PEF3208 - 2016 (Resolução de Exercícios)

1 Treliça

Calcular o valor da força normal nas barras 2,3,7 e 8.

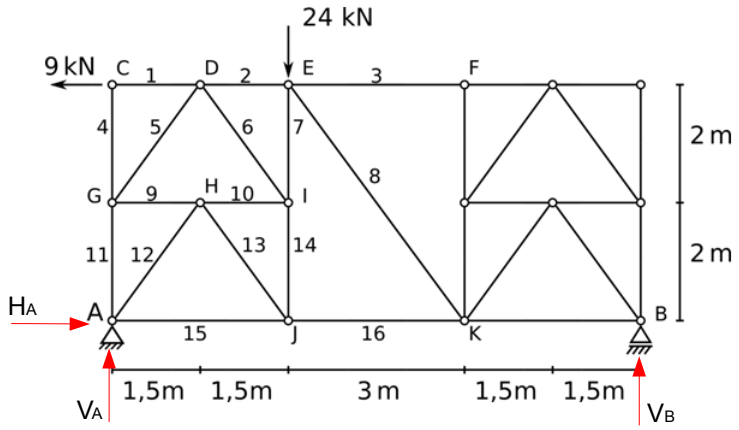

Figure 1: Treliça isostática

1.1 Cálculo das reações de apoio:

$$\sum F_H = 0: \qquad H_A - 9 = 0 \quad \therefore \quad H_A = 9\text{kN} \tag{1}$$

$$\sum F_V = 0: \qquad V_A + V_B - 24 = 0 \quad \therefore \quad V_A + V_B = 24\text{kN} \tag{2}$$

$$\sum M_A = 0: \qquad 9 \times 4 - 24 \times 3 + V_B \times 9 = 0 \quad \therefore \quad V_B = 4\text{kN} \quad \Rightarrow \quad V_A = 20\text{kN} \tag{3}$$

1.2 Cálculo das forças normais nas barras

Fazendo um corte na estrutura - método de Ritter ou método das seções:

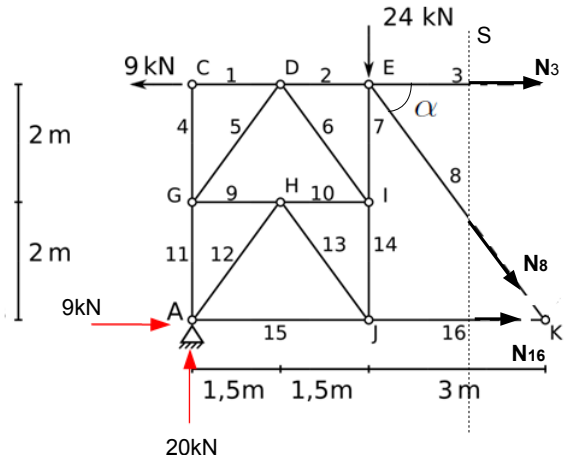

Figure 2: Corte na seção S

$$\sum M_K = 0: \qquad -20 \times 6 + 9 \times 4 + 24 \times 3 - N_3 \times 4 = 0 \quad \therefore \quad N_3 = -3\text{kN} \tag{4}$$

$$\sum F_V = 0: \qquad 20 - 24 - N_8 \times sen\alpha = 0 \quad \text{e} \quad sen\alpha = \frac{4}{5} \quad \therefore \quad N_8 = -5\text{kN} \tag{5}$$

Fazendo o equilíbrio do nó E:

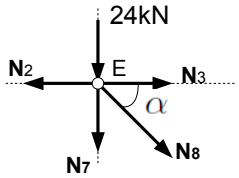

Figure 3: Nó E

$$\sum F_H = 0: \qquad -N_2 + N_3 + N_8 \times cos\alpha = 0 \quad \text{e} \quad cos\alpha = \frac{3}{5} \quad \therefore \quad N_2 = -6\text{kN} \tag{6}$$

$$\sum F_V = 0: \qquad -24 - N_7 - N_8 \times sen\alpha = 0 \quad \text{e} \quad sen\alpha = \frac{4}{5} \quad \therefore \quad N_7 = -20\text{kN} \tag{7}$$

2 Características geométricas de figuras planas

(H. Britto, v.2008, pág 67) - Determinar os momentos de Inércia I_Y e I_Z

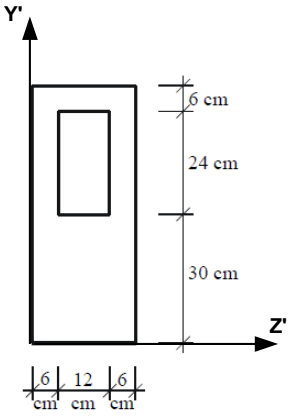

Figure 4: Figura plana

Cálculo do centro de gravidade:

