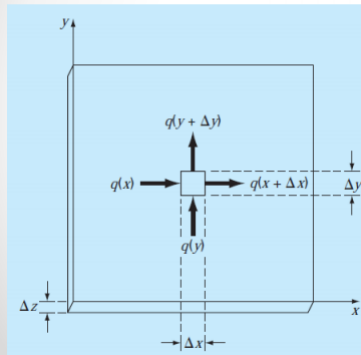


# PHA 3002

Aula 12

## Fluxo de Calor Bidimensional

$$q(x) \Delta y \Delta z \Delta t + q(y) \Delta x \Delta z \Delta t = q(x + \Delta x) \Delta y \Delta z \Delta t + q(y + \Delta y) \Delta x \Delta z \Delta t$$



$$q_t = -k\rho C \frac{\partial T}{\partial i}$$

$$T = \frac{H}{\rho C V}$$

# Equação de Poisson

$$[q(x) - q(x + \Delta x)]\Delta y + [q(y) - q(y + \Delta y)]\Delta x = 0$$

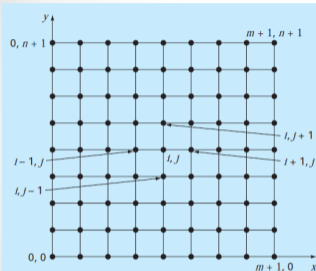
$$\frac{q(x) - q(x + \Delta x)}{\Delta x} \Delta x \Delta y + \frac{q(y) - q(y + \Delta y)}{\Delta y} \Delta y \Delta x = 0$$

$$\rightarrow -\frac{\partial q}{\partial x} - \frac{\partial q}{\partial y} = 0$$



Equação de Poisson  $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$

# Solução Numérica



$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2}$$

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} = 0$$

$$\Delta x = \Delta y \rightarrow$$

$$T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} - 4T_{i,j} = 0$$

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$$

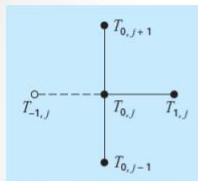
## Exemplo

- Calcular as temperaturas na placa plana ao lado

	0	1	2	3	4	5	6	7	8	9	10
10		75	75	75	75	75	75	75	75	75	
9	100	86	80	77	75	73	72	70	68	63	50
8	100	90	83	78	74	71	69	66	63	57	50
7	100	91	84	78	73	69	66	63	59	55	50
6	100	91	83	76	71	66	63	59	56	53	50
5	100	90	81	73	67	63	59	56	54	52	50
4	100	88	77	69	63	58	54	52	51	50	50
3	100	84	72	63	56	52	49	47	47	48	50
2	100	77	63	53	48	44	42	41	42	45	50
1	100	63	48	41	37	35	34	34	35	39	50
0		25	25	25	25	25	25	25	25	25	
	0	1	2	3	4	5	6	7	8	9	10

$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$$

## Condições de Contorno



$$T_{1,j} + T_{-1,j} + T_{0,j+1} + T_{0,j-1} - 4T_{0,j} = 0$$

Ponto  
imaginário

$$\frac{\partial T}{\partial x} \cong \frac{T_{1,j} - T_{-1,j}}{2 \Delta x}$$

$$T_{-1,j} = T_{1,j} - 2 \Delta x \frac{\partial T}{\partial x}$$

$$2T_{1,j} - 2 \Delta x \frac{\partial T}{\partial x} + T_{0,j+1} + T_{0,j-1} - 4T_{0,j} = 0$$

$$T_{i+1,0} + T_{i-1,0} + 2T_{i,1} - 4T_{i,0} = 0$$

```
Sub prepara()
```

```
Columns("A:BM").Select  
Selection.ColumnWidth = 0.2  
Rows("1:65").Select  
Selection.RowHeight = 2  
Range(Cells(1, 1), Cells(65, 65)).Interior.Color = RGB(0, 255, 255)  
Range("a1").Select
```

```
End Sub
```

## Exercício

- Resolver o problema da temperatura variável no tempo da placa com dois lados isolados utilizando 100 elementos..

