Particle-in-Cell (PIC) kinetic simulations
05. Poisson's equation

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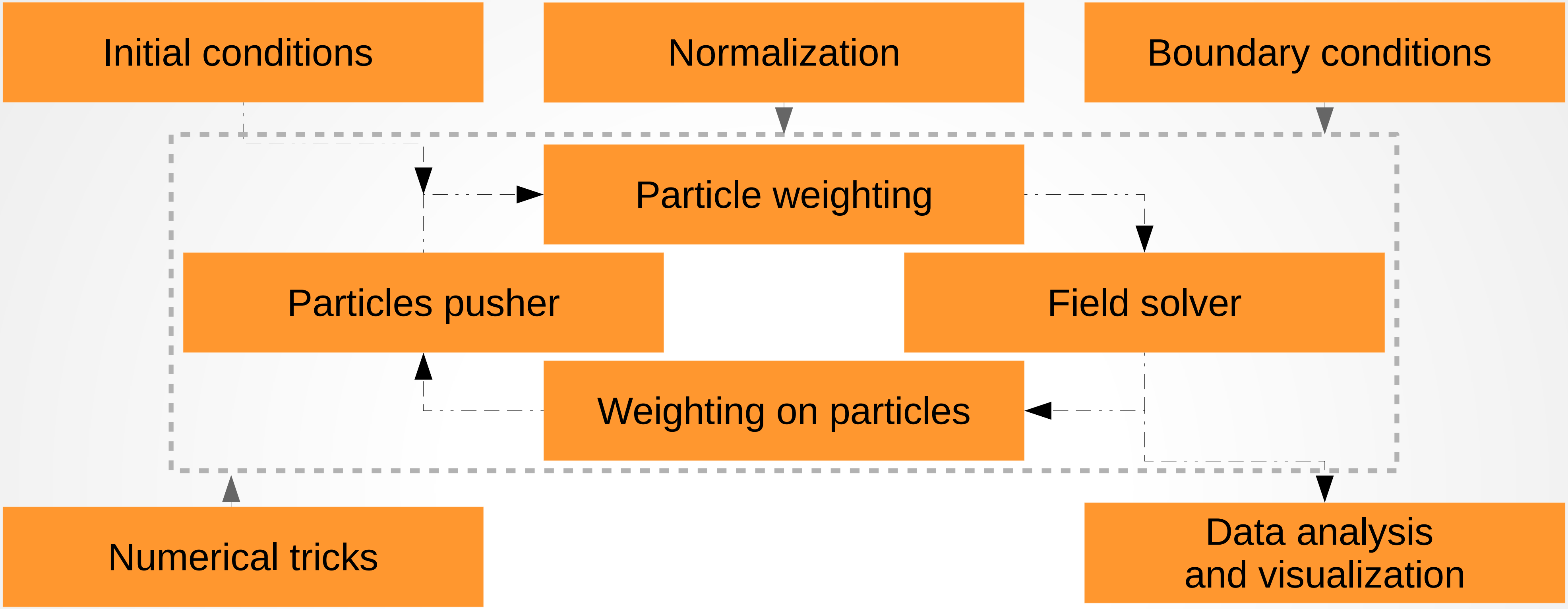
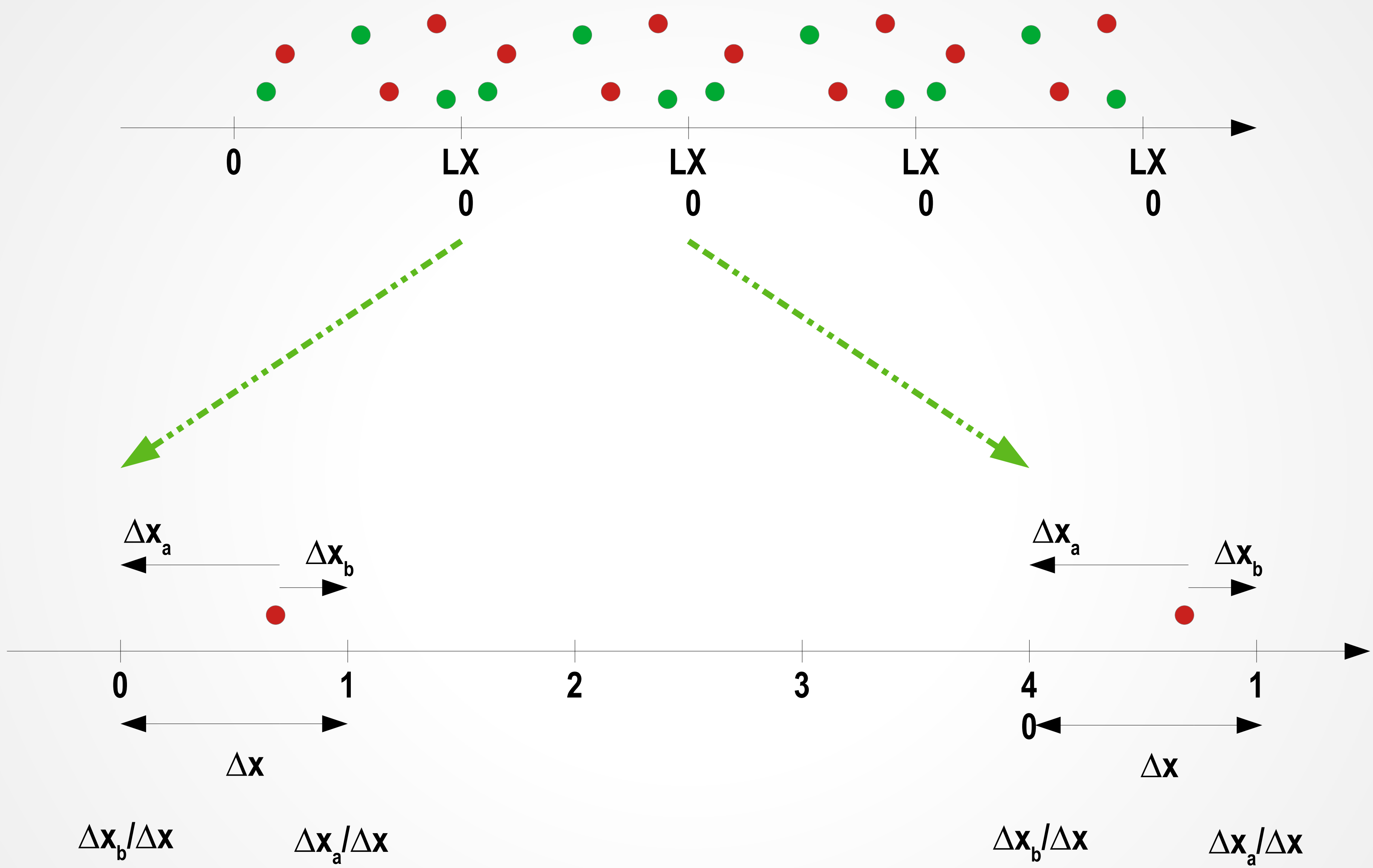
www.slido.com code: #P320

