PROBLEMS

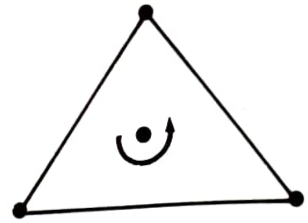
1. a. N identical spin $\frac{1}{2}$ particles are subjected to a one-dimensional simple harmonic oscillator potential. What is the ground-state energy? What is the Fermi energy?

   b. What are the ground state and Fermi energies if we ignore the mutual interactions and assume N to be very large?

2. It is obvious that two nonidentical spin1 particles with no orbital angular momenta (that is, *s*-states for both) can form $j=0$, $j=1$, and $j=2$. Suppose, however, that the two particles are *identical*. What restrictions do we get?

3. Discuss what would happen to the energy levels of a helium atom if the electron were a spinless boson. Be as quantitative as you can.

4. Three spin 0 particles are situated at the corners of an equilateral triangle. Let us define the *z*-axis to go through the center and in the direction normal to the plane of the triangle. The whole system is free to rotate about the *z*-axis. Using statistics considerations, obtain restrictions on the magnetic quantum numbers corresponding to J_z.



5. Consider three weakly interacting, identical spin 1 particles.
   a. Suppose the space part of the state vector is known to be symmetric under interchange of *any* pair. Using notation $|+\rangle|0\rangle|+\rangle$ for particle 1 in $m_s = +1$, particle 2 in $m_s = 0$, particle 3 in $m_s = +1$, and so on, construct the normalized spin states in the following three cases:
      (i) All three of them in $|+\rangle$.
      (ii) Two of them in $|+\rangle$, one in $|0\rangle$.
      (iii) All three in different spin states.
      What is the total spin in each case?
   b. Attempt to do the same problem when the space part is antisymmetric under interchange of any pair.
6. Suppose the electron were a spin $\frac{3}{2}$ particle obeying Fermi-Dirac statistics. Write the configuration of a hypothetical Ne ($Z = 10$) atom made up of such "electrons" [that is, the analog of $(1s)^2(2s)^2(2p)^6$]. Show that the configuration is highly degenerate. What is the ground state (the lowest term) of the hypothetical Ne atom in spectroscopic notation ($^{2S+1}L_J$, where S, L, and J stand for the total spin, the total orbital angular momentum, and the total angular momentum, respectively) when exchange splitting and spin-orbit splitting are taken into account?
7. Two identical spin $\frac{1}{2}$ fermions move in one dimension under the influence of the infinite-wall potential $V = \infty$ for $x < 0$, $x > L$, and $V = 0$ for $0 \leq x \leq L$.
   a. Write the ground-state wave function and the ground-state energy when the two particles are constrained to a triplet spin state (*ortho* state).
   b. Repeat (a) when they are in a singlet spin state (*para* state).
   c. Let us now suppose that the two particles interact mutually via a very short-range attractive potential that can be approximated by

$$V = -\lambda\delta(x_1 - x_2) \quad (\lambda > 0).$$

Assuming that perturbation theory is valid even with such a singular potential, discuss semiquantitatively what happens to the energy levels obtained in (a) and (b).