Particle-in-Cell (PIC) kinetic simulations
04. Particle pusher

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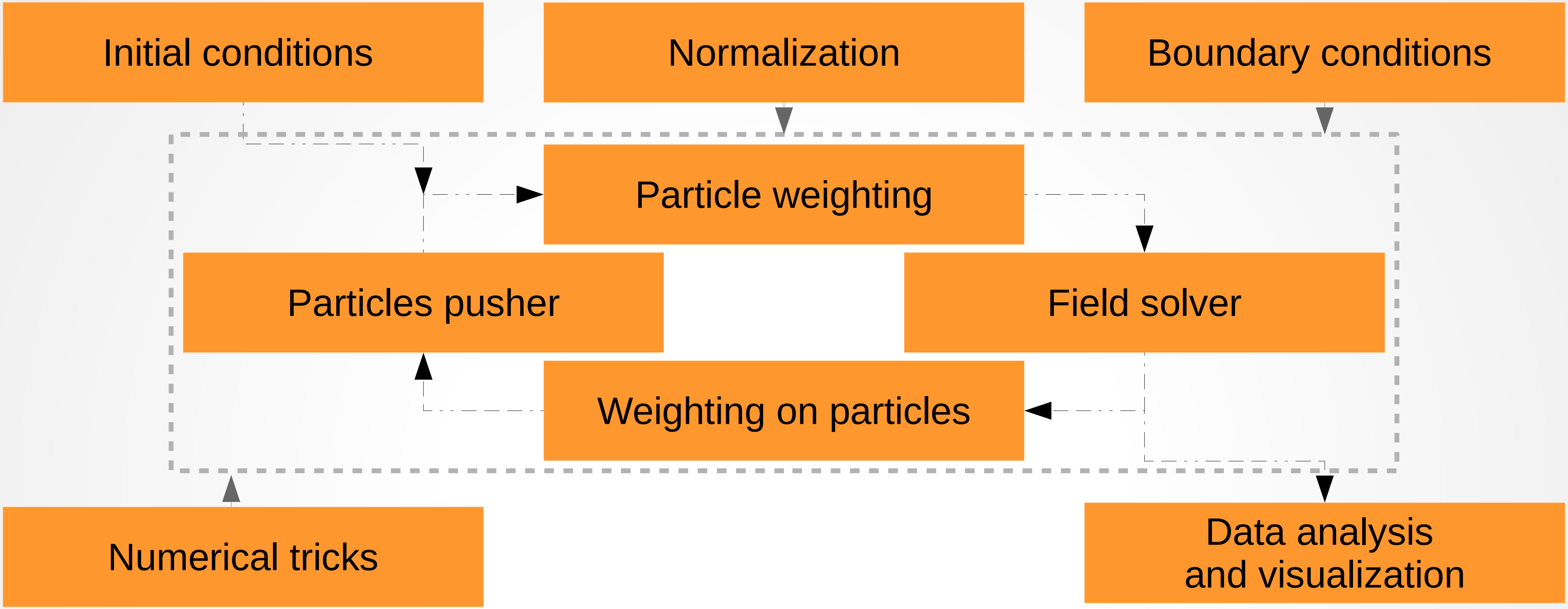
University of São Paulo, 2019.11.25-12.06

www.slido.com code: #P320

Concept and basic structure



Equations of motion

$$m\frac{d\vec{v}}{dt}=q\vec{E}$$

$$\frac{d\vec{x}}{dt}=\vec{v}$$

Extrapolation

$$\frac{\vec{v}_{t+\Delta t}-\vec{v}_t}{\Delta t}=\frac{q}{m}\vec{E}_t$$

$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_t$$

$$\vec{E}_t=\vec{E}_t(\vec{x})$$

$$\vec{v}_{t+\Delta t}=\vec{v}_t+\frac{q}{m}\vec{E}_t\times\Delta t$$

$$\vec{x}_{t+\Delta t}=\vec{x}_t+\vec{v}_t\times\Delta t$$

While time is not continue but discrete……

t	$t+\Delta t$	$t+2\Delta t$
$\vec{E}_t$	$\vec{E}_{t+\Delta t}$	$\vec{E}_{t+2\Delta t}$
$\vec{x}_t$	$\vec{x}_{t+\Delta t}$	$\vec{x}_{t+2\Delta t}$
$\vec{v}_t$	$\vec{v}_{t+\Delta t}$	$\vec{v}_{t+2\Delta t}$

