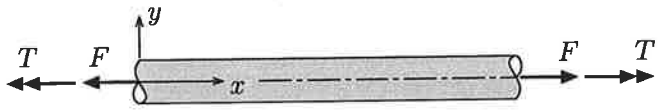
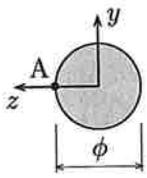


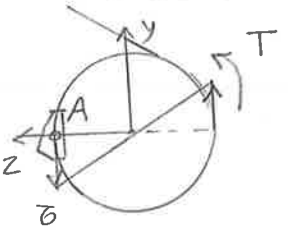
Seção transversal:



Ferro fundido
ASTM A-47
$f_t = 34 \ kN/cm^2$
$f_c = 62 \ kN/cm^2$

Aço S355
$f_y = 35{,}5 \ kN/cm^2$

1. Tensões na seção transv.

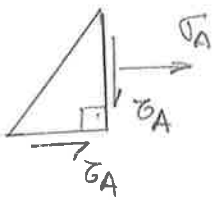


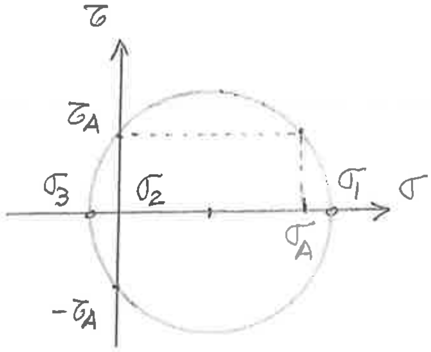
$$A = \pi R^2 = \pi 2^2 = 12{,}57 cm^2$$

$$I_p = \frac{\pi R^4}{2} = \frac{\pi 2^4}{2} = 25{,}13 \ cm^4$$

$$\sigma_A = \frac{N}{A} = \frac{F}{12{,}57} = 7{,}96 \times 10^{-2} F$$

$$\tau_A = \frac{M_T}{I_p} R = \frac{200}{25{,}13} \times 2 = 15{,}92$$





2. Tensões principais em A

$$\sigma_{1,3} = \frac{\sigma_A}{2} \pm \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} = 3{,}98 \times 10^{-2} F \pm \sqrt{(3{,}98 \times 10^{-2} F)^2 + 15{,}92^2}$$

$$\therefore \begin{cases} \sigma_3 = 3{,}98 \times 10^{-2} F - \sqrt{15{,}83 \times 10^{-4} F^2 + 253{,}3} \\ \sigma_2 = 0 \\ \sigma_1 = 3{,}98 \times 10^{-2} F + \sqrt{15{,}83 \times 10^{-4} F^2 + 253{,}3} \end{cases}$$

$$\tau_{máx} = (\sigma_1 - \sigma_3)/2 = \sqrt{15{,}83 \times 10^{-4} F^2 + 253{,}3}$$

3. Critérios de resistência

a) Ferro fundido ASTM-47 (material frágil)

– $\boxed{\text{Critério de Rankine}}$ $\quad \sigma_{eq} = \sigma_1 < f_t$

$$\sigma_{eq} = 3{,}98 \times 10^{-2} F + \sqrt{15{,}83 \times 10^{-4} F^2 + 253{,}3} = 34$$ ← condição limite de ruptura

$$15{,}83 \times 10^{-4} F^2 + 253{,}3 = (34 - 3{,}98 \times 10^{-2} F)^2$$
$$= 1156 - 2{,}706 F + 15{,}83 \times 10^{-4} F^2$$

$$2{,}706 F = 902{,}7 \Rightarrow F = 333{,}6 \ kN$$

– $\boxed{\text{Critério de Mohr - Coulomb}}$ $\sigma_{eq} = \sigma_1 - \frac{\sigma_3}{m} < f_t$ $m = \frac{f_c}{f_t}$

ou

$$\sigma_{eq} = \left(1 - \frac{1}{m}\right)\frac{\sigma_A}{2} + \left(1 + \frac{1}{m}\right)\sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2}$$

