LEI DE HALLEY

$\vec{f}$: FORÇA DE UM GÁS POR UNIDADE DE VOLUME

$$F_z^{(1)} = f_z \Delta V = f_z \, dS \, dz$$

PRESSÃO DO GÁS NO VOLUME ΔV: $P' = -p(z+dz)$, $P = p(z)$

$$F_z^{(2)} = [P - P'] \, dS = -[p(z+dz) - p(z)] \, dS = -\frac{\partial p}{\partial z} \, dS \, dz$$

$$F_z^{(1)} + F_z^{(2)} = \left[f_z - \frac{\partial p}{\partial z}\right] dS \, dz = 0 \Rightarrow \boxed{f_z = \frac{\partial p}{\partial z}}$$

MAS $F_z^{(1)} = -mg = \underbrace{-\rho g}_{f_z} \Delta S \, dz \Rightarrow \boxed{\frac{\partial p}{\partial z} = -\rho g}$ (I)

- LEI DOS GASES PERFEITOS: $pV = \text{CONST} \Rightarrow p = \text{CONST} \times \rho$

$$\boxed{\frac{p(z)}{p(0)} = \frac{\rho(z)}{\rho(0)}} \quad \text{(II)}$$

(I) + (II): $\frac{dp}{\rho} = -g \, dz = \frac{dp}{\rho_0 \, p(z)} p_0 \Rightarrow \frac{dp}{p(z)} = -g \frac{\rho_0}{p_0} dz$

$$\therefore \boxed{p(z) = p_0 \, e^{-g \frac{\rho_0}{p_0} z}}$$

$$F^{(1)} + F^{(2)} = 0$$

$$F^{(1)} = f_z \, dS \, dz$$

$$F^{(2)} = -[p(z+dz) - p(z)] \, dS = -\frac{\partial p}{\partial z} \, dS \, dz$$

(1)

Noções de Mecânica Estatística

- Descrições: macro × micro

$p, V, T, C_v, \ldots$ × $\sim 10^{24}$ variáveis

• Teoria cinética dos gases: valores médios ⇒ estatística

Distribuição de velocidades

- Método de Boltzmann: campo gravitacional

$$\frac{p(z)}{p(0)} = \frac{\rho(z)}{\rho(0)} = e^{-g\frac{\rho(0)}{p(0)}z}$$

(Lei de Halley)

$$pV = mRT = \frac{M}{M_M}RT \Rightarrow p = \frac{M}{V}\frac{RT}{M_M} = \rho\frac{RT}{M_M} = \rho\frac{RT}{N_0 m} = \frac{\rho k T}{m}$$

$\therefore \frac{\rho(0)}{p(0)} = \frac{m}{kT}$

$\tilde{m}$ ... nº de part / unid de vol

$$\rho(z) = \tilde{m}(z)\times m \Rightarrow \boxed{\frac{\rho(z)}{\rho(0)} = \frac{\tilde{m}(z)}{\tilde{m}(0)} = e^{-\frac{mg}{kT}z}}$$

• Partida: coluna de gás à temp. constante T (eq. térmico)

• Hipótese: distribuição de velocid/s depende somente de T (c.f. TCG)

• Objetivo: obter $f(v_z)$

$$\langle v_z^+\rangle = \int_0^\infty f(u)\,u\,du$$

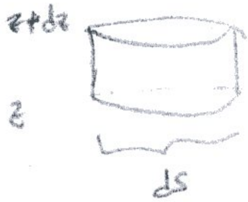
(Nota: $\langle v_z\rangle = 0$)

dS

$\chi(z)$ ... nº de part que atravessam dS "por baixo" no interv de tempo dt

$$\chi(z) = \tilde{m}(z)\langle v_z^+\rangle\, dS\,dt \Rightarrow \boxed{\frac{\chi(z)}{dS\,dt} = \tilde{m}(z)\langle v_z^+\rangle}$$

(2)



Nº DE PART. QUE "FICAM" ENTRE z E $z+dz$:

$$\frac{\chi(z)-\chi(z+dz)}{dS\,dt} = -\frac{d\tilde{m}(z)}{dz}\langle v_z^+\rangle\, dz$$

• IDÉIA DO CAMPO GRAVITACIONAL:

$$mgz = \frac{1}{2} m v_z^2$$

$$\frac{d}{dz}(mgz) = \frac{d}{dz}\left(\frac{1}{2} m v_z^2\right)$$

$$mg = m v_z \frac{dv_z}{dz} \Rightarrow \boxed{v_z \frac{dv_z}{dz} = g}$$

$$\frac{\chi(0)}{dS\,dt} = \tilde{m}(0)\, f(v_z)\, v_z\, dv_z$$

$$\Longrightarrow \frac{\chi(0)}{dS\,dt} = \frac{\chi(z)-\chi(z+dz)}{dS\,dt} \Rightarrow -\frac{d\tilde{m}(z)}{dz}\langle v_z^+\rangle = \tilde{m}(0)\, f(v_z)\, v_z \frac{dv_z}{dz} = \tilde{m}(0)\, f(v_z)\, g$$

$$\frac{d\tilde{m}(z)}{dz} = -\frac{mg}{kT}\tilde{m}(0)\, e^{-\frac{mg}{kT}z} \Rightarrow f(v_z) = \frac{m}{kT}\langle v_z^+\rangle e^{-\frac{mg}{kT}z} = \boxed{\frac{m}{kT}\langle v_z^+\rangle e^{-\frac{1}{2}\frac{m v_z^2}{kT}}}$$

$$\boxed{\text{FAZER AS GAUSSIANAS} \quad \int_{-\infty}^{\infty} e^{-\alpha x^2}\, dx \quad \text{E} \quad \int_{0}^{\infty} x\, e^{-\alpha x^2}\, dx}$$