Equations of motion

$$m\frac{d\vec{v}}{dt}=q\vec{E}$$

$$\frac{d\vec{x}}{dt}=\vec{v}$$

Interpolation

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}\vec{E}_t$$

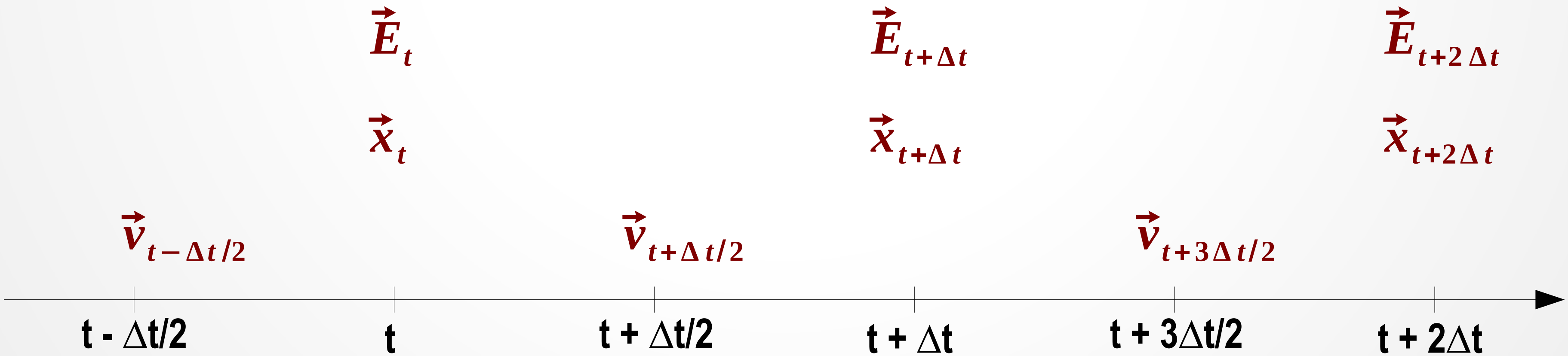
$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2}$$

$$\vec{E}_t=\vec{E}_t(\vec{x})$$

$$\vec{v}_{t+\Delta t/2}=\vec{v}_{t-\Delta t/2}+\frac{q}{m}\vec{E}_t\times\Delta t$$

$$\vec{x}_{t+\Delta t}=\vec{x}_t+\vec{v}_{t+\Delta t/2}\times\Delta t$$

While time is not continue but discrete……



Equations of motion

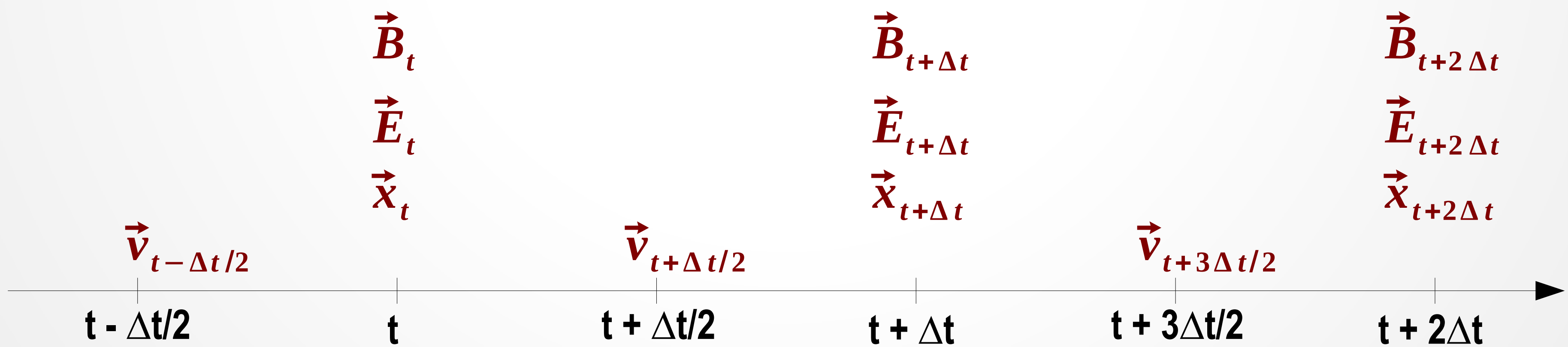
$$m\frac{d\vec{v}}{dt}=q(\vec{E}+\vec{v}\times\vec{B}) \qquad \frac{d\vec{x}}{dt}=\vec{v}$$