$$m = \frac{62}{34} = 1{,}824$$

$$1/m = 0{,}548$$

$$\sigma_{eq} = 0{,}452 \times 3{,}98 \times 10^{-2} F + 1{,}548 \sqrt{15{,}83 \times 10^{-4} F^2 + 253{,}3} = 34$$

$$15{,}83 \times 10^{-4} F^2 + 253{,}3 = (-1{,}162 \times 10^{-2} F + 21{,}96)^2$$

$$= 482{,}4 - 0{,}510 F + 1{,}35 \times 10^{-4} F^2$$

$$14{,}48 \times 10^{-4} F^2 + 0{,}510 F - 229{,}1 = 0$$

$F = 258{,}8$ kN

$\cancel{F = -610{,}6 \text{ kN}}$

b) Aço estrutural S355 (dúctil)

– $\boxed{\text{Critério de Tresca}}$ $\sigma_{eq} = \sigma_1 - \sigma_3 < f_y = 35{,}5$

Para vigas ($\sigma_2 = 0$): $\sigma_{eq} = \sqrt{\sigma_A^2 + 4\tau^2}$

$$\sigma_{eq} = 2\sqrt{15{,}83 \times 10^{-4} F^2 + 253{,}3} = 35{,}5$$

$$15{,}83 \times 10^{-4} F^2 + 253{,}3 = \left(\frac{35{,}5}{2}\right)^2$$

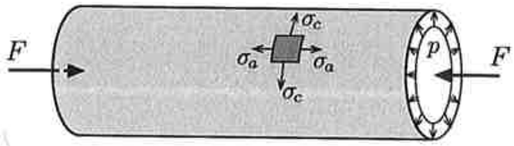
$$15{,}83 \times 10^{-4} F^2 = 61{,}76 \Rightarrow F = 197{,}5 \text{ kN}$$

– $\boxed{\text{Critério de von Mises}}$ $\sigma_{eq} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$

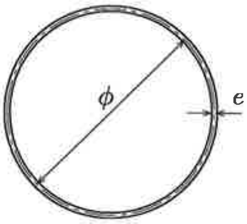
$$\sigma_{eq} = \sqrt{\frac{1}{2}\left[\sigma_1^2 + \sigma_3^2 + (\sigma_3 - \sigma_1)^2\right]} = \sqrt{\sigma_A^2 + 3\tau_A^2}$$ } caso particular p/ vigas ($\sigma_2 = 0$)

$$(7{,}96 \times 10^{-2} F)^2 + 3 \times 15{,}92^2 = 35{,}5^2$$

$$63{,}4 \times 10^{-4} F^2 = 499{,}9 \Rightarrow F = 280{,}8 \text{ kN}$$



Seção transversal:



$$R = \frac{\emptyset_m}{2} \approx \frac{\emptyset_{int}}{2} = 30\,cm$$

$$A = 2\pi R e = 60\pi e$$

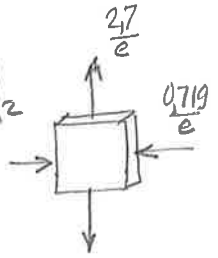
1. Estado de tensão na parede do tubo

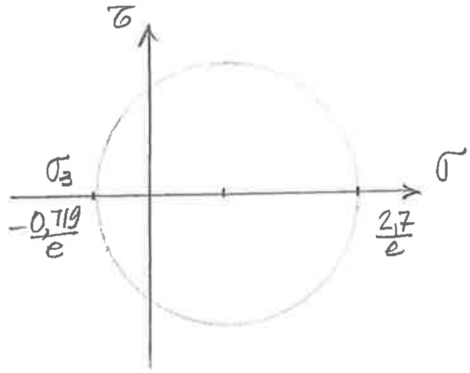
$$\sigma_a = \frac{N}{A} + \frac{pR}{2e} = -\frac{390}{60\pi e} + \frac{0{,}09 \times 30}{2e} = -\frac{2{,}069}{e} + \frac{1{,}35}{e} = -\frac{0{,}719}{e}$$

$$\sigma_b = \frac{pR}{e} = \frac{0{,}09 \times 30}{e} = \frac{2{,}7}{e}$$

– Tensões principais (admite-se que $p < \frac{0{,}719}{e}$ ou $\sigma_2 = -p$)

$$\begin{cases} \sigma_1 = \frac{2{,}7}{e} \\ \sigma_2 = -p = -0{,}09\ \frac{kN}{cm^2} \\ \sigma_3 = -\frac{0{,}719}{e} \end{cases}$$





