7,0

6° Trabalho em Grupo – Comportamento dinâmico de sistemas de 2ª. ordem

Grupo: 8

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1) Uma mudança em degrau na pressão de um vaso de 15 para 24 psi resulta em uma resposta no medidor de pressão de acordo com a Figura 1. Assuma dinâmica de segunda ordem, calcule todos os parâmetros importantes e escreva a FT na forma:

$$\frac{\overline{Rm}(s)}{\overline{P}(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

Em que Rm → instrumento de medida (mm); P → pressão (psi)

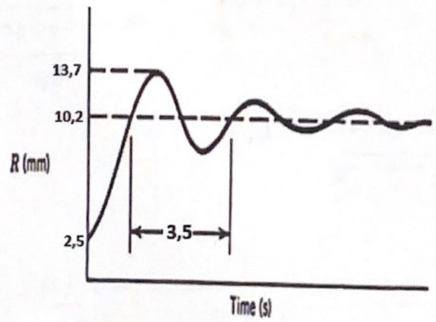

Figura 1: Resposta dinâmica do sistema de medida

2) Encontre a função de transferência, de um sistema composto por 2 tanques em série como mostrado na Figura 2, que descreva o comportamento do sistema ($h_2(t)$) frente a uma perturbação na vazão de entrada (F_0 (t)).

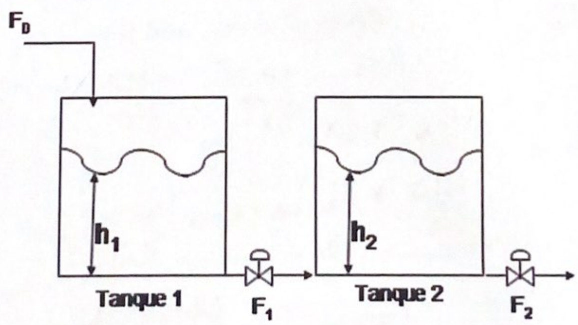

Figura 2: Tanques em série

Considere: $F_1 = \beta\sqrt{h_1 - h_2}$ $F_2 = \beta\sqrt{h_2}$ em que β é o termo associado às resistências aos fluxos volumétricos F_1 e F_2, respectivamente.

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① Pressão → 15 ~ 24 → 9 psi

$T = 3,5$
$B = 10,2 - 2,5 = 7,7$
$A = 13,7 - 10,2 = 3,5$
$\Delta y' = 10,2 - 2,5 = 7,7$
$\Delta C' = 9$

$K_p = \dfrac{\Delta y'}{\Delta C'}$ $\quad K_p = \dfrac{7,7}{9} = \boxed{0,855}$

Overshot $= \dfrac{A}{B} = \exp\left(\dfrac{-\pi \zeta}{\sqrt{1-\zeta^2}}\right) \rightarrow \dfrac{3,5}{7,7} = \exp\left(\dfrac{-\pi \zeta}{\sqrt{1-\zeta^2}}\right)$

$\ln\left(\dfrac{3,5}{7,7}\right) = \ln\left(\exp\left(\dfrac{-\pi \zeta}{\sqrt{1-\zeta^2}}\right)\right) \rightarrow \sqrt{+1-\zeta^2} = \dfrac{-\pi \zeta}{\ln\left(\frac{3,5}{7,7}\right)} \rightarrow 1-\zeta^2 = \left(\dfrac{-\pi \zeta}{-0,7884}\right)^2$

$\rightarrow 1-\zeta^2 = \dfrac{9,8596\ \zeta^2}{0,62157} \rightarrow 1-\zeta^2 = 15,8624 \cdot \zeta^2 \rightarrow (-1 + (-15,8624))\ \zeta^2 + 1 = 0$

$\cancel{+}16,8624 \cdot \zeta^2 = +1 \quad \therefore \quad \zeta^2 = \dfrac{1}{16,8624} \quad \zeta^2 = 0,059303 \quad \boxed{\zeta = 0,2435}$

- $T = \dfrac{2\pi}{\omega} \rightarrow 3,5 = \dfrac{2\pi}{\omega} \rightarrow \boxed{\omega = 1,794}$ $\quad \omega = \dfrac{\sqrt{1-\zeta^2}}{\tau_p} \rightarrow 1,794 = \sqrt{1-(0,2435)^2}$ $\quad \boxed{\tau_p = 0,54}$

- $T_p = \dfrac{\pi\ \tau_p}{\sqrt{1-\zeta^2}} \rightarrow T_p = \dfrac{\pi\ 0,54}{\sqrt{1-0,2435^2}}$ $\quad \boxed{T_p = 1,75}$

Função de Transf.