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}(\vec{E}_t+\vec{v}_t\times\vec{B}_t) \qquad \frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2}$$

$$\vec{E}_t=\vec{E}_t(\vec{x})$$
$$\vec{B}_t=\vec{B}_t(\vec{x})$$

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}[\vec{E}_t+(\frac{\vec{v}_{t+\Delta t/2}+\vec{v}_{t-\Delta t/2}}{2})\times\vec{B}_t]$$

While time is not continue but discrete……



Boris Method

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}[\vec{E}_t+(\frac{\vec{v}_{t+\Delta t/2}+\vec{v}_{t-\Delta t/2}}{2})\times\vec{B}_t]$$

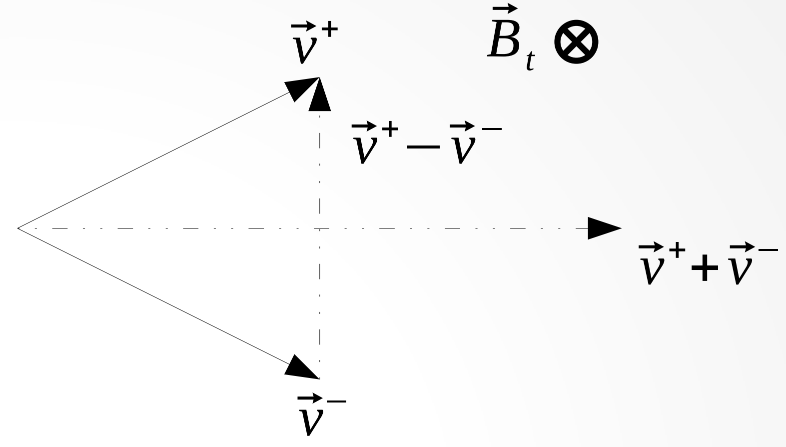
$$\vec{v}_{t+\Delta t/2}=\vec{v}^{+}+\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\vec{v}_{t-\Delta t/2}=\vec{v}^{-}-\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\frac{\vec{v}^{+}-\vec{v}^{-}}{\Delta t}=\frac{q}{2m}(\vec{v}^{+}+\vec{v}^{-})\times\vec{B}_t$$

Boris Method

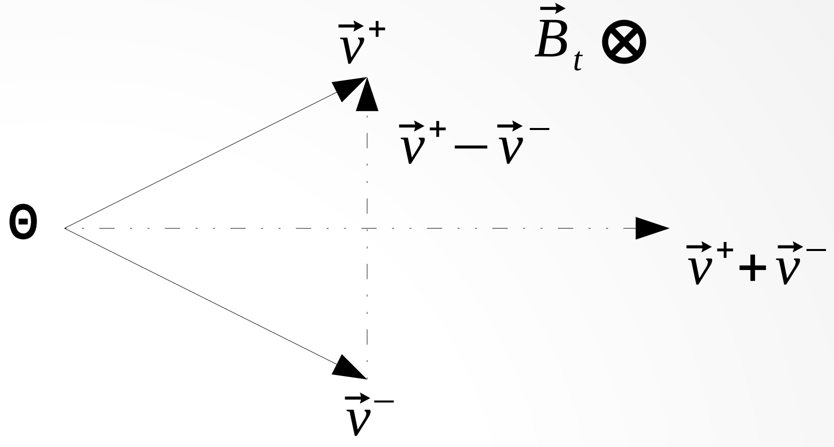
$$\frac{\vec{v}^{+}-\vec{v}^{-}}{\Delta t}=\frac{q}{2m}\left(\vec{v}^{+}+\vec{v}^{-}\right)\times\vec{B}_t$$