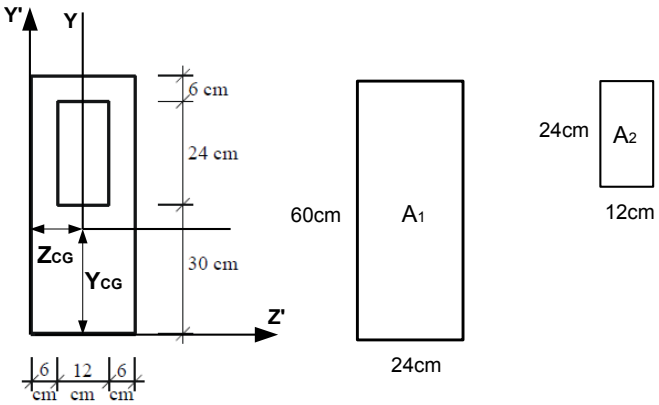

Figure 5: Cálculo do C.G.

Por simetria: $Z_{CG} = 12cm$

$$Y_{CG} = \frac{y_{CG1} \times A_1 - y_{CG2} \times A_2}{A_1 - A_2} \tag{8}$$

$$A_1 = \text{Área Total} \tag{9}$$

$$A_2 = \text{Área do Furo} \tag{10}$$

$$y_{CG1} = 30cm \tag{11}$$

$$y_{CG2} = 42cm \tag{12}$$

$$A_1 = 24 \times 60 = 1440cm^2 \tag{13}$$

$$A_2 = 12 \times 24 = 288cm^2 \tag{14}$$

$$Y_{CG} = 27cm \tag{15}$$

Encontrando o momento de Inércia em relação ao eixo $I_{Z_{CG}}$:

$$I_Z = I_{Z_1} - I_{Z_2} \tag{16}$$

$$I_{Z_1} = I_{z_1} + d_1^2 \times A_1 \tag{17}$$

$$I_{Z_2} = I_{z_2} + d_2^2 \times A_2 \tag{18}$$

$$d_1 = |Y_{CG} - y_{CG1}| = 3cm \tag{19}$$

$$d_2 = |Y_{CG} - y_{CG2}| = 15cm \tag{20}$$

$$I_{Z_1} = I_{z_1} + d_1^2 \times A_1 = 444960cm^4 \tag{21}$$

$$I_{Z_2} = I_{z_2} + d_2^2 \times A_2 = 78624cm^4 \tag{22}$$

$$I_Z = I_{Z_1} - I_{Z_2} = 366336cm^4 \tag{23}$$

Encontrando o momento de Inércia em relação ao eixo $I_{Y_{CG}}$:

$$I_Y = I_{Y_1} - I_{Y_2} \tag{24}$$

$$I_{Y_1} = I_{y_1} + d_1^2 \times A_1 \tag{25}$$

$$I_{Y_2} = I_{y_2} + d_2^2 \times A_2 \tag{26}$$

$$d_1 = |Z_{CG} - z_{CG1}| = 0 \tag{27}$$

$$d_2 = |Z_{CG} - z_{CG2}| = 0 \tag{28}$$

$$I_{Y_1} = I_{y_1} + d_1^2 \times A_1 = I_{y_1} = 69120cm^4 \tag{29}$$

$$I_{Y_2} = I_{y_2} + d_2^2 \times A_2 = I_{y_2} = 3456cm^4 \tag{30}$$

$$I_Y = I_{Y_1} - I_{Y_2} = 65664cm^4 \tag{31}$$

3 Flexão Composta Normal

(H. Britto, v.2008, pág 67) - Calcular as tensões normais extremas ($\sigma_{máx}$ e $\sigma_{mín}$) na seção transversal:

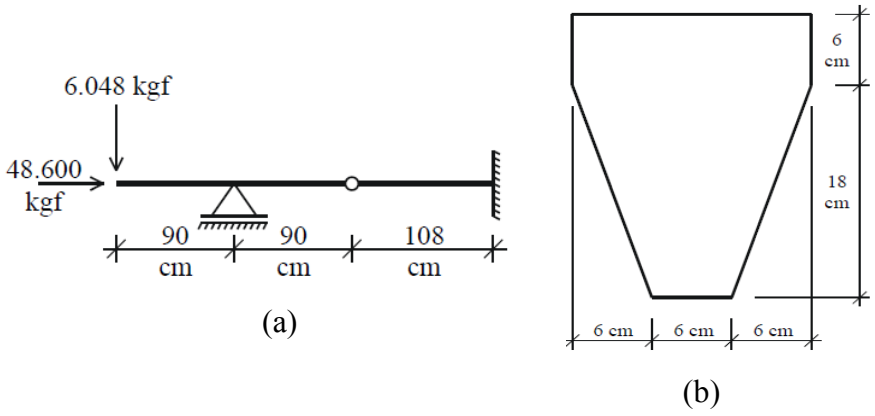

Figure 6: viga isostática

3.1 Cálculo das reações de apoio (esquações de equilíbrio da estática)

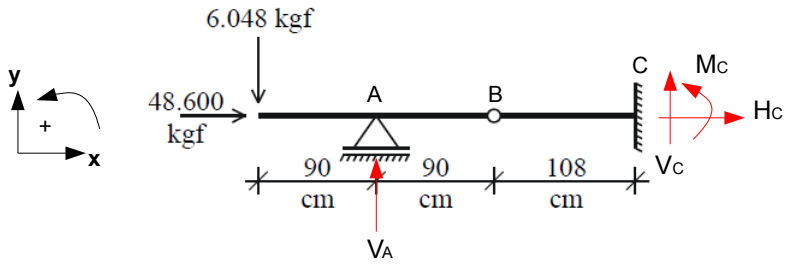

Figure 7: Cálculo das reações de apoio

$$\sum F_H = 0: \qquad 48600 + H_c = 0 \quad \therefore \quad H_C = -48600\text{kgf} \tag{32}$$

$$\sum F_V = 0: \qquad -6048 + V_A + V_C = 0 \quad \therefore \quad V_A + V_C = 6048\text{kgf} \tag{33}$$

$$\sum M_B(esq) = 0: \qquad 6048 \times (90+90) - V_A \times 90 = 0 \quad \therefore \quad V_A = 12096\text{kgf} \quad \Rightarrow \quad V_C = -6048\text{kgf} \tag{34}$$

$$\sum M_B(dir) = 0: \qquad V_C \times 108 + M_c = 0 \quad \therefore \quad M_C = 653184\text{kgf.cm} \tag{35}$$

