(I) - (II):

$$0 \equiv (\cancel{x+1})\left\{\Delta_f[-1,3] - \Delta_f[-1,1]\right\} + \Delta_f[-1,3,1](\cancel{x+1})\left\{(x-3)-(x-1)\right\}$$

$$(3-1)\,\Delta_f[-1,3,1] = \Delta_f[-1,3] - \Delta_f[-1,1]$$

$$\Delta_f[-1,3,1] = \frac{\Delta_f[-1,3] - \Delta_f[-1,1]}{(3-1)}$$

loucura!

Em geral:

$$\Delta_f[\sim, w, z] = \frac{\Delta_f[\sim, w] - \Delta_f[\sim, z]}{w - z}$$

23/03 **INTERPOLAÇÃO POLINOMIAL - forma de Newton / diferenças divididas**

caso $n=3$ (mas vale p/ qualquer n)

TABELA

x_0	x_1	x_2	x_3
$f(x_0)$	$f(x_1)$	$f(x_2)$	$f(x_3)$

Objetivo: determinar $P_f[x_0, x_1, x_2, x_3]$ = único polinômio em P_3 que interpola f nos pontos dados.

$$P_f[x_0,x_1,x_2,x_3] = \underbrace{\overbrace{\underbrace{\Delta_f[x_0]\,x^0}_{P_f[x_0]} + \Delta_f[x_0,x_1](x-x_0)}^{P_f[x_0,x_1]} + \Delta_f[x_0,x_1,x_2](x-x_0)(x-x_1)}_{P_f[x_0,x_1,x_2]} + \Delta_f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2)$$

coef. de x^2 de $P_f[x_0,x_1,x_2]$

coef. de x^3 de $P_f[x_0,x_1,x_2,x_3]$

Problema: como achar os coeficientes $\Delta_f[x_0]$, $\Delta_f[x_0,x_1]$, $\Delta_f[x_0,x_1,x_2]$, $\Delta_f[x_0,x_1,x_2,x_3]$?

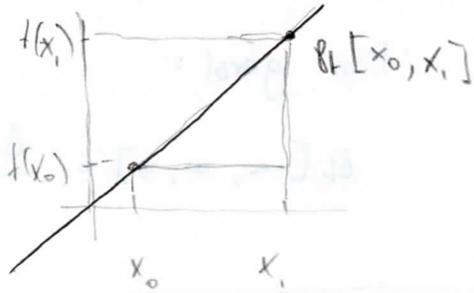
Na aula passada...

1 pt $\Delta_f[x_0] = f(x_0)$

$\Delta_f[x_1] = f(x_1)$

$\vdots$

2 pts: $$\Delta_f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{\Delta_f[x_1] - \Delta_f[x_0]}{x_1 - x_0}$$ coef. angular



· **Regra geral:** diferenças divididas!

ñ depende da ordem

$$\Delta_f[\ldots\ldots, w_1, w_2] = \Delta_f[w_1, \ldots\ldots, w_2] = \frac{\Delta_f[\ldots\ldots, w_2] - \Delta_f[w_1, \ldots\ldots]}{w_2 - w_1}$$

vários pts

P. ex:

$$\Delta_f[x_0, x_1, x_2] = \frac{\Delta_f[x_1, x_2] - \Delta_f[x_0, x_1]}{x_2 - x_0}$$

$$\Delta_f[x_0, x_1, x_2, x_3] = \frac{\Delta_f[x_1, x_2, x_3] - \Delta_f[x_0, x_1, x_2]}{x_3 - x_0}$$

TABELA DE DIFERENÇAS DIVIDIDAS

x_i	$f(x_i)$	ordem 1	ordem 2	ordem 3
x_0	$f(x_0)$			
		$\Delta_f[x_0,x_1] = \dfrac{f(x_1)-f(x_0)}{x_1-x_0}$		
x_1	$f(x_1)$		$\Delta_f[x_0,x_1,x_2] = \dfrac{\Delta f[x_1,x_2]-\Delta f[x_1,x_0]}{x_2-x_0}$	
		$\Delta_f[x_1,x_2] = \dfrac{f(x_1)-f(x_2)}{x_2-x_1}$		
x_2	$f(x_2)$		$\Delta_f[x_1,x_2,x_3] = \dfrac{\Delta f[x_2,x_3]-\Delta_f[x_1,x_2]}{x_3-x_1}$	
		$\Delta_f[x_2,x_3] = \dfrac{f(x_3)-f(x_2)}{x_3-x_2}$		
x_3	$f(x_3)$			

$$\Delta_f[x_0,x_1,x_2,x_3] = \frac{\Delta f[x_1,x_2,x_3]-\Delta f[x_0,x_1,x_2]}{x_3-x_0}$$