Boris Method

$$\frac{\vec{v}^{+}-\vec{v}^{-}}{\Delta t}=\frac{q}{2m}\left(\vec{v}^{+}+\vec{v}^{-}\right)\times\vec{B}_t$$

$$\tan\left(\frac{\Theta}{2}\right)=\frac{\vec{v}^{+}-\vec{v}^{-}}{\vec{v}^{+}+\vec{v}^{-}}=\frac{qB_t}{m}\frac{\Delta t}{2}$$



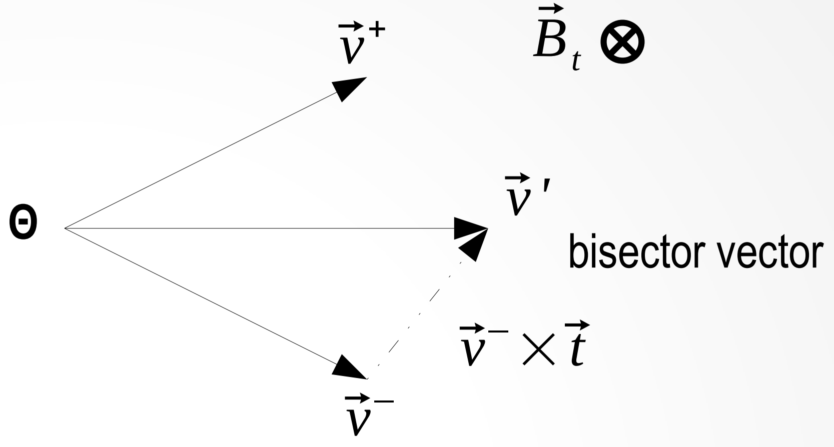
Boris Method

$$\frac{\vec{v}^{+}-\vec{v}^{-}}{\Delta t}=\frac{q}{2m}(\vec{v}^{+}+\vec{v}^{-})\times\vec{B}_t$$

$$\tan\left(\frac{\Theta}{2}\right)=\frac{\vec{v}^{+}-\vec{v}^{-}}{\vec{v}^{+}+\vec{v}^{-}}=\frac{qB_t}{m}\frac{\Delta t}{2}$$

$$\vec{v}\,'=\vec{v}^{-}+\vec{v}^{-}\times\vec{t}$$

$$\vec{t}=\frac{qB_t}{m}\frac{\Delta t}{2}\hat{b}$$



Boris Method

$$\frac{\vec{v}^+ - \vec{v}^-}{\Delta t} = \frac{q}{2m}(\vec{v}^+ + \vec{v}^-) \times \vec{B}_t$$

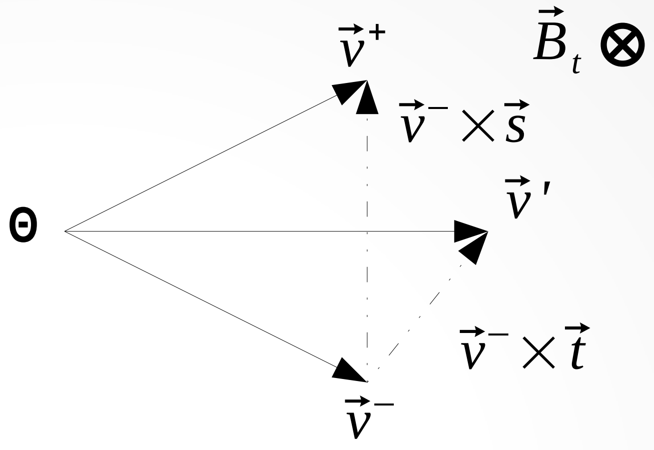
$$\tan\left(\frac{\Theta}{2}\right) = \frac{\vec{v}^+ - \vec{v}^-}{\vec{v}^+ + \vec{v}^-} = \frac{q B_t}{m}\frac{\Delta t}{2}$$