Concept and basic structure



Two-stream instability https://www.youtube.com/watch?v=Cz5tRSMY3Kc

Periodic boundary condition



Electrostatic system

$\vec{B}=\vec{B}(\vec{x}_t)$

$\vec{E}=\vec{E}(\vec{x}_t)$

$\vec{v}_{t-\Delta t/2}$

$\vec{B}_t(\vec{x}_t)$

$\vec{E}_t(\vec{x}_t)$

$\vec{x}_t$

$\vec{v}_{t+\Delta t/2}$

$\vec{B}_{t+\Delta t}(\vec{x}_{t+\Delta t})$

$\vec{E}_{t+\Delta t}(\vec{x}_{t+\Delta t})$

$\vec{x}_{t+\Delta t}$

$\vec{v}_{t+3\Delta t/2}$

$\vec{B}_{t+2\Delta t}(\vec{x}_{t+2\Delta t})$

$\vec{E}_{t+2\Delta t}(\vec{x}_{t+2\Delta t})$

$\vec{x}_{t+2\Delta t}$

t - Δt/2 | t | t + Δt/2 | t + Δt | t + 3Δt/2 | t + 2Δt

Self consistent electrostatic system

$\vec{B}=\vec{B}(\vec{x}_t)$

$\vec{E}=\vec{E}(\vec{x}_t)$

$t-\Delta t/2$	t	$t+\Delta t/2$	$t+\Delta t$	$t+3\Delta t/2$	$t+2\Delta t$
	$\vec{B}_t$		$\vec{B}_{t+\Delta t}$		$\vec{B}_{t+2\Delta t}$
	$\vec{E}_t$		$\vec{E}_{t+\Delta t}$		$\vec{E}_{t+2\Delta t}$
	$\vec{x}_t$		$\vec{x}_{t+\Delta t}$		$\vec{x}_{t+2\Delta t}$
$\vec{v}_{t-\Delta t/2}$		$\vec{v}_{t+\Delta t/2}$		$\vec{v}_{t+3\Delta t/2}$	