$\dfrac{\bar{R}m(s)}{\bar{P}(s)} = \dfrac{K_p}{\tau_p^2 s^2 + 2\zeta\tau_p \cdot s + 1}$ $\quad \dfrac{\bar{R}m(s)}{\bar{P}(s)} = \dfrac{0,855}{0,2916 \cdot s^2 + 2 \cdot 0,2434 \cdot 0,545 + 1}$

$\boxed{\dfrac{\bar{R}m(s)}{\bar{P}(s)} = \dfrac{0,855}{0,2916\ s^2 + 0,2635 + 1}}$

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② Hipóteses p/ tanque ①
- mistura completa
- Densidade constante
- área de seção transversal

Balanço de massa p/ o tanque ①

$F_1 = \beta\sqrt{h_1 - h_2}$
$F_2 = \beta\sqrt{h_2}$

$\cancel{\rho_0} F_0 - \cancel{\rho_1} F_1 = \dfrac{d\,m\,\cancel{(t)}}{dt} = d(\cancel{\rho}.\ A_1\ h_1)(t) = \rho_1 . A_1 . \dfrac{d(h_1(t))}{dt}$

$F_0(t) - F_1(t) = A_1 \dfrac{dh_1}{dt}$

$F_0(t) - \beta\sqrt{h_1(t) - h_2(t)} = A\dfrac{dh_1}{dt}$ (EDO ñ linear) ①

· Linearização por Taylor

$\sqrt{h_1(t) - h_2(t)} \simeq \sqrt{h_{1M} - h_{2M}} + \dfrac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_1(t) - h_{1M}) - \dfrac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_2(t) - h_{2M})$ ②

substituindo ② em ①

$F_0(t) - \beta\left(\sqrt{h_{1M} - h_{2M}} + \dfrac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_1(t) - h_{1M}) - \dfrac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_2(t) - h_{2M})\right) = A\dfrac{dh_1}{dt}$ ③

estado estacionário

$F_{0M} - \beta\sqrt{h_{1M} - h_{2M}} = 0$ ④ $\quad$ subtrair ④ de ③

$(F_0(t) - F_{0M}) - \beta . \left(\sqrt{h_{1M} - h_{2M}} + \dfrac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_1(t) - h_{1M}) - \dfrac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_2(t) - h_{2M}) + \right.$

$\beta\sqrt{h_{1M} - h_{2M}} = A . \dfrac{dh_1}{dt}$ $\quad$ continuação

$$(F_0(t) - F_{0M}) - \beta \left(\frac{1}{2\sqrt{h_{1M} - h_{2M}}} (h_1(t) - h_{1M}) - \frac{1}{2\sqrt{h_{1M} - h_{2M}}} (h_2(t) - h_{2M}) \right)$$

$$= A \cdot \frac{dh_1}{dt} \quad (5)$$

Forma da variável desvio

$$F_0'(t) - \beta \cdot \frac{1}{2\sqrt{h_{1M} - h_{2M}}} (h_1'(t) - h_2')(t) = A \cdot \frac{dh_1'}{dt} \quad (6)$$