2. Critérios de Resistência

a) ASTM A-48

– $\boxed{\text{Critério de Rankine}}$ $\quad \sigma_{eq} = \sigma_1 \leq \bar{f}_t$

$$\sigma_1 = \frac{2{,}7}{e} = \frac{17}{2{,}3} \Rightarrow e \leq 0{,}365\ cm$$

– $\boxed{\text{Critério de Mohr-Coulomb}}$ $\quad \sigma_{eq} = \sigma_1 - \frac{\sigma_3}{m} \leq \bar{f}_t \qquad m = \frac{f_c}{f_t} = \frac{65}{17} = 3{,}82$

$$\sigma_1 - \frac{\sigma_3}{m} = \frac{2{,}7}{e} - \frac{\left(-\frac{0{,}719}{e}\right)}{3{,}82} = \frac{17}{2{,}3} \Rightarrow \frac{2{,}888}{e} = \frac{17}{2{,}3} \Rightarrow e \leq 0{,}390\ cm$$

b) ASTM A-36

- $\boxed{\text{Critério de Tresca}}$; $\sigma_{eq} = \sigma_1 - \sigma_3 = \overline{f_y}$

$$\sigma_1 - \sigma_3 = \frac{2,7}{e} + \frac{0,719}{e} = \frac{25}{2,3} \Rightarrow e \geqslant 0,315 \text{ cm}$$

- $\boxed{\text{Critério de von Mises}}$ $\sigma_{eq} = \sqrt{\frac{1}{2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} = \overline{f_y}$

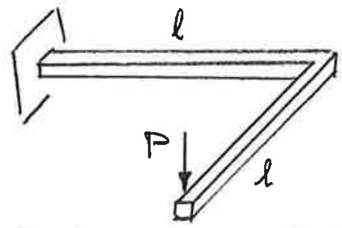
$$\left(\frac{2,7}{e}+0,09\right)^2 + \left(-0,09+\frac{0,719}{e}\right)^2 + \left(-\frac{0,719}{e}-\frac{2,7}{e}\right)^2 = 2\left(\frac{25}{2,3}\right)^2$$

resolvendo: $\underline{e = 0,288 \text{ cm}}$

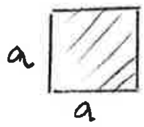
<u>Obs.</u> considerando $\sigma_2 = 0$ obteríamos $e = 0,287$ cm.

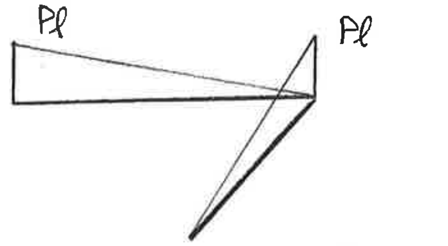
2014 jun 18

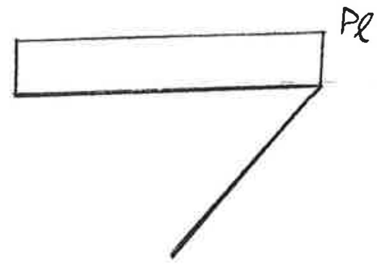
Ex. [Mário] Determine o máximo valor de P usando diferentes critérios de resistência. São dados: $\sigma_{rt} = 15\ \frac{kN}{cm^2}$, $\sigma_{rc} = 33\ \frac{kN}{cm^2}$, $\gamma = 3$, $l = 30\,cm$ e $a = 3\,cm$



ST:

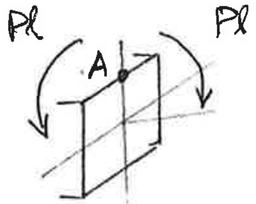






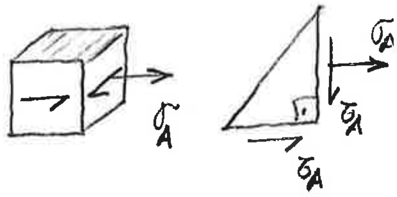
$Pl = 30P$ (kNcm)