$$\vec{v}\,' = \vec{v}^- + \vec{v}^- \times \vec{t}$$

$$\vec{t} = \frac{q B_t}{m}\frac{\Delta t}{2}\hat{b}$$

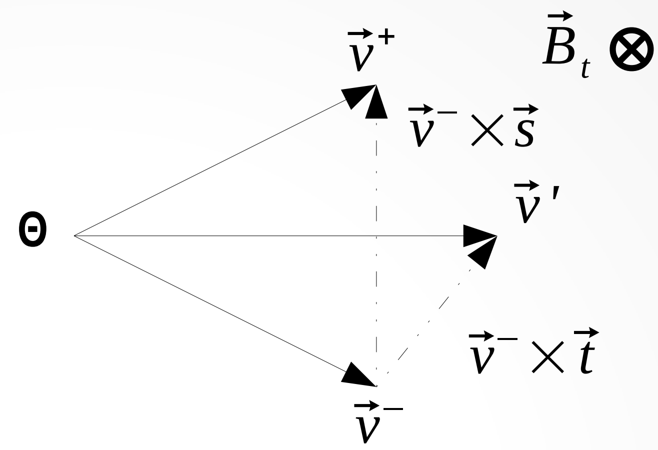
$$\vec{v}^+ = \vec{v}^- + \vec{v}\,' \times \vec{s}$$

$$\vec{s} = \frac{2}{1+t^2}\vec{t}$$



Boris Method

$$\frac{\vec{v}^+ - \vec{v}^-}{\Delta t} = \frac{q}{2m}(\vec{v}^+ + \vec{v}^-) \times \vec{B}_t$$



$$\tan\left(\frac{\Theta}{2}\right) = \frac{\vec{v}^+ - \vec{v}^-}{\vec{v}^+ + \vec{v}^-} = \frac{q B_t}{m}\frac{\Delta t}{2}$$

$$\vec{v}\,' = \vec{v}^- + \vec{v}^- \times \vec{t}$$

$$\vec{t} = \frac{q B_t}{m}\frac{\Delta t}{2}\hat{b}$$

$$\vec{v}^+ = \vec{v}^- + \vec{v}\,' \times \vec{s}$$

$$\vec{s} = \frac{2}{1+t^2}\vec{t}$$

$$\frac{\vec{v}_{t+\Delta t/2} - \vec{v}_{t-\Delta t/2}}{\Delta t} = \frac{q}{m}\left[\vec{E}_t + \left(\frac{\vec{v}_{t+\Delta t/2} + \vec{v}_{t-\Delta t/2}}{2}\right) \times \vec{B}_t\right]$$

$$\vec{v}_{t-\Delta t/2} = \vec{v}^- - \frac{q\vec{E}_t}{m}\frac{\Delta t}{2} \qquad \vec{v}_{t+\Delta t/2} = \vec{v}^+ + \frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\frac{\vec{v}^+ - \vec{v}^-}{\Delta t} = \frac{q}{2m}(\vec{v}^+ + \vec{v}^-) \times \vec{B}_t$$

