Vibração de Sistema com 2 Graus de Liberdade

Sistemas Dinâmicos II

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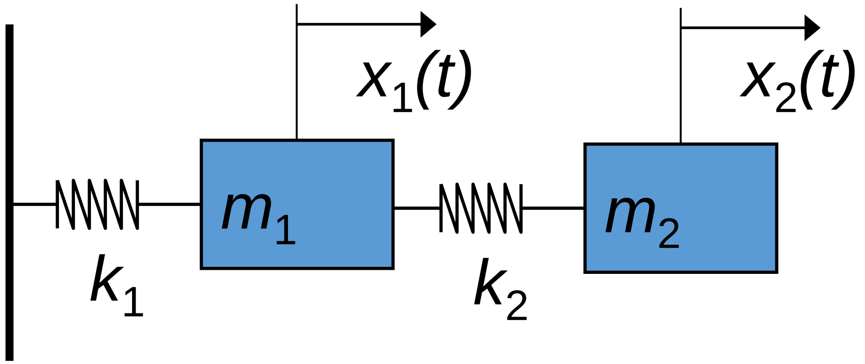
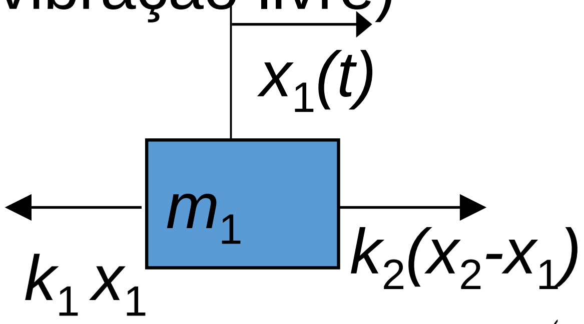
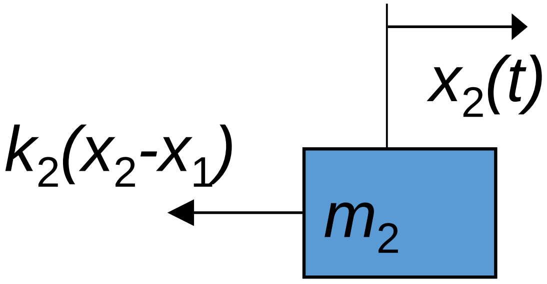
2 graus de liberdade

(sem amortecimento e vibração livre)







$$m_1\ddot{x}_1 + (k_1+k_2)x_1 - k_2x_2 = 0$$

$$m_2\ddot{x}_2 - k_2x_1 + k_2x_2 = 0$$

condições iniciais:

$$\dot{\mathbf{x}}(0) = \begin{bmatrix} \dot{x}_{10} \\ \dot{x}_{20} \end{bmatrix}$$

$$\mathbf{x}(0) = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix}$$

Forma matricial:

$$M\ddot{\mathbf{x}} + K\mathbf{x} = 0$$

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}; \; K = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}; \; \mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

solução...

$$\mathbf{x}(t) = \mathbf{u}e^{i\omega t}$$

$$\left(-\omega^2 M + K\right)\mathbf{u} = \mathbf{0}$$

$$\det\left(-\omega^2 M + K\right) = 0$$

$$m_1 m_2 \omega^4 + \left(m_1 k_2 + m_2 k_1 + m_2 k_2\right)\omega^2 + k_1 k_2 = 0$$

$$\pm\omega_1, \pm\omega_2$$

$$\left(-\omega_1^2 M + K\right)\mathbf{u}_1 = \mathbf{0}$$

$$\left(-\omega_2^2 M + K\right)\mathbf{u}_2 = \mathbf{0}$$

$$\mathbf{x}(t) = A_1 \sin\left(\omega_1 t + \phi_1\right)\mathbf{u}_1 + A_2 \sin\left(\omega_2 t + \phi_2\right)\mathbf{u}_2$$

