How to teach friction: Experiments and models

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Students generally have difficulty understanding friction and its associated phenomena. High school and introductory college-level physics courses usually do not give the topic the attention it deserves. We have designed a sequence for teaching about friction between solids based on a didactic reconstruction of the relevant physics, as well as research findings about student conceptions. The sequence begins with demonstrations that illustrate different types of friction. Experiments are subsequently performed to motivate students to obtain quantitative relations in the form of phenomenological laws. To help students understand the mechanisms producing friction, models illustrating the processes taking place on the surface of bodies in contact are proposed.

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I. TEACHING FRICTION: TRADITIONAL TREATMENTS AND STUDENT DIFFICULTIES

In standard physics courses friction is usually presented as a marginal topic in a cursory, abstract, and schematic manner. The typical presentation focuses on the simplicity of the classic laws of static and kinetic friction between solids, rolling friction, and friction in fluids and obscures the complexity and variety of phenomena involving friction. As Hahner and Spencer (1998) write, "Though simply expressed, the laws of friction encapsulate a host of microscopic and nanoscopic phenomena whose elucidation has become one of the most fascinating pursuits in applied physics."[1] Yet in the standard treatment, apart from a brief mention of the effect of the roughness of surfaces, the solid bodies between which friction takes place are nearly always taken to be rigid and are often represented by rectangles moving on horizontal planes depicted by a line. This representation hinders attempts at creating an image of the underlying microscopic phenomena as the explanation of friction. Research has demonstrated[2] that the construction of an image is required by students to understand physical situations. It may be sufficient to use simplified laws to calculate the physical quantities necessary for solving a problem, but such laws cannot produce an understanding of the physical situation.[3]

Research on student conceptions has highlighted several difficulties that students encounter in understanding friction between solids.[4] For example, students rarely acknowledge that friction can play a motive role, and consider it almost exclusively as resistive. Friction force is often conceived as opposed to "actual" motion and not to the relative motion between two solids in contact. Consider the horizontal motion involving two objects, one placed on top of the other, and with an external force applied to the upper one. Students generally think the friction force acts only on the upper object and not also on the lower one (over-under effect). For vertical motion, such as when an object is in contact with a vertical surface or a cylinder is tightly placed inside a ring, the vast majority of students believe that there is only a single friction force at work, acting on the object in motion or stimulated to move. It is commonly thought that a solid can be dragged by another solid by "adhesion," without necessarily requiring the presence of a force to act on it explicitly (dragging effect). There also is a tendency to identify normal force with weight. This misconception is encouraged by examples that focus too much on cases where the normal force is equal to the weight or a component of the weight, such as horizontal motion or motion on an inclined plane.

In the following we will present an alternative to the standard approach to teaching friction between solids. Our sequence is designed to address the student difficulties noted in the literature, as well as to help students acquire the elements of an explanatory model necessary for the construction of an image of the mechanisms producing friction. The design of the sequence is based on the results of our preliminary analysis of didactic research on the topic, an overview of the usual approaches in textbooks, and an analysis of the scientific content, considered also in its historical development.

II. RECONSTRUCTING THE PHYSICS FROM A DIDACTIC PERSPECTIVE

Due to friction's near omnipresence in everyday life and technology, numerous examples and connections are possible. However, friction is not a fundamental topic and its phenomenology depends on the materials involved and the particular conditions of use. The study of friction has a long history, as well as recent developments, and many issues have yet to be resolved.[5] It represents a good example of the interface between abstract and formal theories and the reality of daily experience.

We are impressed by the complexity of the topic and the wide-ranging nature of its problems, applications, and theories.[6] We have to select content matter, models, and examples suitable for teaching purposes. This choice is common to all scientific topics, but it takes on particular importance in the case of friction because most teachers are not familiar with recent developments in the field and because many research results conflict with the laws given in most physics textbooks. For example, for sliding friction between solids, textbooks generally present three classic laws attributed to Amontons (1699) and Coulomb (1785), according to which the magnitude F of the friction force is proportional to the magnitude F_n of the normal force, independent of the area A of the contact surfaces and, in the dynamic case, independent of the relative speed u between the two surfaces. However, for some materials (for example, rubber, diamond, textile fibers, polymers, and numerous rocks) relations have been found of the type $F \propto F_n^k$, with $k<1$.[7] In many cases, the relation between the friction force and normal force cannot

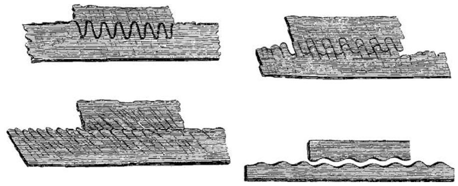

Fig. 1. The Coulomb model of friction due to interlocking and deformation of roughness on opposing surfaces (Ref. 11). The fibers of the wood surfaces penetrate each other, like the bristles of a brush. When one tries to slide the two objects past each other, the fibers will deform and bend each other until they separate and slip off.