PRÓX. PÁG.

$$f(v_z) = \frac{m}{kT}\langle v_z^+\rangle e^{-\frac{m v_z^2}{2kT}} = A\, e^{-\alpha v_z^2} \qquad \alpha = \frac{m}{2kT}$$

$$\int_{-\infty}^{\infty} f(v_z)\, dv_z = A\int_{-\infty}^{\infty} e^{-\alpha v_z^2}\, dv_z = 1$$

• ESTATÍSTICA:
- NOTAS DE ALUNOS
- AVALIAÇÃO DE PROFESSORES
- TEMPO DE VIDA MÉDIO (ABELHA: ~ 1 DIA)

INSTITUTO DE FÍSICA - USP

NOTA 2

NOME: ______________________
PROFESSOR: ______________________
DATA: ______________________

(3) $I = \int_{-\infty}^{\infty} e^{-\alpha x^2} dx \Rightarrow I^2 = \int_{-\infty}^{\infty} e^{-\alpha x^2} dx \int_{-\infty}^{\infty} e^{-\alpha y^2} dy$

$$I^2 = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\, e^{-\alpha(x^2+y^2)} = \int_0^{2\pi} d\theta \int_0^{\infty} dr\, r\, e^{-\alpha r^2} = \pi \int_0 dr\, 2r\, e^{-\alpha r^2}$$

$$= -\frac{\pi}{\alpha}\int_0^{\infty} dr\,(-2\alpha r)\, e^{-\alpha r^2} = -\frac{\pi}{\alpha}\int_0^{\infty} dr \frac{d}{dr}\left[e^{-\alpha r^2}\right] = -\frac{\pi}{\alpha}\left[e^{-\alpha r^2}\right]_0^{\infty} = -\frac{\pi}{\alpha}(0-1)$$

$$I^2 = \frac{\pi}{\alpha} \Rightarrow \boxed{I = \sqrt{\frac{\pi}{\alpha}}}$$

$$\therefore \int_{-\infty}^{\infty} A\, e^{-\alpha v_z^2} dv_z = A\sqrt{\frac{\pi}{\alpha}} = 1 \Rightarrow \boxed{A = \sqrt{\frac{\alpha}{\pi}}}$$

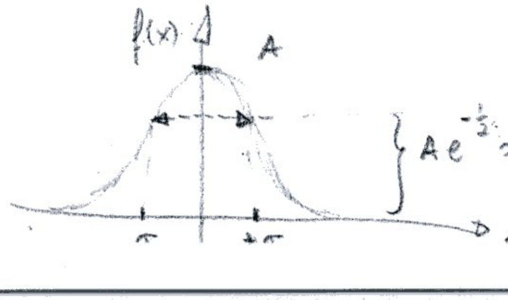
$$\boxed{f(v_z) = \sqrt{\frac{\alpha}{\pi}}\, e^{-\alpha v_z^2} = \sqrt{\frac{m}{2\pi kT}}\, e^{-\frac{m v_z^2}{2kT}}}$$

$\alpha = \frac{1}{2\sigma^2}$ σ ... DESVIO PADRÃO

$$f(x) = \sqrt{\frac{1}{2\pi\sigma^2}}\, e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2} \qquad \langle x^2\rangle = \int_{-\infty}^{\infty} f(x)\, x^2 dx$$

$$\langle x^2\rangle = \sqrt{\frac{\alpha}{\pi}}\int_{-\infty}^{\infty} x^2 e^{-\alpha x^2} dx = \sqrt{\frac{\alpha}{\pi}}\left(-\frac{d}{d\alpha}\right)\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\alpha}{\pi}}\,\frac{\sqrt{\pi}}{2\alpha^{3/2}} = \frac{1}{2\alpha} = \sigma^2$$

$$f(v_z) = \sqrt{\frac{m}{2\pi kT}}\, \exp\left[-\frac{1}{2}\left(\frac{\frac{1}{2} m v_z^2}{\frac{1}{2}kT}\right)\right]$$



$\} A e^{-\frac{1}{2}} \approx 0,61A$

INSTITUTO DE FÍSICA - USP

NOTA 3

NOME:
PROFESSOR:
DATA:

4 DISTRIBUIÇÃO DE MAXWELL

$$f(v_x)\,dx = f(-v_x)\,dx \Rightarrow f(v_x) = f(|v_x|) = f(v_x^2)$$

$$f(v_x, v_y, v_z)\,dv_x\,dv_y\,dv_z = f(v_x^2+v_y^2+v_z^2)\,dv_x\,dv_y\,dv_z$$

NÃO DEPENDEM DA DIREÇÃO...

