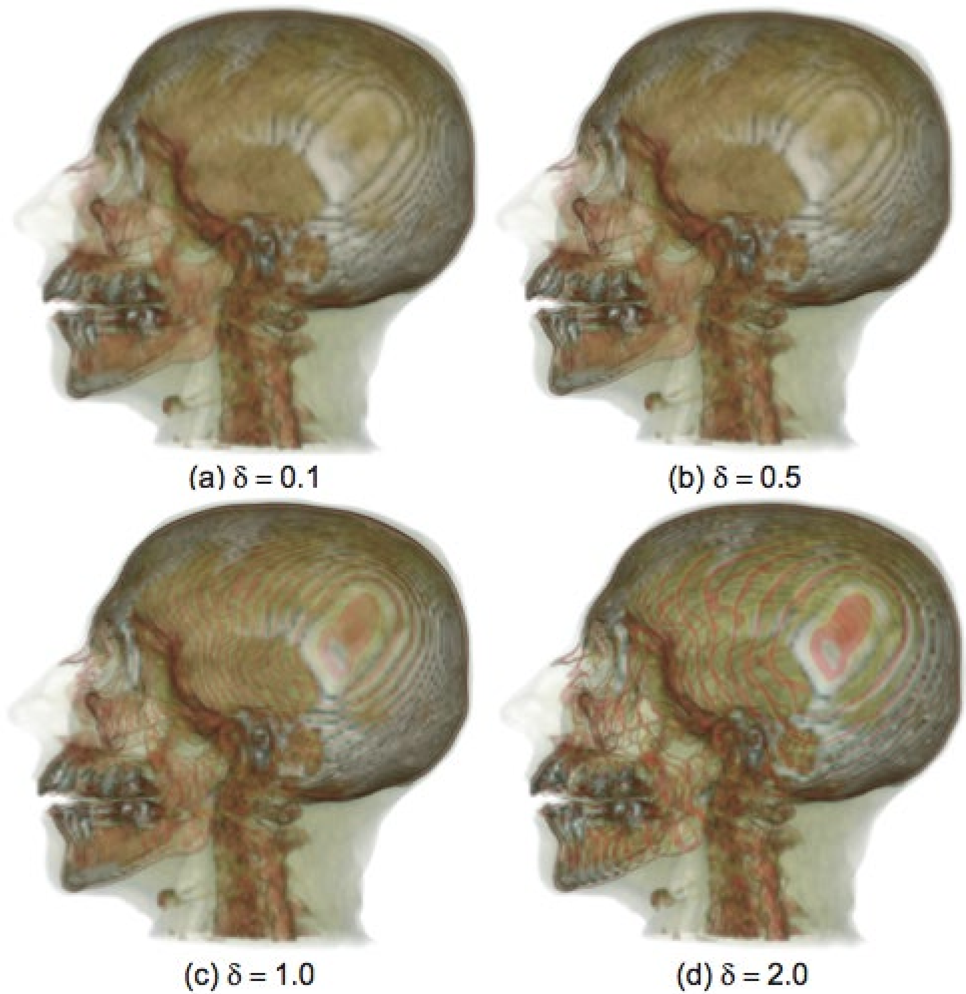
Beautiful result…

…but not directly usable by e.g. clinicians

Implementation issues

Sampling density

- recall the ray parameterization $q(t) = (1-t)q_0 + tq_1, \quad t \in [0,1]$
- we need to sample along the ray (e.g. integrate, compute min/max, etc)
- how small should we take the sampling step $\delta = \mathrm{d}t$?



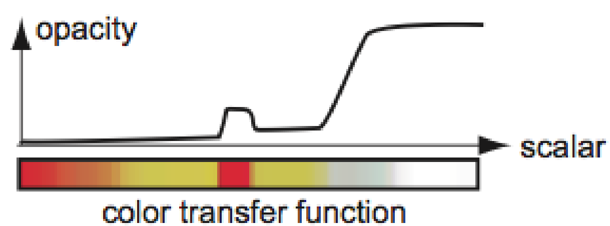
(a) $\delta = 0.1$ (b) $\delta = 0.5$ (c) $\delta = 1.0$ (d) $\delta = 2.0$

Human head CT, four different δ values

•smaller δ: more accuracy
•too small δ: slow rendering

Practical guideline

□δ should never exceed a voxel size (otherwise we skip voxels while traversing the ray…)



Summary

Volume visualization (book Chapter 10)

- Extends classical scalar visualization to ‘see through’ 3D volumes
  - ray functions and transfer functions

- Evaluation
  - produces highly realistic, easy to interpret images
  - requires quite some computational power
  - can be easily accelerated using GPUs (e.g. pixel shaders, CUDA)
  - good transfer function design: critical, application-dependent, hard

Thank you for your interest!