be expressed by a simple expression. Moreover, there are various sticky materials, such as plasticine, putty, and resin, which are adhesive and present friction even without a load or with a "negative" load.[8] Similar behavior has been observed in nanotribology experiments, where many measurements yield the relation $F=\mu F_n+kA$, where kA is a purely adhesive term proportional to the area.[9]

The dependence of kinetic friction on velocity is even more problematic.[10] For many materials (for example, steel, copper, and lead), the friction force decreases as the velocity increases. A common example is friction between automobile tires and the road, for which the friction force also increases with the tire width and thus the area of contact. In contrast, for materials such as the polymer Teflon™, friction increases with velocity. In many cases only complicated empirical graphs are known. For example, for steel sliding on polymers such as polypropylene and butadiene acrylonitrile, a peak in the graph of friction versus speed is observed (see Ref. 10).

The mechanisms for the origin of friction have long been the object of controversy. According to Amontons and Coulomb (see Fig. 1), the origin of sliding friction lies mainly in the interlocking and deforming of the surface asperities of the materials. However, Desaguliers (1734) and Vince (1785) emphasized the importance of adhesion, and Tomlinson (1929) emphasized the role of energy dissipation due to phonons.[11]

It is presently believed that a variety of mechanisms are at work, with the relevance of each depending on the situation (adhesion, deformation, and plowing of surfaces, elastic hysteresis, abrasion, the effect of impurities and of absorbed layers). In 1992, the American Society of Mechanics Handbook reported: "Universal agreement as to what truly causes friction does not exist…Much still remains to be done before a complete picture can emerge."[12]

III. DIDACTIC CHOICES

We have made a few fundamental decisions regarding the design of the teaching sequence. They can be summarized as follows:

(a) Introduce friction as an omnipresent set of phenomena crucial for most everyday activities, phenomena which vary greatly while maintaining certain common traits. Teachers should offer an overview of the wide-ranging phenomenology of friction, including friction between solids, drag in fluids, and internal friction.

(b) Start the teaching sequence by giving examples in which friction is presented as an important phenomenon from the pragmatic point of view and as a positive resource, rather than merely as an obstacle or loss. These examples should present friction as a central object of study and not as a phenomenon to be eliminated.

(c) Emphasize the crucial role of friction in establishing equilibrium after a stress or motion. Such an observation serves to focus attention on a fundamental aspect of friction whose relevance in practical life is often neglected. Moreover, it is propaedeutic to the study of energy dissipation.

(d) Refute from the start the idea that friction always has a resistive effect, generating a force that invariably opposes motion and acts only on an object that is in motion or induced to move.

(e) Avoid an overemphasis on situations with horizontal friction forces, which can favor the identification of normal force with weight. For this purpose we recommend presenting examples from the start with a vertical friction force where the normal force is not related to or is equal to weight. Even for a "pressing" force equal to weight, it should be emphasized that this force is not the weight. For this reason, the symbol F_n is used instead of W, which can suggest the idea of weight.

(f) Formal models conceived in terms of functional relations are inadequate for the students' needs of understanding. We suggest using appropriate structural models, that is, models describing some aspects of the material structure of solid surfaces and of the physical processes producing friction. These models, involving visual representations and stimulating intuition, can help students overcome common difficulties concerning this topic and build mental models of mechanisms producing friction. Even presented in a simplified and qualitative way, the models allow reasoning, interpretations, and predictions concerning friction phenomena and are cognitively fertile because they stimulate questioning about the entities and processes presumed to exist within the material system. The incompleteness of these models should be discussed immediately, as well as the degree to which they fit physical reality.

IV. DESCRIPTION OF THE TEACHING LEARNING SEQUENCE

The sequence of topics is organized into five parts: (1) introductory experiments and observations; (2) vertical friction force: definition of descriptive quantities and first qualitative relations; (3) static and kinetic friction and phenomenological laws; (4) topography of surfaces and mechanisms producing friction; and (5) friction phenomena from the point of view of energy.

Rolling friction is not treated explicitly, but is mentioned in the first and fourth parts of the sequence. Part (5) can be treated after the other parts, after the fundamental concepts about energy have been introduced. The sequence has been tested for two years in the post-graduate school for physics teacher education at the University of Pavia, and subsequently, in an adapted format, in high school classes.

In the following we describe the main activities of the sequence, paying special attention to the experiments involved.

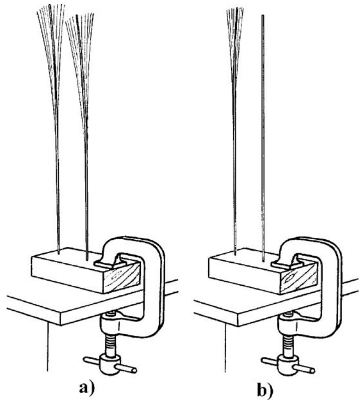

Fig. 2. Example of internal friction. (a) Two steel wires of the same size begin oscillating at the same time. (b) A little later, one wire has stopped vibrating, and the other is still oscillating. The former wire had been heated with a flame to about 800 °C and then slowly cooled, dramatically increasing its damping.