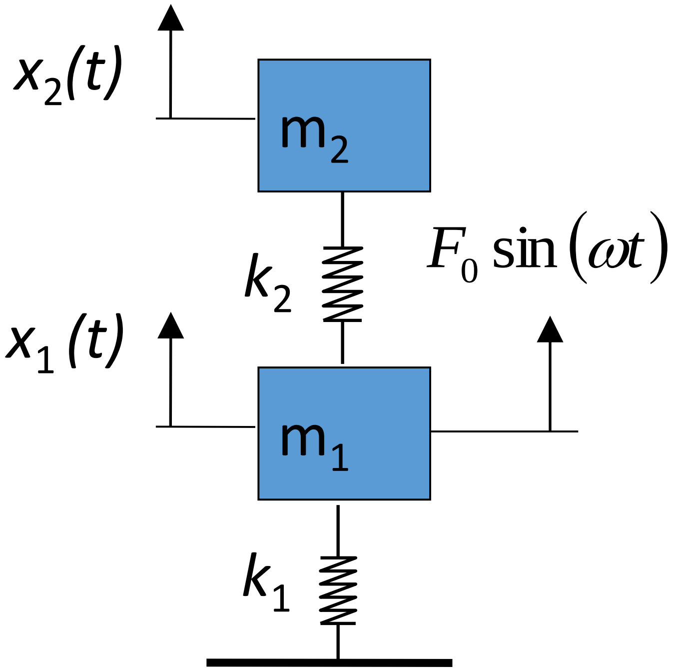
ω_1, ω_2 *freqüências naturais* $\mathbf{u}_1, \mathbf{u}_2$ *modos* de vibrar

Absorvedores de Vibrações

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix}\sin(\omega t)$$

Calcule k_2 e m_2 de modo que $x_1(t)$ seja o menor possível na resposta permanente.

$$\boxed{\begin{aligned} x_1(t) &= A_1 \sin \omega t \\ x_2(t) &= A_2 \sin \omega t \end{aligned}}$$