Boris Method

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}\left[\vec{E}_t+\left(\frac{\vec{v}_{t+\Delta t/2}+\vec{v}_{t-\Delta t/2}}{2}\right)\times\vec{B}_t\right]$$

$$\vec{v}_{t-\Delta t/2}=\vec{v}^- -\frac{q\vec{E}_t}{m}\frac{\Delta t}{2} \qquad \vec{v}_{t+\Delta t/2}=\vec{v}^+ +\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\frac{\vec{v}^+-\vec{v}^-}{\Delta t}=\frac{q}{2m}\left(\vec{v}^+ +\vec{v}^-\right)\times\vec{B}_t$$

$$\vec{v}\,'=\vec{v}^- +\vec{v}^-\times\vec{t} \qquad \vec{t}=\frac{qB_t}{m}\frac{\Delta t}{2}\hat{b}$$

$$\vec{v}^+=\vec{v}^- +\vec{v}\,'\times\vec{s} \qquad \vec{s}=\frac{2}{1+t^2}\vec{t}$$

Boris Method

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}[\vec{E}_t+(\frac{\vec{v}_{t+\Delta t/2}+\vec{v}_{t-\Delta t/2}}{2})\times\vec{B}_t]$$

$$\vec{v}^-=\vec{v}_{t-\Delta t/2}+\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\vec{v}_{t-\Delta t/2}=\vec{v}^- -\frac{q\vec{E}_t}{m}\frac{\Delta t}{2} \qquad \vec{v}_{t+\Delta t/2}=\vec{v}^+ +\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\vec{v}\,'=\vec{v}^- +\frac{q}{m}\frac{\Delta t}{2}\vec{v}^-\times\vec{B}_t$$

$$\frac{\vec{v}^+-\vec{v}^-}{\Delta t}=\frac{q}{2m}(\vec{v}^+ +\vec{v}^-)\times\vec{B}_t$$

$$\vec{v}\,'=\vec{v}^- +\vec{v}^-\times\vec{t} \qquad \vec{t}=\frac{qB_t}{m}\frac{\Delta t}{2}\hat{b}$$

$$\vec{v}^+=\vec{v}^- +\frac{q}{m}\frac{\Delta t}{2}\frac{2}{1+(\frac{q}{m}\frac{\Delta t}{2}\vec{B}_t)^2}\vec{v}\,'\times\vec{B}_t$$

$$\vec{v}^+=\vec{v}^- +\vec{v}\,'\times\vec{s} \qquad \vec{s}=\frac{2}{1+t^2}\vec{t}$$

$$\vec{v}_{t+\Delta t/2}=\vec{v}^+ +\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

Equation of motion

$$m\frac{d\vec{v}}{dt}=q\left(\vec{E}+\vec{v}\times\vec{B}\right)$$

$$\frac{d\vec{x}}{dt}=\vec{v}$$

$$\vec{E}_t=\vec{E}_t(\vec{x})$$

$$\vec{B}_t=\vec{B}_t(\vec{x})$$

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}\left[\vec{E}_t+\left(\frac{\vec{v}_{t+\Delta t/2}+\vec{v}_{t-\Delta t/2}}{2}\right)\times\vec{B}_t\right]$$

$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2}$$

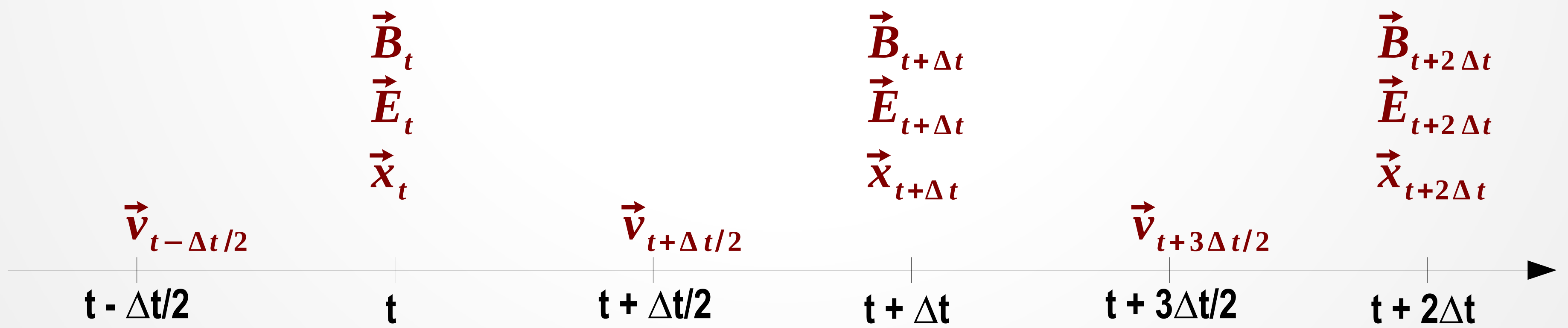
$$\vec{v}^{-}=\vec{v}_{t-\Delta t/2}+\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\vec{v}\,'=\vec{v}^{-}+\frac{q}{m}\frac{\Delta t}{2}\vec{v}^{-}\times\vec{B}_t$$