Laplace

$$L\{F_0'(t)\} - L\left\{ \beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}} \cdot h_1'(t) \right\} + L\left\{ \beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}} \right.$$

$$= L\left\{ A \cdot \frac{dh_1'}{dt} \right\}$$

NÃO LINEARIZOU

$$\bar{F}_0(s) - \beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}} \cdot \bar{h}_1(s) + \beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}} \bar{h}_2(s) = A \cdot \bar{h}_1 \cdot S$$

$$\bar{F}_0(s) + \beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}} \bar{h}_2(s) = \left(A \cdot S \cdot \beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}} \right) \bar{h}_1(s)$$

$$\bar{h}_1(s) = \frac{1}{A \cdot S + \beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}}} \cdot \bar{F}_0(s) + \frac{\beta / 2\sqrt{h_{1M} - h_{2M}}}{A \cdot S + \beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}}} \cdot \bar{h}_2(s)$$

$$\bar{h}_1(s) = \frac{2\sqrt{h_{1M} - h_{2M}} / \beta}{\frac{2 \cdot A\sqrt{h_{1M} - h_{2M}}}{\beta} \cdot s + 1} \bar{F}_0(s) + \frac{1}{\frac{2A\sqrt{h_{1M} - h_{2M}}}{\beta} s + 1} \bar{h}_2(s) \quad (7)$$

$$K_{p1} = \frac{2\sqrt{h_{1M} - h_{2M}}}{\beta} \qquad \tau_{p1} = \frac{2 . A . \sqrt{h_{1M} - h_{2M}}}{\beta}$$

· Balanço de massa p/ Tanque (2)

$$F_1(t) . \rho_1 - F_2(t) . \rho_2 = \frac{dm(t)}{dt} = \frac{d(\rho . A . h_2(t))}{dt} = \rho . A \frac{d(h_2(t))}{dt}$$

$$F_1(t) - F_2(t) = A . \frac{dh_2(t)}{dt}$$

$$\beta\sqrt{h_1(t) - h_2(t)} - \beta\sqrt{h_2(t)} = A \frac{dh_2(t)}{dt} \quad (8) \text{ EDO } \bar{n} \text{ linear}$$

Linearizar

$$\sqrt{h_1(t) - h_2(t)} \therefore \sqrt{h_{1M} - h_{2M}} + \frac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_1(t) - h_{1M}) - \frac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_2(t) - h_{2M}) \quad (9) \rightarrow \sqrt{h_2(t)} \therefore \sqrt{h_{2M}} + \frac{1}{2\sqrt{h_{2M}}}(h_2(t) - h_{2M}) \quad (10)$$

Substituindo (9) e (10) na eq (8)

$$\beta . \left(\sqrt{h_{1M} - h_{2M}} + \frac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_1(t) - h_{1M}) - \frac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_2(t) - h_{2M})\right) - \beta\left(\sqrt{h_{2M}} + \frac{1}{2\sqrt{h_{2M}}}(h_2(t) - h_{2M})\right) = A \frac{dh_2(t)}{dt} \quad (11)$$

Estado Estacionário

$$\beta\sqrt{h_{1M} - h_{2M}} - \beta\sqrt{h_{2M}} = 0 \quad (12)$$

Subtraindo (12) da (11)

$$\beta\left(\frac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_1(t) - h_{1M}) - \frac{1}{2\sqrt{h_{1M} - h_{2M}}}(h_2(t) - h_{2M})\right) - \beta\left(\frac{1}{2\sqrt{h_{2M}}}(h_2(t) - h_{2M})\right) = A . \frac{dh_2(t)}{dt} \quad (13)$$

Variável desvio

$$\beta\left(\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\cdot h_1'(t)-\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\cdot h_2'(t)-\beta\left(\frac{1}{2\sqrt{h_{2M}}}\,h_2'(t)\right)\right)=A\cdot\frac{dh_2(t)}{dt} \quad (14)$$

Laplace

$$\beta\left(\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\cdot \bar{h}_1(s)-\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\cdot \bar{h}_2(s)-\beta\left(\frac{1}{2\sqrt{h_{2M}}}\,\bar{h}_2(s)\right)\right)=A\cdot\bar{h}_2\cdot s \quad (15)$$

$$\beta\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\cdot\bar{h}_1(s)-\beta\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\cdot\bar{h}_2(s)-\beta\frac{1}{2\sqrt{h_{2M}}}\cdot\bar{h}_2(s)=A\cdot\bar{h}_2(s)\cdot s$$