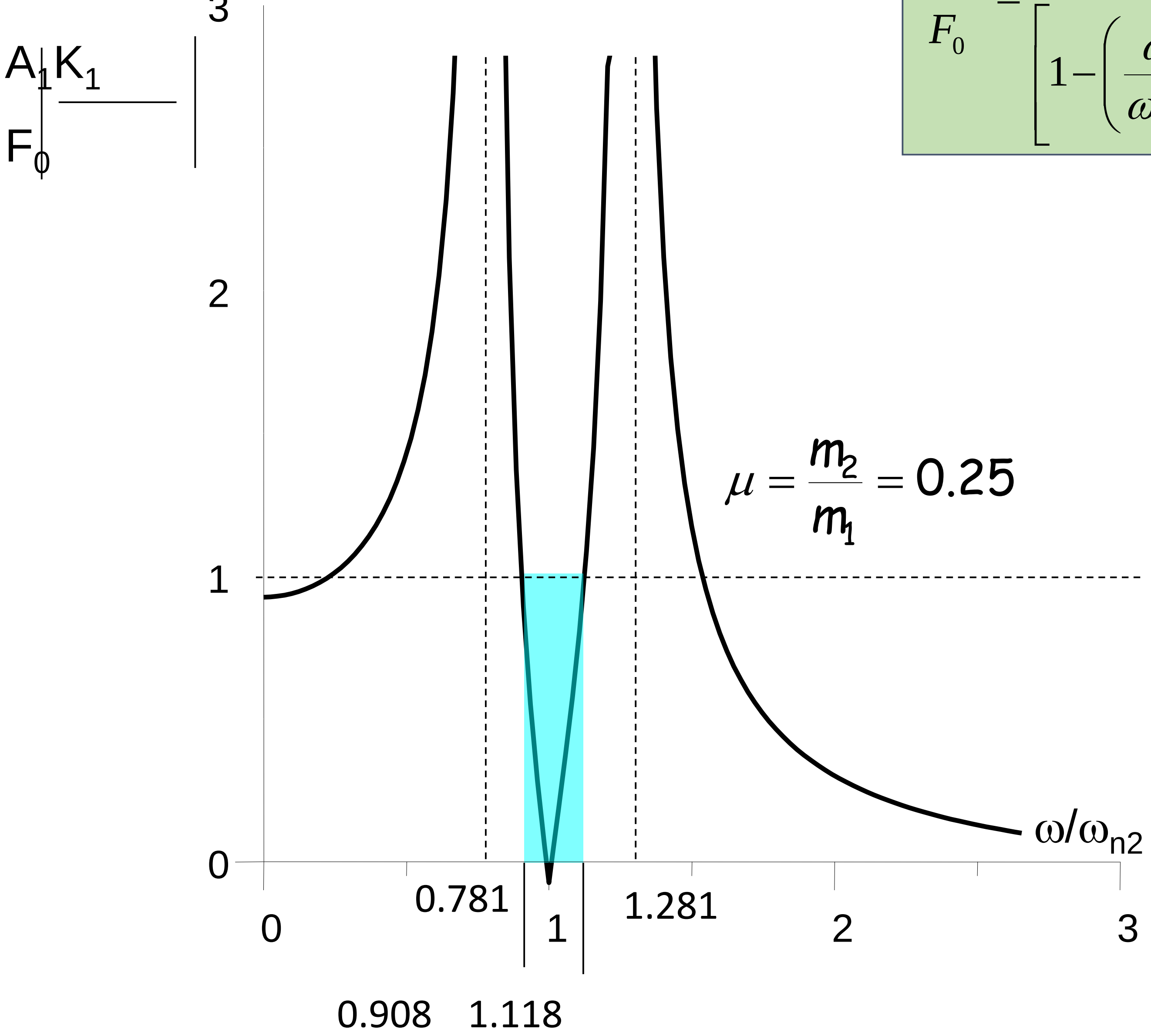
$$A_1 = \frac{\left(1-\omega^2/\omega_{n2}{}^2\right)F_0/k_1}{\left(1-\frac{\omega^2}{\omega_{n2}^2}\right)\left(1+\frac{k_2}{k_1}-\frac{\omega^2}{\omega_{n1}^2}\right)-\frac{k_2}{k_1}}$$

$$A_2 = \frac{F_0/k_1}{\left(1-\frac{\omega^2}{\omega_{n2}^2}\right)\left(1+\frac{k_2}{k_1}-\frac{\omega^2}{\omega_{n1}^2}\right)-\frac{k_2}{k_1}}$$

$$\omega_{n1}{}^2 = \frac{k_1}{m_1};\ \omega_{n2}{}^2 = \frac{k_2}{m_2}$$

$$\boxed{A_1 = 0 \Rightarrow \omega^2 = \frac{k_2}{m_2}} \quad \boxed{\Rightarrow} \quad \boxed{x_2 = -\frac{F_0}{k_2}\sin(\omega t)}$$

$$\frac{A_1 k_1}{F_0} = \frac{\left(1 - \omega^2/\omega_{n2}\right)}{\left[1 - \left(\frac{\omega}{\omega_{n2}}\right)^2\right]\left[1 + \mu\left(\frac{\omega_{n2}}{\omega_{n1}}\right)^2 - \left(\frac{\omega}{\omega_{n1}}\right)^2\right] - \mu\left(\frac{\omega_{n2}}{\omega_{n1}}\right)^2}$$



World Trade Center

Citicorp Center

Some buildings already use advanced wind-compensating dampers. The Citicorp Center in New York, for example, uses a **tuned mass damper**. In this complex system, oil hydraulic systems push a 400-ton concrete weight back and forth on one of the top floors, shifting the weight of the entire building from side to side. A sophisticated computer system carefully monitors how the wind is shifting the building and moves the weight accordingly. Some similar systems shift the building's weight based on the movement of giant pendulums.

From: www.howstuffworks.com https://www.youtube.com/watch?v=-6KnVhVvuB0 absorvedor torsional

https://www.youtube.com/watch?v=ohKqE_mwMmo Taipei absorvedor