PROBABILIDADE DE EVENTOS NÃO-CORRELACIONADOS:

$$f(v_x^2+v_y^2+v_z^2)\,dv_x\,dv_y\,dv_z = \left[f(v_x^2)\,dv_x\right] \times \left[f(v_y^2)\,dv_y\right] \times \left[f(v_z^2)\,dv_z\right]$$

$$f(v_x^2+v_y^2+v_z^2) = f(v_x^2)\,f(v_y^2)\,f(v_z^2)$$

$$e^{-\alpha(v_x^2+v_y^2+v_z^2)} = e^{-\alpha v_x^2}\,e^{-\alpha v_y^2}\,e^{-\alpha v_z^2}$$

$$\boxed{f(v_x, v_y, v_z) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{m(v_x^2+v_y^2+v_z^2)}{2kT}\right]}$$

- COORDENADAS POLARES

$$\int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{-\infty}^{\infty} dv_z\, f(v_x, v_y, v_z) = 1 = \int_0^{\infty} dv \int_0^{\pi} d\theta \int_0^{2\pi} d\varphi\, v^2 \operatorname{sen}\theta\, f = \int_0^{\infty} dv \left[4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}}\right]$$

$$\boxed{= \int_0^{\infty} dv\, F(v)}$$

INSTITUTO DE FÍSICA - USP

NOTA 4

NOME: ________
PROFESSOR: ________
DATA: ________

5 VELOCIDADES CARACTERÍSTICAS

- $v_{qm}^2 = \langle v^2 \rangle = \int_0^\infty v^2 F(v)\, dv = \frac{3kT}{m} = \frac{3RT}{M_{MOL}}$

$$v_{qm} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

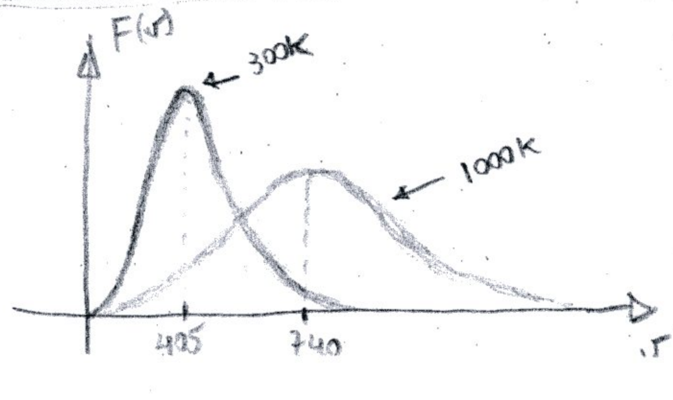
- VELOCIDADE (MÓDULO) MÉDIA:

$$\langle v \rangle = \langle |\vec{v}| \rangle = \int_0^\infty v F(v)\, dv = \sqrt{\frac{8kT}{\pi m}}$$

- VELOCIDADE MAIS PROVÁVEL:

$$\frac{d}{dv}\Big[F(v) \Big]_{v_p} = 0 \implies v_p = \sqrt{\frac{2kT}{m}}$$

$$\begin{pmatrix} v_{qm} \\ \langle v \rangle \\ v_p \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ \sqrt{8/\pi} \\ \sqrt{2} \end{pmatrix} \sqrt{\frac{kT}{m}} \approx \begin{pmatrix} 1,732 \\ 1,596 \\ 1,414 \end{pmatrix} \sqrt{\frac{kT}{m}}$$



- VERIFICAÇÃO EXPERIMENTAL

(6) GENERALIZAÇÃO: MAXWELL-BOLTZMANN

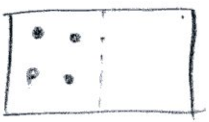
PROBAB. $\longrightarrow F(x,y,z;v_x,v_y,v_z)\,dx\,dy\,dz\;dv_x\,dv_y\,dv_z$

$$F(\vec{r},\vec{v}) = C\exp\left(-\frac{m\vec{v}^2}{2kT}\right)\exp\left(-\frac{mgz}{kT}\right)$$

$$= C\exp\left(-\frac{E}{kT}\right)$$

ENTROPIA: INTERPRETAÇÃO MICROSCÓPICA

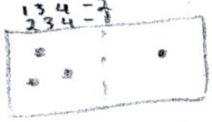
1234-



(1) MICROESTADO

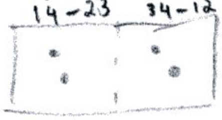
ANALOGIA: $(x+y)^{4} = \sum_{k=0}^{4} \frac{4!}{k!(4-k)!} x^{4-k} y^{k}$

123-4
124-3
134-2
234-1



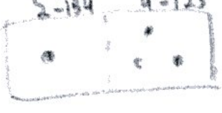
(4) MICROESTADOS

12-34 23-14
13-24 24-13
14-23 34-12



(6) MICROESTADOS

1-234 3-124
2-134 4-123



(4) MICROESTADOS

-1234

(1) MICROESTADO