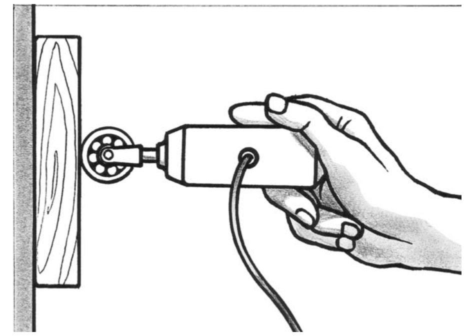

Fig. 3. A small wooden board is pressed against a wall by a force sensor. By pressing more or less, we can either prevent or allow the board to slide along the wall. The force sensor is equipped with a small ball bearing on its tip to minimize friction between the force sensor and the board.

A. Introductory examples and experiments

Some simple qualitative experiments can illustrate the various types of friction in different situations, presenting friction both as an obstacle and a disturbance, as well as a useful and desired phenomenon.

An initial motivating question is “What would happen if there were no friction?” How could simple daily activities such as picking up a bottle, walking, weighing with a spring scale, pouring liquid into a container, rounding a curve in a car, carrying glasses and cups on a tray, and playing baseball take place without friction? To elicit the notion of rolling friction, we use a metal cylinder rolling first on the floor and then on a strip of foam rubber. The idea of drag (friction on solid objects moving in fluids) can be introduced by observing light objects falling through the air and metal pellets falling in tubes full of water or another liquid such as glycerin or oil. The observation of damped oscillations of various liquids (for example, water and oil or glycerin) in transparent containers can introduce internal friction in fluids. Analogous observations of the different damping times of the oscillations of two metal wires of the same size can suggest the presence of internal friction in elastic solids as well (see Fig. 2).

The role of adhesion and the behavior of sticky materials can be illustrated by two simple demonstrations: a small block of plasticine pressed against a vertical wooden board which does not slide down when released, and a small wet polystyrene board applied against a door, which also sticks.

At the end of this phase, students should be aware of the vastness and significance of the topic and be able to distinguish among the various types of friction: drag, internal friction, friction between solids, sliding and rolling, and static and kinetic friction. They should also be sufficiently motivated to study the topic further, or at least be convinced of the utility of an in-depth study of these phenomena.

B. Experiments involving vertical friction force

In this part we use Newton’s laws to analyze experiments in terms of forces, and introduce the necessary descriptive quantities such as the normal force F_n, the friction force F, and the contact area A. We aim for students to formulate preliminary hypotheses regarding the relations between these quantities and to think about the interdependence of the three types of force involved: normal force, tangential force, and friction force. To keep students from identifying normal force with weight, experiments are done in which a vertical friction force is present and the normal force is not related to weight.

In the first experiment small wooden boards are pressed against a wall. We provide wooden boards of equal width and length but varying thickness, to vary the weight while maintaining the area constant. One side of the boards is covered with cloth or paper. Students observe that by varying the horizontal force exerted, they can either prevent or allow the board to slide along the wall. At first students push with their finger, so as to feel the subjective physical sensation of the variation in the thrust and its effects on the movement of the board. By repeating the experiment with a thicker and heavier board or by pressing the side covered with cloth against the wall, students observe that they must press more or less with their finger in order to stop the board from falling. The presence of non-negligible friction between the fingertip and the board adds a tangential force, which alters the evaluation of the friction force on the board due to the wall. To minimize this effect, we suggest that students use the surface of their fingernail, which causes a much weaker friction force than does their fingertip. In this way, they feel that they have to push more forcefully to prevent the board from sliding.

The next step is to repeat the experiments by pushing with a force sensor to measure the force. To minimize the friction between the force sensor and the board, we equip the force sensor with a small ball bearing on its tip (Fig. 3). We use PASCO PS-2104 force sensors. This experiment is an initial exploration of the phenomenon. Students do it without systematic data collection, but they write down the values of the force that they find, as well as their ongoing observations and conclusions. Afterward the teacher shows other ways of producing a normal force on the contact surfaces. A piece of iron is pressed against a glass by the attractive force of a magnet (Fig. 4). A wooden paddle, accelerating horizontally, is pressed against a block. For sufficiently large acceleration, the block does not fall (Fig. 5).

The experiments in this phase highlight some qualitative properties, such as the increase of friction force with the pressing force, and the dependence of friction on the nature and state of the contact surfaces. These initial observations

Fig. 4. A magnet presses a piece of iron against a glass. The attractive force between the magnet and the iron produces horizontal normal forces, which trigger vertical friction forces counteracting the weight of the iron pieces.

provide the starting hypotheses and motivation for a more detailed quantitative study, which will be the aim of the subsequent experiments.