3.2 Diagramas de esforços solicitantes

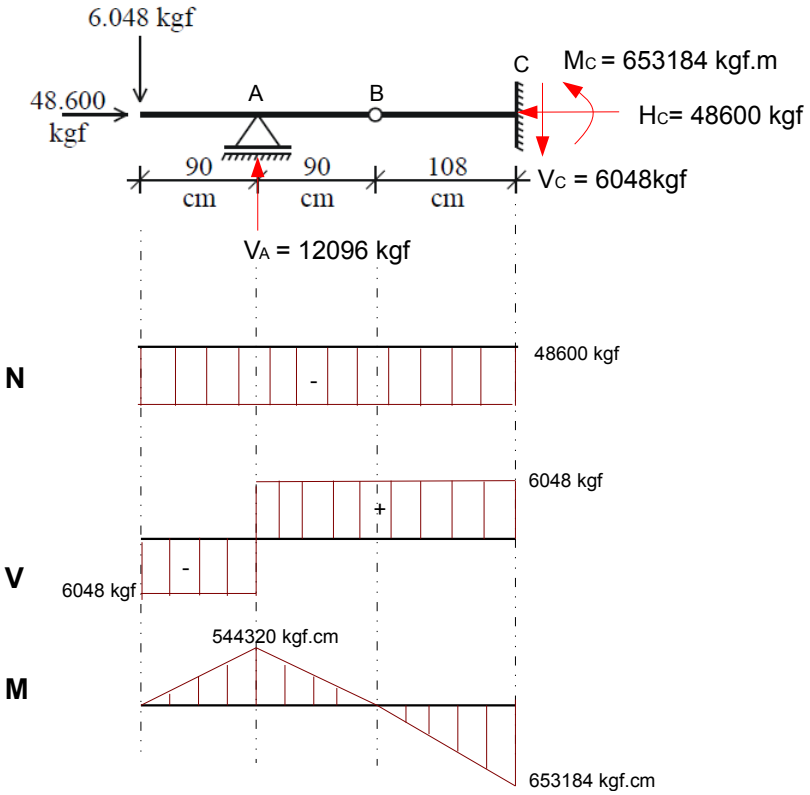

Figure 8: Diagramas de esforços solicitantes

3.3 Características Geométricas

3.3.1 Centro de gravidade

Por simetria: $Z_{CG} = 9cm$

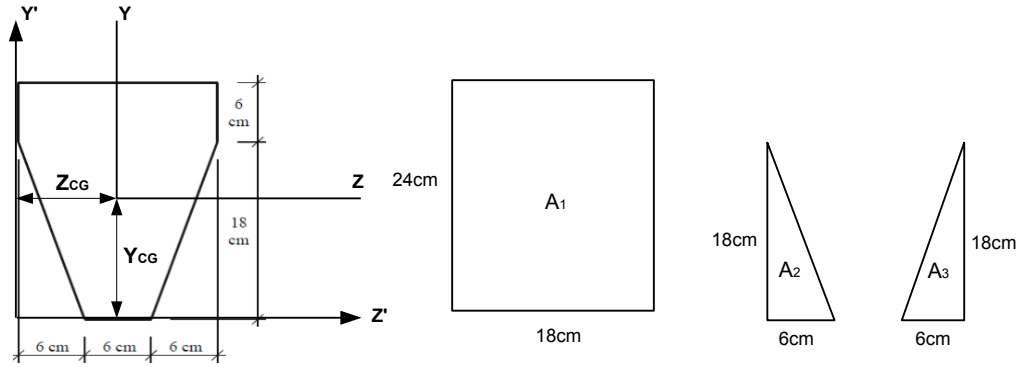

Figure 9: Cálculo do C.G.

$$Y_{CG} = \frac{y_{CG1} \times A_1 - y_{CG2} \times A_2 - y_{CG3} \times A_3}{A_1 - A_2 - A_3} \tag{36}$$

$$A_1 = 18 \times 24 = 432cm^2 \tag{37}$$

$$A_2 = \frac{18 \times 6}{2} = 54cm^2 \tag{38}$$

$$A_3 = A_2 = 54cm^2 \tag{39}$$

$$y_{CG1} = 12cm \tag{40}$$

$$y_{CG2} = 6cm \tag{41}$$

$$y_{CG3} = 6cm \tag{42}$$

$$Y_{CG} = \frac{12 \times 432 - 2 \times 6 \times 54}{432 - (2 \times 54)} = 14cm \tag{43}$$

3.3.2 Inércia em relação ao eixo Z (I_Z)

$$d_1 = |Y_{CG} - y_{CG1}| = 2cm \tag{44}$$

$$d_2 = |Y_{CG} - y_{CG2}| = 8cm \tag{45}$$

$$d_3 = |Y_{CG} - y_{CG3}| = 8cm \tag{46}$$

$$I_Z = I_{Z_1} - I_{Z_2} - I_{Z_3} \tag{47}$$

$$I_{Z_1} = I_{z_1} + d_1^2 \times A_1 = \frac{18 \times 24^3}{12} + 2^2 \times 432 = 22464cm^4 \tag{48}$$

$$I_{Z_2} = I_{z_2} + d_2^2 \times A_2 = \frac{6 \times 18^3}{36} + 8^2 \times 54 = 4428cm^4 \tag{49}$$

$$I_{Z_{CG3}} = I_{Z_{CG2}} = 4428cm^4 \tag{50}$$

$$I_{Z_{CG}} = 22464 - (2 \times 4428) = 13608cm^2 \tag{51}$$

3.3.3 Tensões normais extremas

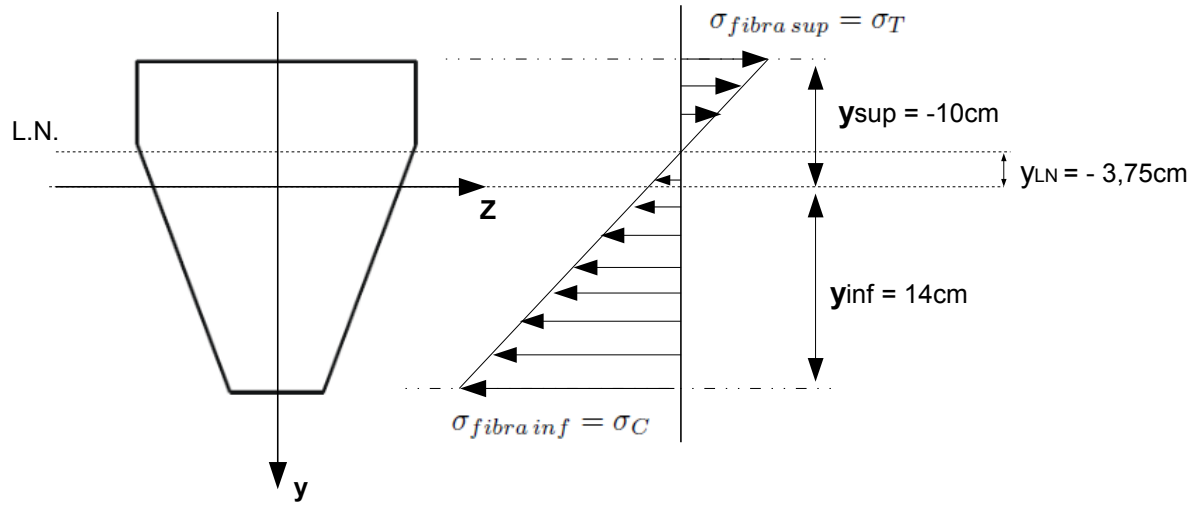

Figure 10: Seção A

Para Seção A (fibras superiores tracionando e inferiores comprimindo)