$\vec{B}=\vec{B}(\vec{x}_t)$

$\vec{E}=\vec{E}(\vec{x}_t, t)$

$t-\Delta t/2$	t	$t+\Delta t/2$	$t+\Delta t$	$t+3\Delta t/2$	$t+2\Delta t$
	$\vec{B}_t$		$\vec{B}_{t+\Delta t}$		$\vec{B}_{t+2\Delta t}$
	$\vec{E}_t$		$\vec{E}_{t+\Delta t}$		$\vec{E}_{t+2\Delta t}$
	φ_t		$\varphi_{t+\Delta t}$		$\varphi_{t+2\Delta t}$
	ρ_{ct}		$\rho_{ct+\Delta t}$		$\rho_{ct+2\Delta t}$
	$\vec{x}_t$		$\vec{x}_{t+\Delta t}$		$\vec{x}_{t+2\Delta t}$
$\vec{v}_{t-\Delta t/2}$		$\vec{v}_{t+\Delta t/2}$		$\vec{v}_{t+3\Delta t/2}$	

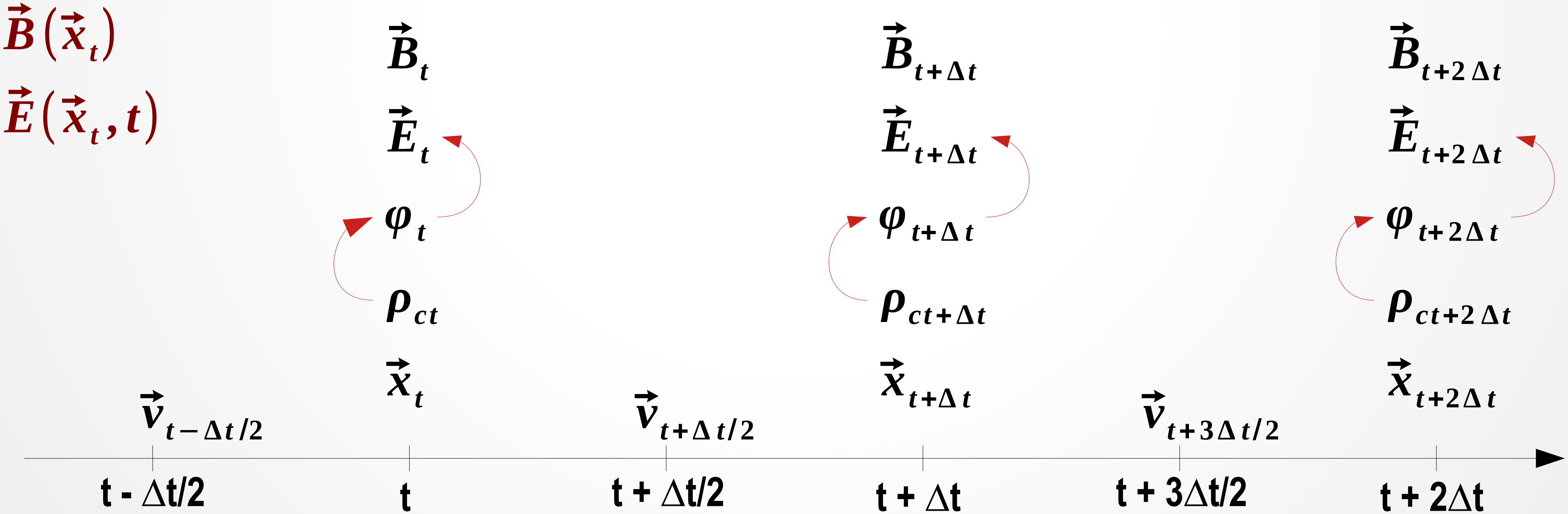
Self consistent electrostatic system

$$-\nabla^2 \varphi = \frac{\rho_c}{\varepsilon_0}$$

$$\vec{E} = -\nabla \varphi$$

$$\vec{B} = \vec{B}(\vec{x}_t)$$

$$\vec{E} = \vec{E}(\vec{x}_t, t)$$



Self consistent electrostatic system

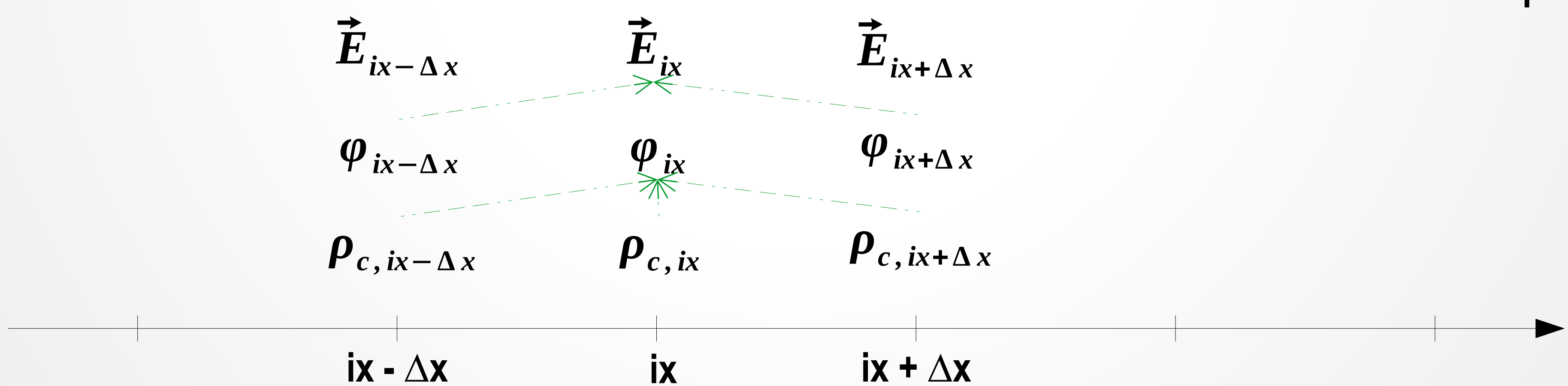
$$-\nabla^2 \varphi = \frac{\rho_c}{\varepsilon_0}$$

$$\vec{E} = -\nabla \varphi$$

$$\frac{\vec{\varphi}_{ix+\Delta x} - 2\vec{\varphi}_{ix} + \vec{\varphi}_{ix-\Delta x}}{(\Delta x)^2} = -\frac{\rho_{c,ix}}{\varepsilon_0}$$