C. Static and kinetic friction: Phenomenological laws

This section begins with three simple demonstrations designed to reinforce the realization that action-reaction friction forces occur on two surfaces in contact, as specified by Newton's third law. In these demonstrations a block is pulled while placed on three different media: a long strip of paper, a woolen scarf, and on a small cart. Students observe that the paper, scarf, and cart are dragged by the block, due to the friction force exerted by it.

We next do a more systematic experiment involving horizontal motion, with the aid of a computer data acquisition system. Students are divided into groups of twos or threes, and each group is provided with a motion detector, a force sensor, and wooden boards of varying surface area. The force sensor is applied to a board, and small blocks of different

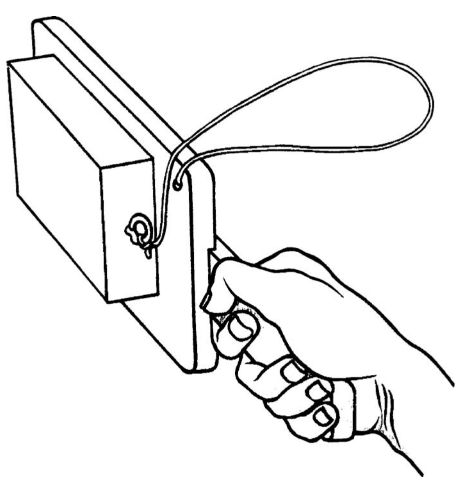

Fig. 5. A wooden paddle, which is accelerating horizontally, presses against a wooden block, thus provoking a vertical friction force, which can counteract the weight of the block.

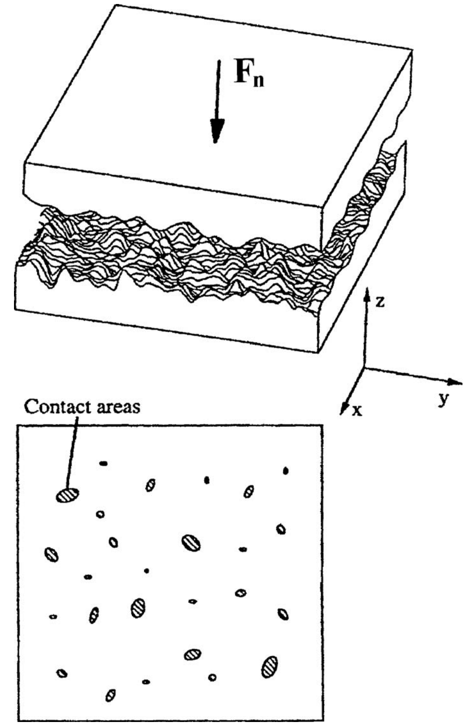

Fig. 6. The image from Bhushan, Ref. 6, p. 147, shows that the real area of contact is a small fraction of the apparent (macroscopic) area of contact.

mass are then fixed to the board to vary the normal force. By analyzing graphs of the applied force versus time, students can evaluate the static friction force at breakaway when the block starts to move and the maximum static force occurs, the dependence of the maximum friction force on the normal force, the dependence of the kinetic friction force on the normal force, and the dependence of the friction force on the area of the board.

The static and kinetic friction coefficients μ_s and μ_k are defined as the ratio of the maximum static friction force and the kinetic friction force, respectively, to the normal force. The role and the validity of the relations $F \leqslant \mu_s F_n$, $F = \mu_k F_n$, and F independent of the contact area are then discussed in light of their observations. We stress that these relations are phenomenological laws which are valid in many cases but not in all, and require an explanation on the basis of the properties of bodies in contact. Some examples are given that exhibit alternate relations, for example, $F \propto F_n^k$ with $k < 1$, or patterns which cannot be described by a mathematical formula.[13]

For this purpose, students repeat the same experiments after having covered the surfaces in contact by transparency films and/or rubber sheets. They find in these cases that the friction force is clearly not proportional to normal force (it increases less than proportionally), and that it depends on the contact area of the block. We extend this point to sticky materials such as putty and plasticine, also showing that for these materials there can be a friction force with zero or negative load (see Ref. 8).