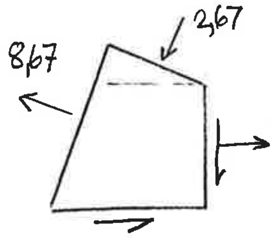
- a ST do engaste é a mais solicitada



$$\sigma_A = \frac{M}{W} = \frac{Pl}{a^3/6} = \frac{6Pl}{a^3}$$

$$\tau_A = \frac{M_T}{W_T} = \frac{Pl}{0{,}208\,a^3} = 4{,}81\,\frac{Pl}{a^3}$$





$$\sigma_{I,II} = \frac{\sigma_A}{2} \pm \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} = \left(3 \pm \sqrt{9 + 4{,}81^2}\right)\frac{Pl}{a^3} = (3 \pm 5{,}67)\frac{Pl}{a^3}$$

$$tg\,\alpha_1 = \frac{\sigma_I - \sigma_x}{\tau_{xy}} = \frac{8{,}67 - 6}{+4{,}81} = 0{,}555 \Rightarrow \alpha_1 = 29^\circ$$

$$\tau_{máx} = 5{,}67\,\frac{Pl}{a^3}$$

Estado Triplo: $\sigma_3 = -2{,}67\,\frac{Pl}{a^3}$; $\sigma_2 = 0$; $\sigma_1 = 8{,}67\,\frac{Pl}{a^3}$

Ferro fundido

a) Critério da máx. tensão normal (Rankine) $\quad \sigma_{eq} = \sigma_1 \leq \bar{\sigma}_t$

$$8{,}67 \cdot \frac{Pl}{a^3} \leq \frac{\sigma_{rt}}{3} \Rightarrow P \leq \frac{15}{3} \times \frac{3^3}{8{,}67 \times 30} = 0{,}52\ kN$$

b) Critério da máx. deformação linear ($\nu = 0{,}25$) $\quad \sigma_{eq} = \sigma_1 - \nu(\sigma_2 + \sigma_3) \leq \bar{\sigma}_t$

$$[8{,}67 - 0{,}25(0 - 2{,}67)]\,\frac{Pl}{a^3} \leq \frac{\sigma_{rt}}{3} \Rightarrow P \leq \frac{15}{3} \times \frac{3^3}{9{,}34 \times 30} = 0{,}48\ kN$$

c) Critério de Mohr-Coulomb $\quad \sigma_{eq} = \sigma_1 - \frac{\sigma_3}{m} \leq \bar{\sigma}_t \qquad m = \frac{|\bar{\sigma}_c|}{\bar{\sigma}_t}$

$$\left[8{,}67 - \frac{15}{33} \times (-2{,}67)\right]\frac{Pl}{a^3} \leq \frac{\sigma_{rt}}{3} \Rightarrow P \leq \frac{15}{3} \times \frac{3^3}{9{,}88 \times 30} = 0{,}46\ kN$$

2014 jun 18

Barra de aço: $\sigma_y = 21 \frac{kN}{cm^2}$ $\gamma = 2$

d) Critério de Tresca $\sigma_{eq} = \sigma_1 - \sigma_3 \leq \sigma_y$

$$\left[8{,}67 - (-2{,}67)\right] \frac{Pl}{a^3} \leq \frac{\sigma_y}{2} \Rightarrow P \leq \frac{21}{2} \times \frac{3^3}{11{,}34 \times 30} = \underline{0{,}83 \text{ kN}}$$

e) Critério de von Mises $\sigma_{eq} = \left\{\frac{1}{2}\left((\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right)\right\}^{1/2} \leq \sigma_t$

$$\left\{\frac{1}{2}\left[8{,}67^2 + 2{,}67^2 + 11{,}34^2\right]\right\}^{1/2} \frac{Pl}{a^3} \leq \frac{21}{2} \Rightarrow P \leq \frac{21}{2} \times \frac{3^3}{10{,}27 \times 30} = \underline{0{,}92 \text{ kN}}$$