$$\vec{E}_{ix} = \frac{-\vec{\varphi}_{ix+\Delta x} + \vec{\varphi}_{ix-\Delta x}}{2\Delta x}$$

At each time step



Self consistent electrostatic system

$$-\nabla^2 \varphi = \frac{\rho_c}{\varepsilon_0}$$

$$\vec{\varphi}_{ix+\Delta x} - 2\vec{\varphi}_{ix} + \vec{\varphi}_{ix-\Delta x} = -(\Delta x)^2 \frac{\rho_{c,ix}}{\varepsilon_0}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ .. \\ \varphi_{NX-1} \\ \varphi_{NX} \end{bmatrix} = \frac{-(\Delta x)^2}{\varepsilon_0} \begin{bmatrix} 0 \\ \rho_{c,1} \\ \rho_{c,2} \\ \rho_{c,3} \\ .. \\ \rho_{c,NX-1} \\ 0 \end{bmatrix}$$

$$A\varphi = w \qquad \varphi = A^{-1} w$$

inverse matrix:05_01_Inverse.f90

inverse matrix

Poisson's equation

$$-\nabla^2 \varphi = \frac{\rho_c}{\varepsilon_0} \qquad \vec{E} = -\nabla \varphi$$

$$\vec{\varphi}_{ix+\Delta x} - 2\vec{\varphi}_{ix} + \vec{\varphi}_{ix-\Delta x} = (\Delta x)^2 \frac{\rho_{c,ix}}{\varepsilon_0} \qquad \vec{E}_{ix} = \frac{\vec{\varphi}_{ix+\Delta x} - \vec{\varphi}_{ix-\Delta x}}{2\Delta x}$$

$$A\varphi = w \qquad \varphi = A^{-1} w$$

At each time step