We emphasize the nature of the inequality in the static friction law, as well as the fact that this force varies in magnitude and direction depending on the external force, and so inhibits the relative motion of the two surfaces in contact. We also stress that although the static friction force inhibits the

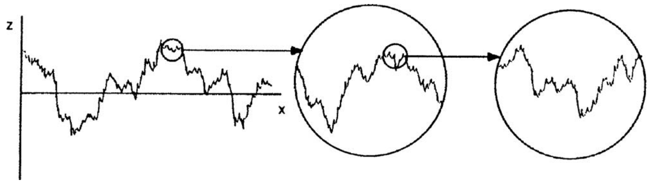

Fig. 7. The multiscale level of asperities, showing a rough fractal structure; the figure from Bhushan, Ref. 6, p. 49, represents a surface profile viewed at different magnifications. Note that the slope of the asperities is exaggerated and increases with magnification.

relative motion of the two surfaces in contact (or, better, just for that reason), it can play the role of motive force in many cases. For example, when we place a box on a cart and then accelerate the cart, the static friction force prevents the motion of the box relative to the cart, and so accelerates the box in the direction of the cart acceleration.

We point out that the friction coefficient is almost constant for certain materials and velocity ranges, but we also discuss examples where friction decreases or increases with velocity, or exhibits an even more complicated pattern (see Sec. II and Ref. 10). We illustrate the case of friction between automobile tires and the road, using friction coefficient tables for car breaking distances, showing that in this case friction decreases as velocity increases.

D. Structural models: Surface topography and friction-producing mechanisms

We present here the main characteristics of the generally accepted models of friction, as well as some methods of investigation and research results involving the topography of surfaces. The aim is to help students understand the distinction between apparent (or nominal) area and the real area of contact. Figures from the literature can be used to illustrate the irregular nature of the surface of apparently smooth objects when viewed on a micrometric scale, on which we observe asperities (see Fig. 6). It is important to help students to understand that asperities exist on many different scales, from micrometer to nanometer (see Fig. 7).[14]

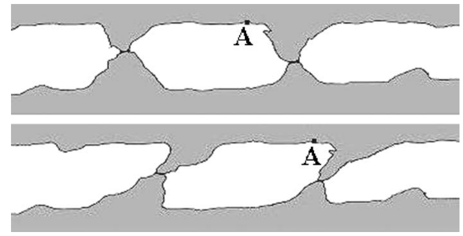

Fig. 9. A description of the adhesive junction model of Bowden and Tabor (Ref. 6). The slope of the asperities is magnified; in reality they are much smoother. Similar figures can be found in Ref. 20.

We next address the mechanisms producing friction. We stress that there are a variety of phenomena, the relevance of which varies according to the situations and materials considered. A number of mechanisms are presented in a descriptive and intuitive form. These include adhesion between the asperities of surfaces, the deformation, tracking, or scratching of surfaces, the impact and interlocking among asperities, wear due to the relative motion of the two contact surfaces, and the effect due to particles trapped between the surfaces (third body).

Some historical explanatory models of sliding friction phenomena are considered: Bélidor's model of spherical asperities[15] (Fig. 8), Coulomb's model of interlocking asperities (Fig. 1), and the Bowden and Tabor model of adhesive junctions (Fig. 9). According to this last model, because the surfaces are irregular, the contact takes place only between highest asperities. Thus the real contact area A_r is much smaller than the apparent macroscopic area A and increases with load. The pressure at the small contact areas is very high and causes deformations, high temperatures, and local junctions. The total force needed to separate all junctions (that is, the opposite of the friction force) is proportional to the real contact area and depends on the deformation (plastic or elastic) of asperities.

We contrast these older models to more recent ones, such as spring-like models (Fig. 10) and atomic interaction models based on computer simulation.[16] Although partial and limited in applicability, these models have the advantage of