exemplo

interpolar a tabela

x_i	-2	0	1	3
$f(x_i)$	0	1	-2	-1

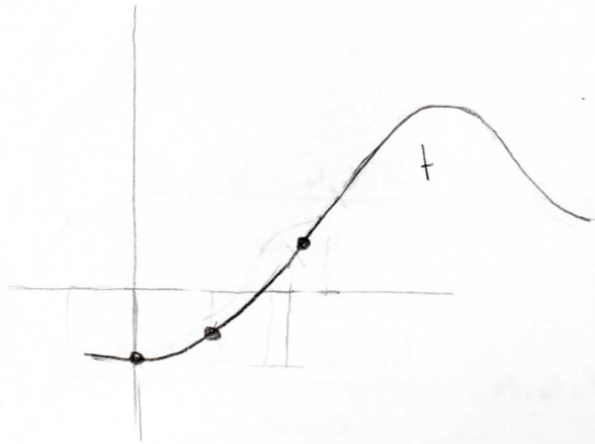
x_i	$f(x_i)$				
-2	0				
		$\frac{1-0}{0-(-2)} = \frac{1}{2}$			
0	1		$\frac{-3-1/2}{1+2} = -7/6$		
		$\frac{-2-1}{1-0} = -3$		$\frac{\frac{7}{6}+\frac{7}{6}}{3+2} = \frac{7}{15}$	
1	-2		$\frac{1/2+3}{-1-1} = \frac{7}{6}$		$-\frac{9}{20}$
		$\frac{-1+2}{2} = \frac{1}{2}$		$-\frac{4}{3}$	
3	-1		$-\frac{3}{2}$		
		-1			
2	0				

$$P_f[x_0,x_1,x_2,x_3] = 0 + \frac{1}{2}(x+2) - \frac{7}{6}(x+2)(x-0) + \frac{7}{15}(x+2)(x-0)(x-1)$$

(*) Suponha que queiramos acrescentar um 5° ponto, digamos: $P(2,0)$ (x_3, $f(x_3)$)

$$P_f[-2,0,1,3,2](x) = \underbrace{(x+2)\left[\frac{1}{2} - \frac{7}{6}x + \frac{7}{15}x(x-1)\right]}_{P[-2,0,1,3] \text{ simplificado}} + \left(\frac{-9}{20}\right)(x+2)x(x-1)(x-3)$$

30/03 PROBLEM. 0



x_i			
0	$f(0)$		
		$\Delta f[0,1]$	
1	$f(1)$		$f[0,1,2]$
		$\Delta f[1,2]$	
2	$f(2)$		

$$\boxed{P_f[0,1,2] = f(0) + \Delta f[0,1]\,x + \Delta f[0,1,2]\,x(x-1)}$$

PROBLEM. 1

x_i			
0	$f(0)$		
		$\Delta f[0,h]$	
h	$f(h)$		$\Delta f[0,h,2]$
		$\Delta f[h,2]$	
2	$f(2)$		

$$\boxed{P_f[0,h,2] = f(0) + \Delta f[0,h]\,x + \Delta f[0,h,2]\,x(x-h)}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Limite $h \to 0$

$$\lim P_f[0,h,2](x) = f(0) + x\cdot\underbrace{\lim_{h\to 0} \Delta f[0,h]}_{f'(0)} + x^2 \underbrace{\lim \Delta f[0,h,2]}_{\frac{\frac{f(2)-f(0)}{2} - f'(0)}{2}}$$

x_i	$f(x_i)$		
0	$f(0)$		
		$\Delta f[0,h] = \frac{f(h)-f(0)}{h-0} \to f'(0)$	
h	$f(h) \to f(0)$		$\Delta f[0,h,2] = \frac{\Delta f[2,h] - \Delta f[0,h]}{2}$
		$\Delta f[2,h] = \frac{f(2)-f(h)}{2-h} \to \Delta f[0,2]$	
2	$f(2)$		

$$\Delta f[0,h,2] = \frac{\frac{f(2)-f(h)}{2-h} - \frac{f(h)-f(0)}{h}}{2}$$

$$= \frac{\frac{f(2)-f(0)}{2-0} - f'(0)}{2}$$

23/03 - Gabriela Duarte

$f(x) = \frac{1}{1+x}$ { interpolar c/ grau 3 os pts. $x = 0, 1, 2, 3$ NL

x_i	$f(x_i)$			
0	1			
		$-\frac{1}{2}$		
1	1/2		$\frac{1}{6}$	
		$-\frac{1}{6}$		$-\frac{1}{24}$
2	1/3		$\frac{1}{24}$	
		$-\frac{1}{12}$		
3	1/4			

$$P_f[0,1,2,3] = 1 - \frac{1}{2}x + \frac{1}{6}x(x-1) - \frac{1}{24}x(x-1)(x-2)$$

graf. do polinômio:

· pontos críticos: derivada = 0

$$P_f[0,1,2,3]' = -\frac{1}{2} + \frac{1}{6}[1\cdot(x-1) + x\cdot 1] - \frac{1}{24}[1(x-1)(x-2) + x(1)(x-2) + x(x-1)(1)]$$

$$= -\frac{1}{2} + \frac{2x-1}{6} - \frac{3x^2-6x+2}{24} = \frac{1}{24}(-3x^2+14x-13)$$

$$\frac{1}{24}\cdot(-3x^2+14x-13) = 0 \qquad \Delta = (14)^2 - 4(-3)(-13) = 196 - 216 = -20$$

$$-3x^2+14x-13=0$$

não há raízes ∇ ∴

· pt. de inflexão: 2ª derivada = 0

$$\frac{1}{24}(-5x+14) = 0$$

$$-5x+14=0$$

$$5x = 14$$

$$x = \frac{14}{5}$$

$x = -1 \Rightarrow P_x = 25/12$

$x = 4 \Rightarrow P_x =$

x	$p(x)$	$f(x$
-1	2,03	+∞
-0,5	1,45	2
0	1	1
0,5	0,69	0,66
1	0,5	0,5
1,5	0,39	0,4
2	0,33	0,33
2,5	0,29	0,2
3	0,25	0,25
3,5	0,16	0,22
4	0	0,2