$$\beta\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\cdot\bar{h}_1(s)=A\cdot\bar{h}_2(s)\,s+\beta\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\cdot\bar{h}_2(s)+\beta\frac{1}{2\sqrt{h_{2M}}}\cdot\bar{h}_2(s)$$

$$\bar{h}_2(s)=\frac{\left(\beta\dfrac{1}{2\sqrt{h_{1M}-h_{2M}}}\right)}{A\cdot s+\beta\dfrac{1}{2\sqrt{h_{1M}-h_{2M}}}+\beta\dfrac{1}{2\sqrt{h_{2M}}}}\cdot\bar{h}_1(s)$$

$$\bar{h}_2(s)=\frac{\left(\beta\dfrac{1}{2\sqrt{h_{1M}-h_{2M}}}\right)}{\left(A\cdot s+\beta\dfrac{\sqrt{h_{2M}}}{2\sqrt{h_{2M}}}+\dfrac{\sqrt{h_{1M}-h_{2M}}}{\sqrt{h_{1M}-h_{2M}}}\right.}\cdot\bar{h}_1(s)$$

$$\bar{h}_2(s)=\frac{\beta\dfrac{1}{2\sqrt{h_{1M}-h_{2M}}}}{\left(\dfrac{2A\sqrt{h_{2M}}\sqrt{h_{1M}-h_{2M}}}{\beta\left(\sqrt{h_{2M}}+\sqrt{h_{1M}-h_{2M}}\right)}\cdot s+1\right)}\cdot\bar{h}_1(s) \quad (16)$$

$$K_{p2} = \left(\beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}}\right) \qquad \tau_{p2} = \frac{2 \cdot A \sqrt{h_{2M}} \sqrt{h_{1M} - h_{2M}}}{\beta\left(\sqrt{h_{2M}} + \sqrt{h_{1M} - h_{2M}}\right)}$$

Relacionar (7) com (16)

$$\bar{h}_2(s) = \frac{K_{p2}}{(\tau_{p2} \cdot S + 1)} \cdot \left[\frac{K_{p1}}{\tau_{p1} \cdot S + 1} \cdot \bar{F}_0(s) + \frac{1}{\tau_{p1} \cdot S + 1} \cdot \bar{h}_2(s)\right]$$

$$\bar{h}_2(s) - \frac{K_{p2}}{(\tau_{p2} \cdot s + 1)} \cdot \frac{1}{(\tau_{p1} \cdot S + 1)} \cdot \bar{h}_2(s) = \frac{K_{p2}}{(\tau_{p2}\, s + 1)} \cdot \frac{K_{p1}}{(\tau_{p1}\, S + 1)} \cdot \bar{F}_0(s)$$

$$\bar{h}_2(s) \cdot \left[1 - \frac{K_{p2}}{\tau_{p1} \cdot \tau_{p2} \cdot S^2 + (\tau_{p2} + \tau_{p1}) \cdot S + 1}\right] = \frac{K_{p1} \cdot K_{p2}}{\tau_{p1} \cdot \tau_{p2} \cdot S^2 + (\tau_{p2} + \tau_{p1}) \cdot S + 1} \cdot \bar{F}_0(s)$$

$$\bar{h}_2(s) \cdot \left[\frac{(\tau_{p1}\tau_{p2}\, S^2 + (\tau_{p2} + \tau_{p1}) \cdot S + 1) - K_{p2}}{\tau_{p1} \cdot \tau_{p2}\, S^2 + (\tau_{p2} + \tau_{p1})\, S + 1}\right] = \frac{K_{p1} \cdot K_{p2}}{\tau_{p1} \cdot \tau_{p2} \cdot S^2 + (\tau_{p2} + \tau_{p1}) \cdot S + 1} \cdot \bar{F}_0(s)$$

$$\bar{h}_2(s) = \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot \tau_{p2} \cdot S^2 + (\tau_{p2} + \tau_{p1})S + 1) - K_{p2}} \cdot \bar{F}_0(s)$$

