Resolução teste nº 5

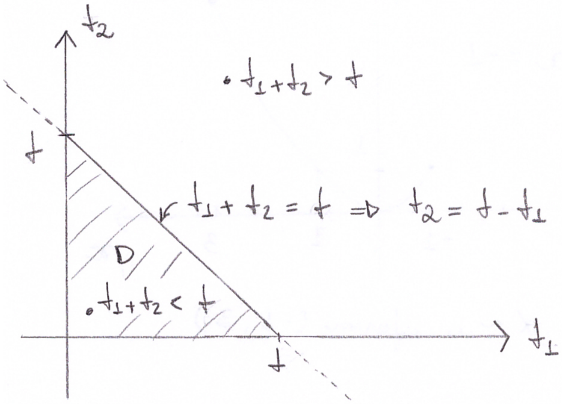
T_1 : tempo para ser atendido no guichê 1

T_2 : " " " " " " 2.

$$T_1 \sim \text{exponencial}(\lambda_1) \Rightarrow f_{T_1}(t_1) = \begin{cases} \lambda_1 e^{-\lambda_1 t_1}, & t_1 \geq 0 \\ 0, & c.c \end{cases}$$

$$T_2 \sim \text{exponencial}(\lambda_2) \Rightarrow f_{T_2}(t_2) = \begin{cases} \lambda_2 e^{-\lambda_2 t_2}, & t_2 \geq 0 \\ 0, & c.c. \end{cases}$$

$P[T_1 + T_2 \leq 10] = ?$



Precisamos de $f_{T_1 T_2}(t_1, t_2)$

Assumindo independência:

$$f_{T_1 T_2}(t_1, t_2) = f_{T_1}(t_1) \cdot f_{T_2}(t_2)$$

$$= \begin{cases} \lambda_1 \lambda_2 e^{-\lambda_1 t_1 - \lambda_2 t_2}, & t_1 \geq 0 \\ & t_2 \geq 0 \\ 0, & c.c. \end{cases}$$

$$P[T_1 + T_2 \leq T] = P[(t_1, t_2) \in D] = \int_0^t \int_0^{t-t_1} f_{T_1 T_2}(t_1, t_2)\, dt_2\, dt_1 =$$

$$= \int_0^t \int_0^{t-t_1} \lambda_1 \lambda_2 e^{-\lambda_1 t_1 - \lambda_2 t_2}\, dt_2 \cdot dt_1 = \lambda_1 \lambda_2 \int_0^t e^{-\lambda_1 t_1} \int_0^{t-t_1} e^{-\lambda_2 t_2}\, dt_2\, dt_1 =$$

$$= \lambda_1 \lambda_2 \cdot \int_0^t e^{-\lambda_1 t_1} \cdot \left. \frac{e^{-\lambda_2 t_2}}{(-\lambda_2)} \right|_0^{t-t_1} dt_1 = \lambda_1 \int_0^t e^{-\lambda_1 t_1} \cdot \left(1 - e^{-\lambda_2 (t - t_1)}\right) dt_1 =$$

$$= \lambda_1 \int_0^t \left(e^{-\lambda_1 t_1} - e^{-\lambda_2 t - \lambda_1 t_1 + \lambda_2 t_1}\right) dt_1 = \lambda_1 \int_0^t \left(e^{-\lambda_1 t_1} - e^{-\lambda_2 t} \cdot e^{-(\lambda_1 - \lambda_2) t_1}\right) dt_1 =$$

$$= \lambda_1 \cdot \left. \left(\frac{e^{-\lambda_1 t_1}}{-\lambda_1} - e^{-\lambda_2 t} \cdot \frac{e^{-(\lambda_1 - \lambda_2) t_1}}{-(\lambda_1 - \lambda_2)} \right) \right|_0^t = 1 - e^{-\lambda_1 t} + \frac{\lambda_1 \cdot e^{-\lambda_2 t}}{\lambda_1 - \lambda_2} \cdot \left(1 - e^{-(\lambda_1 - \lambda_2) t}\right)$$

Para $t = 10$: $P[T_1 + T_2 \leq 10] = 1 - e^{-10\lambda_1} + \frac{\lambda_1}{\lambda_1 - \lambda_2} \left(1 - e^{-10(\lambda_1 - \lambda_2)}\right) \cdot e^{-10\lambda_2}$