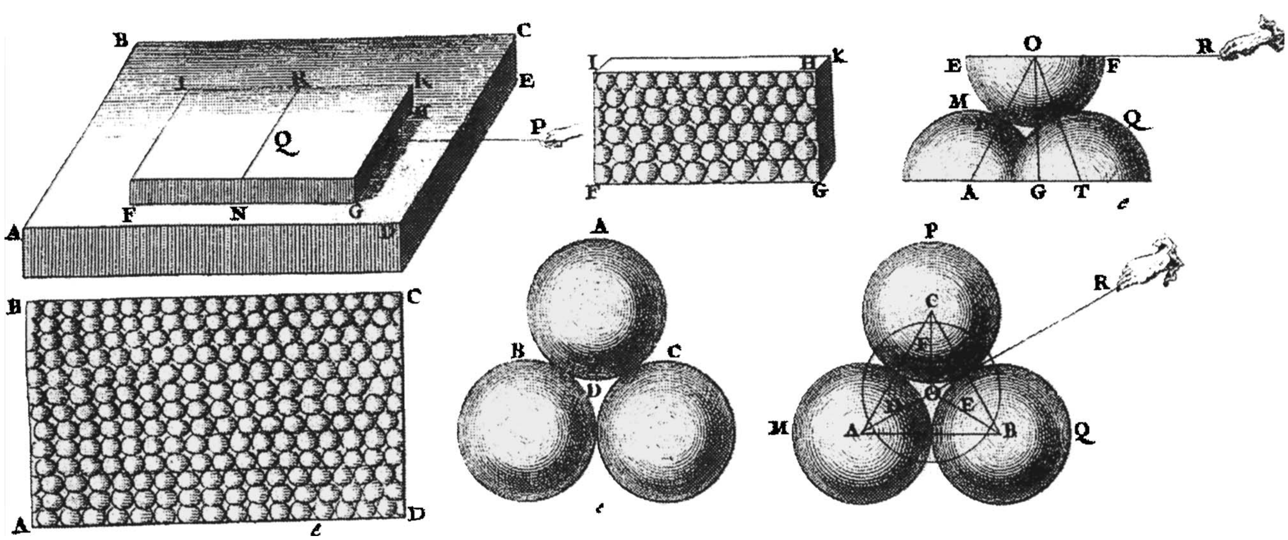

Fig. 8. In Bélidor's model rough surfaces are represented by rigid spherical asperities, which interlock when two surfaces come in contact (Ref. 15). The friction force equals the force needed to move the spheres up and over each other.

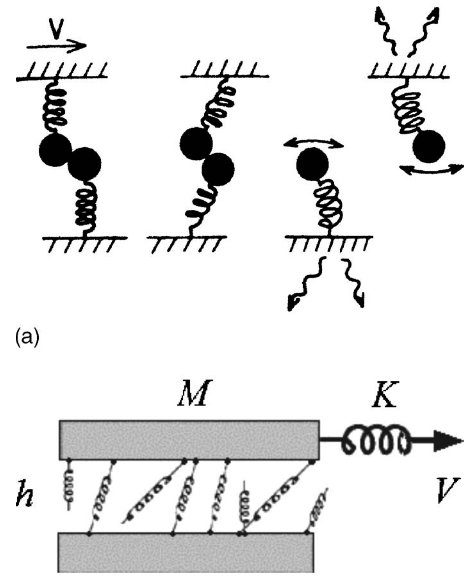

Fig. 10. Spring models for friction interactions. (a) The picture represents two asperities or atomic groups of two surfaces in contact (Persson, Ref. 6, p. 291). As the surfaces slide relative to each other, atomic groups interlock, then deform elastically, and finally slip. The rapid local motion is damped by the emission of sound waves. (b) Bond formation and rupture modeled by elastic springs with damping (Filippov *et al.*, Ref. 19).

presenting visual representations of meso–micro-asperities in interaction. For this reason, they can help students create mental models that are useful for understanding the behavior of friction forces in many physical situations. These models are important in stimulating students to build the causal explanations and operable mechanisms. Although these models do not completely explain current results, it is often necessary to use such approximations in introductory physics courses, where it is rarely possible to give a complete theoretical treatment on the basis of fundamental physical laws or elaborate models. This need is especially true for friction, where a complete understanding and explanation of macroscopic phenomena on the basis of microscopic interactions is not available.

We emphasize that an explanation of friction requires an analysis of phenomena occurring at different scales of magnitude, such as atomic and molecular interactions between surfaces and inside the bodies in contact, as well as mesoscopic structure of surface topography. From a didactic point of view, it is important to promote the idea of multileveled explanations in physics, according to the situation and problem studied. Moreover, pictures representing simulations of atomic interactions and graphs of nanotribology experiments provide students with a useful glimpse of some aspects of modern research.[17]

Our experience has been that the use of qualitative models of asperity interactions is effective in providing students with a tool for understanding the presence and the direction of friction forces in many situations. For example, when asked to respond to questions about the cases sketched in Fig. 11, most students correctly indicated the direction of the friction forces and explained their answers using the proposed models. In particular, for the situation in Fig. 11(b), they produced sketches showing asperities in contact (similar to