$$\sigma_{fibra\,sup} = \sigma_T = \frac{N}{A} + \frac{M_z}{I_z} \times y_{sup} = -\frac{48600}{324} - \frac{544320}{13608} \times (-10) = 250\text{kgf}/cm^2 \tag{52}$$

$$\sigma_{fibra\,inf} = \sigma_C = \frac{N}{A} + \frac{M_z}{I_z} \times y_{inf} = -\frac{48600}{324} - \frac{544320}{13608} \times 14 = -710\text{kgf}/cm^2 \tag{53}$$

posição da linha neutra:

$$-\frac{48600}{324} - \frac{544320}{13608} \times y_{LN} = 0 \quad \Rightarrow \quad y_{LN} = -3,75\text{cm} \tag{54}$$

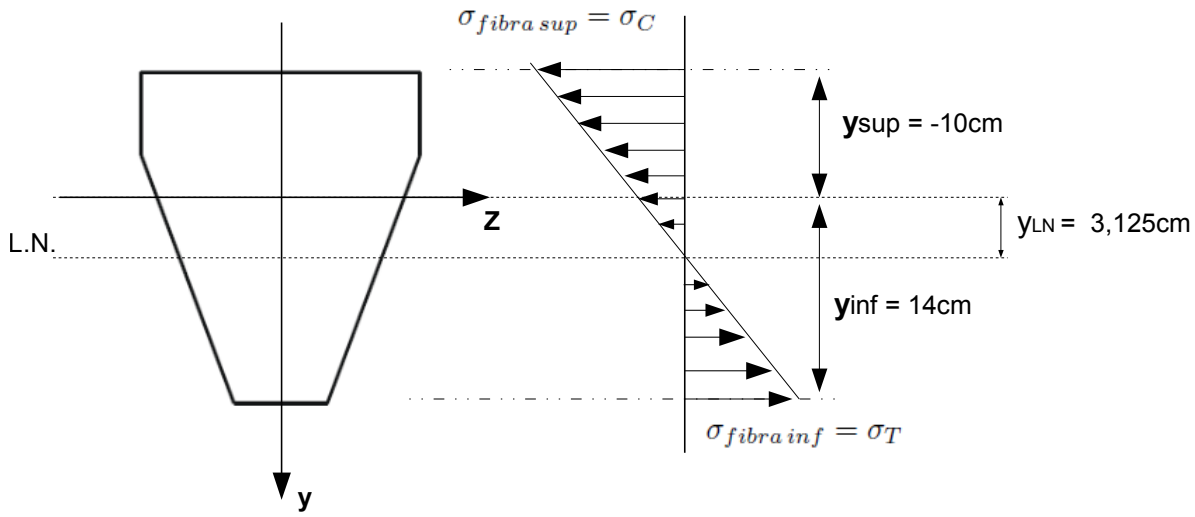

Figure 11: Seção C

Para Seção C (fibras superiores comprimindo e inferiores tracionando)

$$\sigma_{fibra\,sup} = \sigma_C = \frac{N}{A} + \frac{M_z}{I_z} \times y_{sup} = -\frac{48600}{324} + \frac{653184}{13608} \times (-10) = -630\text{kgf}/cm^2 \tag{55}$$

$$\sigma_{fibra\,inf} = \sigma_T = \frac{N}{A} + \frac{M_z}{I_z} \times y_{inf} = -\frac{48600}{324} + \frac{653184}{13608} \times 14 = 522\text{kgf}/cm^2 \tag{56}$$

posição da linha neutra:

$$-\frac{48600}{324} + \frac{653184}{13608} \times y_{LN} = 0 \quad \Rightarrow \quad y_{LN} = 3,125\text{cm} \tag{57}$$

Tensões máximas e mínimas são:

$$\sigma_{máx} = 522\text{kgf}/cm^2(\text{seção C - engastamento}) \tag{58}$$

$$\sigma_{mín} = -710\text{kgf}/cm^2(\text{seção A - apoio simples}) \tag{59}$$