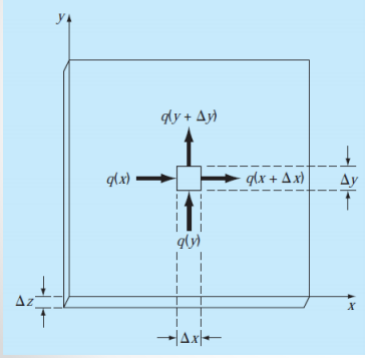
PHA 3002

Aula 12

Fluxo de Calor Bidimensional

$$q(x)\,\Delta y\,\Delta z\,\Delta t + q(y)\,\Delta x\,\Delta z\,\Delta t = q(x+\Delta x)\,\Delta y\,\Delta z\,\Delta t + q(y+\Delta y)\Delta x\,\Delta z\,\Delta t$$



$$q_i = -k\rho C\frac{\partial T}{\partial i}$$

$$T = \frac{H}{\rho C V}$$

Equação de Poisson

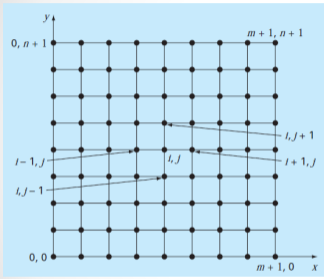
$$[q(x) - q(x+\Delta x)]\Delta y + [q(y) - q(y+\Delta y)]\Delta x = 0$$

$$\frac{q(x) - q(x+\Delta x)}{\Delta x}\Delta x\,\Delta y + \frac{q(y) - q(y+\Delta y)}{\Delta y}\Delta y\,\Delta x = 0$$

$$\rightarrow \quad -\frac{\partial q}{\partial x} - \frac{\partial q}{\partial y} = 0$$

Equação de Poisson $\quad \dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} = 0$

Solução Numérica



$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2}$$

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} = 0$$

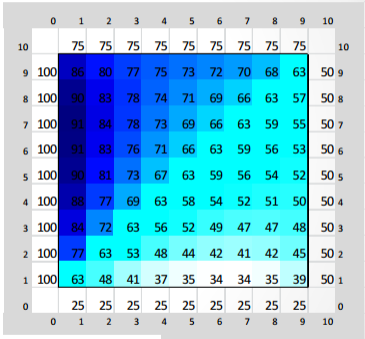
$$\Delta x = \Delta y$$

$$T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} = 0$$

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$$

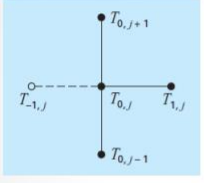
Exemplo

- Calcular as temperaturas na placa plana ao lado



$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$$

Condições de Contorno



$$T_{1,j} + T_{-1,j} + T_{0,j+1} + T_{0,j-1} - 4T_{0,j} = 0$$

Ponto imaginário

$$\frac{\partial T}{\partial x} \cong \frac{T_{1,j} - T_{-1,j}}{2\,\Delta x}$$

$$T_{-1,j} = T_{1,j} - 2\,\Delta x \frac{\partial T}{\partial x}$$

$$2T_{1,j} - 2\,\Delta x \frac{\partial T}{\partial x} + T_{0,j+1} + T_{0,j-1} - 4T_{0,j} = 0$$

$$T_{i+1,0} + T_{i-1,0} + 2T_{i,1} - 4T_{i,0} = 0$$

```
Sub prepara()

    Columns("A:BM").Select
    Selection.ColumnWidth = 0.2
    Rows("1:65").Select
    Selection.RowHeight = 2
    Range(Cells(1, 1), Cells(65, 65)).Interior.Color = RGB(0, 255, 255)
    Range("a1").Select


End Sub
```

Exercício

- Resolver o problema da temperatura variável no tempo da placa com dois lados isolados utilizando 100 elementos..