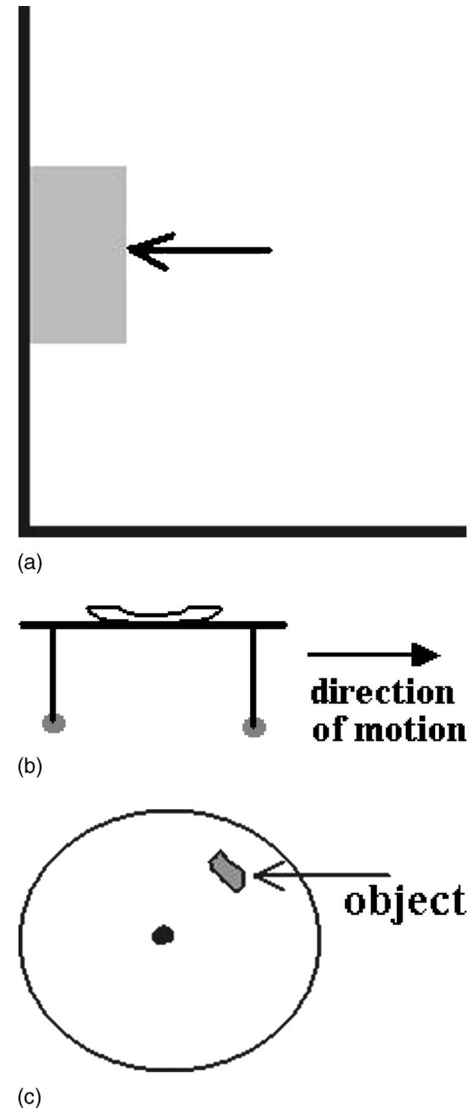

Fig. 11. In each question, students indicate all forces and explain their answers. (a) A wooden block is pushed against a wall by a horizontal force. (b) A cart with a dish placed on it is put in motion with a small acceleration, then moved at uniform motion, and finally slowed down. (c) An object is placed on a merry-go-round rotating at constant speed and remains at rest with respect to the merry-go-round.

those of Fig. 9) that were deformed in different ways according to the acceleration, deceleration, or uniform motion of the dish.

E. Friction phenomena from the energy point of view

In this final part, phenomena that were previously studied in terms of forces are re-examined in terms of energy transformation and dissipation. We present simple qualitative experiments, stressing the transfer of energy to internal parts of the system. To suggest by analogy how the kinetic energy of the center of mass of a system can be transferred to the parts within it, we use a demo in which a special cart equipped with many oscillators collides against a wall (see Fig. 12). The oscillators can be locked by means of a polystyrene bar inserted longitudinally between the two rows of oscillators. With the oscillators locked, the collision of the cart is elastic and the cart rebounds with almost the initial speed. With the

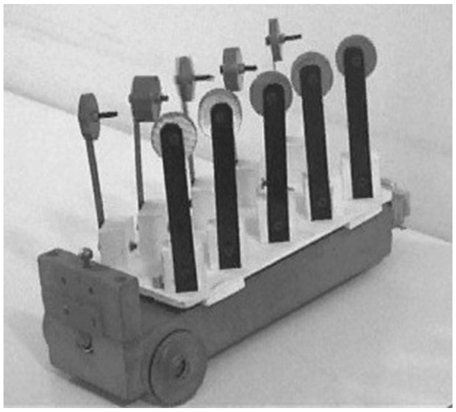

Fig. 12. An analogy for internal energy: A cart equipped with several different freely vibrating oscillators.

oscillators free, most of the translational kinetic energy of the cart is transferred to the oscillators upon a collision, so that the collision becomes strongly inelastic and the rebound speed is almost zero. (A video of the collision experiment is available on EPAPS).[18] This experiment can be used as a model of what happens when kinetic friction is present. The translational kinetic energy of the moving object is transformed into internal energy. Internal energy manifests itself as incoherent motion at the atomic-molecular level, which results in an increase in the temperature.

Although the same interactions can be used to describe static and kinetic friction, dissipation of energy makes an important distinction between them.[19] There are two main points to be made in the explanation of kinetic friction: how a tangential force with a specific direction is generated, and how kinetic energy is dissipated as internal energy in an object. It is not sufficient to give a description of energy alone, because such description fails to take the direction of the interactions into account. For this reason we consider both aspects, first providing descriptions in term of forces and the formation and rupture of bonds, and then in terms of energy balance and dissipation.

Simplified models and pictures have been proposed to help students understand the problems associated with the calculation of the work performed by frictional forces, and to suggest possible mechanisms for energy dissipation.[20]

V. CONCLUSIONS AND IMPLICATIONS

Testing of the sequence with our student teachers and with high school students has provided encouraging results, both from the point of view of overcoming some of the typical difficulties with the topic that emerge in the physics education literature (see Sec. I), and from the perspective of stimulating new and richer approaches and lines of reasoning in relation to physical situations connected with friction. The experiments have encouraged a wider and more critical view of different types of friction phenomena, as well as reflections on the characteristics and possible explanations of these phenomena. As mentioned, we consider structural explanatory models (models describing some aspects of the material structure of solid surfaces and the physical processes producing friction) to be important for going beyond a formal and formula-manipulating approach to physical situations. Their application in this context has had very encouraging results. The lines of reasoning achieved by students concerning the relations between friction force, normal force, and real and nominal contact area, although incomplete, were much more refined and sophisticated than the simple repetition of fixed and abstract rules based on idealized objects.

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[7]See F. P. Bowden and D. Tabor, Ref. 6.

[8]Similar behavior is found in studies on granular materials. See for example, U. Tuzun and O. R. Walton, "Micromechanical modeling of load-dependent friction in contacts of elastic spheres," J. Phys. D **25**, A44–A52 (1992). It has also been observed that a rigid cylinder (for example, plexiglass) can roll on the underside of an inclined rubber surface. See J. C. Charmet and M. Barquins, "Adhesive contact and rolling of a rigid cylinder under the pull of gravity on the underside of a smooth-surfaced sheet of rubber," Int. J. Adhes. Adhes. **16**, 249–254 (1996).