$$\vec{x}_{t+\Delta t}=\vec{x}_t+\vec{v}_{t+\Delta t/2}\times\Delta t$$

$$\vec{v}^{+}=\vec{v}^{-}+\frac{q}{m}\frac{\Delta t}{2}\frac{2}{1+\left(\frac{q}{m}\frac{\Delta t}{2}\vec{B}_t\right)^2}\vec{v}\,'\times\vec{B}_t$$

$$\vec{v}_{t+\Delta t/2}=\vec{v}^{+}+\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$



Particle pusher

Step 1. $\vec{E}_t=\vec{E}_t(\vec{x})$ $\quad$ $\vec{B}_t=\vec{B}_t(\vec{x})$ $\quad$ $\vec{v}_{t-\Delta t/2}$

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}\left[\vec{E}_t+\left(\frac{\vec{v}_{t+\Delta t/2}+\vec{v}_{t-\Delta t/2}}{2}\right)\times\vec{B}_t\right]$$

$$\vec{v}^{-}=\vec{v}_{t-\Delta t/2}+\frac{q\vec{E}_t}{m}\frac{\Delta t}{2} \qquad \vec{v}\,'=\vec{v}^{-}+\frac{q}{m}\frac{\Delta t}{2}\vec{v}^{-}\times\vec{B}_t$$

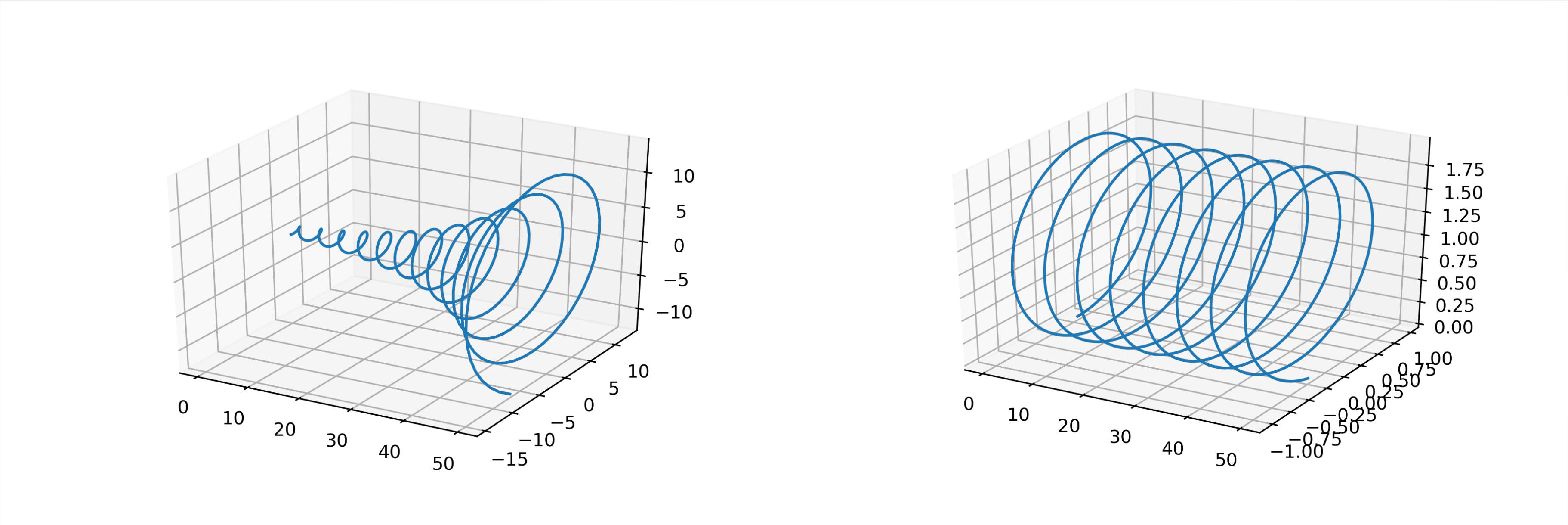
$$\vec{v}^{+}=\vec{v}^{-}+\frac{q}{m}\frac{\Delta t}{2}\frac{2}{1+\left(\frac{q}{m}\frac{\Delta t}{2}\vec{B}_t\right)^2}\vec{v}\,'\times\vec{B}_t \qquad \vec{v}_{t+\Delta t/2}=\vec{v}^{+}+\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