$$\bar{h}_2(s) = \frac{K_{p1} \cdot K_{p2}}{\left(\frac{\tau_{p1} \cdot \tau_{p2}}{1 - K_{p2}}\right) \cdot S^2 + \left(\frac{\tau_{p1} \cdot \tau_{p2}}{1 - K_{p2}}\right) \cdot S + 1} \cdot \bar{F}_0(s)$$

$$K_{p1} \cdot K_{p2} = \left(\frac{2\sqrt{h_{1M} - h_{2M}}}{\beta}\right) \cdot \left(\beta \cdot \frac{1}{2\sqrt{h_{1M} - h_{2M}}}\right) = 1$$

$$\frac{\tau_{p1} \cdot \tau_{p2}}{1 - K_{p2}} = \frac{\left(\frac{2 \cdot A\sqrt{h_{1M} - h_{2M}}}{\beta}\right) \cdot \left(\frac{2 \cdot A\sqrt{h_{2M}}\sqrt{h_{1M} - h_{2M}}}{\beta\left(\sqrt{h_{2M}} + \sqrt{h_{1M} - h_{2M}}\right)}\right)}{1 - \left(\beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}}\right)} = \frac{\frac{4A^2 (h_{1M} - h_{2M})\sqrt{h_{2M}}}{\beta^2\left(\sqrt{h_{2M}} + \sqrt{h_{1M} - h_{2M}}\right)}}{1 - \left(\beta \frac{1}{2\sqrt{h_{1M} - h_{2M}}}\right)}$$

$$\frac{\tau_{p1}+\tau_{p2}}{1-K_{p2}} = \frac{\left(\frac{2A\sqrt{h_{1M}-h_{2M}}}{\beta}\right) + \left(\frac{2A\sqrt{h_{2M}}\sqrt{h_{1M}-h_{2M}}}{\beta\left(\sqrt{h_{2M}}+\sqrt{h_{1M}-h_{2M}}\right)}\right)}{1-\left(\beta\,\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\right)}$$

$$= \frac{\left(\frac{2A\sqrt{h_{1M}-h_{2M}}\left(\sqrt{h_{2M}}+\sqrt{h_{1M}-h_{2M}}\right)+2A\sqrt{h_{2M}}\sqrt{h_{1M}-h_{2M}}}{\beta\left(\sqrt{h_{2M}}+\sqrt{h_{1M}-h_{2M}}\right)}\right)}{1-\left(\beta\,\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\right)}$$

$$\frac{\tau_{p1}+\tau_{p2}}{1-K_{p2}} = \frac{\left(\frac{2A\sqrt{h_{1M}-h_{2M}}\cdot\sqrt{h_{2M}}+2A(h_{1M}-h_{2M})+2A\sqrt{h_{2M}}\cdot\sqrt{h_{1M}-h_{2M}}}{\beta\left(\sqrt{h_{2M}}+\sqrt{h_{1M}-h_{2M}}\right)}\right)}{1-\left(\beta\,\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\right)}$$

$$= \frac{\left(\frac{2A\left(\sqrt{h_{1M}-h_{2M}}\sqrt{h_{2M}}+(h_{1M}-h_{2M})+\sqrt{h_{2M}-h_{2M}}\right)}{\beta\left(\sqrt{h_{2M}}+\sqrt{h_{1M}-h_{2M}}\right)}\right)}{1-\left(\beta\,\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\right)}$$

$$\boxed{\bar{h}_2(s) = \frac{1}{\left(\frac{\frac{4A^2(h_{1M}-h_{2M})\sqrt{h_{2M}}}{\beta^2\left(\sqrt{h_{2M}}+\sqrt{h_{1M}\cdot h_{2M}}\right)}}{1-\left(\beta\cdot\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\right)}\right)\cdot s^2+\left(\frac{\left(\frac{2A\left(\sqrt{h_{1M}-h_{2M}}+\sqrt{h_{2M}}\cdot\sqrt{h_{1M}-h_{2M}}\right)}{\beta\left(\left(\sqrt{h_{2M}}+\sqrt{h_{1M}-h_{2M}}\right)\right)}\right)}{1-\left(\beta\cdot\frac{1}{2\sqrt{h_{1M}-h_{2M}}}\right)}\right)s+1}\cdot\bar{F}_0(s)}$$