[9]See R. W. Carpick and M. Salmeron, "Scratching the surface: Fundamental investigations of tribology with atomic force microscopy," Chem. Rev. (Washington, D.C.) **97**, 1163–1194 (1997); M. L. Gee, P. M. McGuiggan, J. N. Israelachvili, and J. Homola, "Liquid to solid-like transitions of molecularly thin films under shear," Chem. Phys. **93**, 1895–1906 (1990).

[10]B. Bhushan, Ref. 6, Sec. 5.4.3, and K. C. Ludema and D. Tabor, "The friction and viscoelastic properties of polymeric solids," Wear **9**, 329–348 (1966).

[11]G. Amontons, "De la résistance causée dans les machines," Mém. Acad. R. A (1699), in *Histoire de l'Académie Royale des Sciences* (J.-A. Barral, Paris, 1732), pp. 206–227; C. A. Coulomb, "Théories des Machines Simples, en ayant égard au frottement de leurs parties et à la roideur des cordages," Mém. Math. Phys. Acad. R. Sci. Paris **10**, 161–342 (1785), reprinted by (Bachelier, Paris, 1821); J. T. Desaguliers, *A Course of Experimental Philosophy* (Desaguliers, London, 1734), Vol. 1; S. Vince, "The motion of bodies affected by friction," Philos. Trans. R. Soc. London **75**, 165–189 (1785); G. A. Tomlinson, "A molecular theory of friction," Philos. Mag. **7**, 905–939 (1929). See also D. Dowson, *History of Tribology* (Professional Engineering Publishing, London and Bury St.

Edmuns, 1998).
[12] American Society of Mechanics, *Friction, Lubrication and Wear Technology*, ASM Handbook, Vol. 18 (ASM International, OH, 1992), p. 27.
[13] See for example, Bowden and Tabor, Ref. 6, Part II, Chaps. X and XIII; Bhushan, Ref. 6, Sec. 5.2.1; J. F. Archard, "Elastic deformation and the law of friction," Proc. R. Soc. London, Ser. A **243**, 190–205 (1958).
[14] Many authors found a roughly fractal structure. B. Bhushan, "Contact mechanics surfaces in tribology: Multiple asperity contact," Tribol. Lett. **4**, 1–35 (1998), pages 1 and 13 writes "Roughness is found at scales ranging from millimeter to nanometer … a surface is composed of a large number of length scales of roughness that are superimposed on each other … A unique property of rough surfaces is that if a surface is repeatedly magnified, increasing details of roughness are observed right down to the nanoscale.".
[15] B. F. Bélidor, *Architecture Hydraulique, ou l'art de conduire, d'élever et de ménager les eaux* (Jombert, Paris, 1737).
[16] J. Ringlein and M. O. Robbins, "Understanding and illustrating the atomic origins of friction," Am. J. Phys. **72**(7), 884–891 (2004).
[17] See Carpick and Salmeron, Ref. 9, Persson, Ref. 6, Chap. 3, and Bhushan, Ref. 6, Chap. 10.
[18] See EPAPS Document No. E-AJPIAS-75-003711 for this animation and for full color figures of experiments. This document can be reached through a direct link in the online article's HTML reference section or via the EPAPS homepage (http://www.aip.org/pubservs/epaps.html).
[19] For a discussion of the difference between static and kinetic friction, see Ref. 16. Many authors describe kinetic friction as a succession of stick-slip episodes at smaller scales, hence as a formation and rupture of static friction bonds. See F. Al-Bender, V. Lampaert, and J. Swevers, "A novel generic model at asperity level for dry friction force dynamics," Wear **16**, 81–93 (2004). Actually, static friction is not totally static, because there are micro-displacements that are not totally recovered when external force decreases to zero. See Bowden and Tabor, Ref. 6, pp. 64, 65 and Al-Bender *et al.*, p. 87. Given the time dependence of the static friction coefficient, we can say that the observation of static or kinetic friction depends on the time scale, with kinetic friction occurring when the observation time is sufficiently long. See A. E. Filippov, J. Klafter, and M. Urbakh, "Friction through dynamical formation and rupture of molecular bonds," Phys. Rev. Lett. **92**(13), 135503-1–4 (2004).
[20] See B. A. Sherwood and W. H. Bernard, "Work and heat transfer in the presence of sliding friction," Am. J. Phys. **52**(11), 1001–1007 (1984); U. Besson, "Work and energy in the presence of friction: The need for a mesoscopic analysis," Eur. J. Phys. **22**, 613–622 (2001). Al-Bender *et al.* (Ref. 19) propose a model in which an asperity is deformed during contact until slipping occurs, then "it will break loose, vibrate (tangentially and normally) and thereby dissipates (part of) its elastic and inertial energy, by internal hysteresis, until it comes to rest or comes in contact with the next bottom asperity" (p. 86).