Step 2.

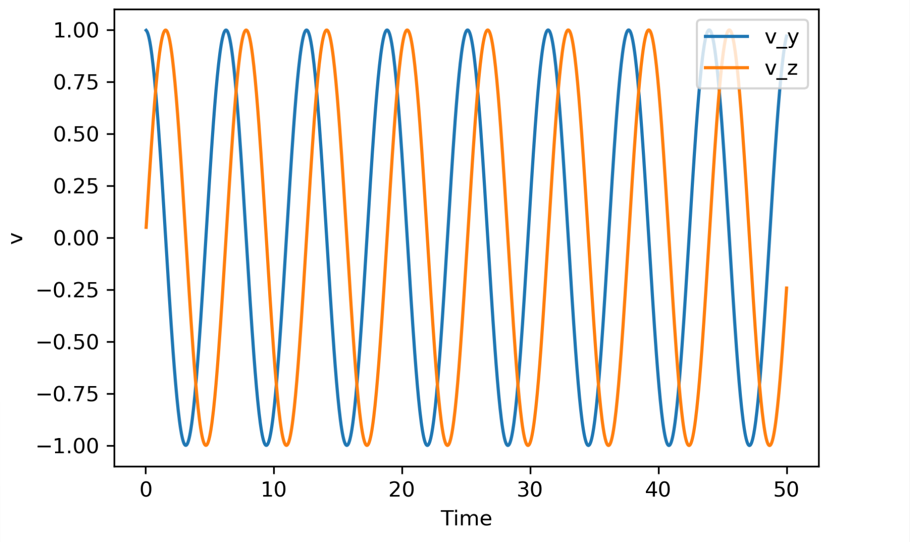
$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2} \qquad \vec{x}_{t+\Delta t}=\vec{x}_t+\vec{v}_{t+\Delta t/2}\times\Delta t$$

$t-\Delta t/2$	t	$t+\Delta t/2$	$t+\Delta t$	$t+3\Delta t/2$	$t+2\Delta t$
	$\vec{B}_t$		$\vec{B}_{t+\Delta t}$		$\vec{B}_{t+2\Delta t}$
	$\vec{E}_t$		$\vec{E}_{t+\Delta t}$		$\vec{E}_{t+2\Delta t}$
	$\vec{x}_t$		$\vec{x}_{t+\Delta t}$		$\vec{x}_{t+2\Delta t}$
$\vec{v}_{t-\Delta t/2}$		$\vec{v}_{t+\Delta t/2}$		$\vec{v}_{t+3\Delta t/2}$	

Particle pusher: example



Example:04_01_BorisPusher_withBug.f90



Particle pusher: hand-on

1. With only the external magnetic field, play the particle charge (+/-) and mass. Plot the interjection of particle…...

2. Try to interpret
   a) E x B drift

   b) gradient-B Drift

   c) a magnetic mirror machine