9,8

6° Trabalho em Grupo – Comportamento dinâmico de sistemas de 2ª. ordem

Grupo: 7

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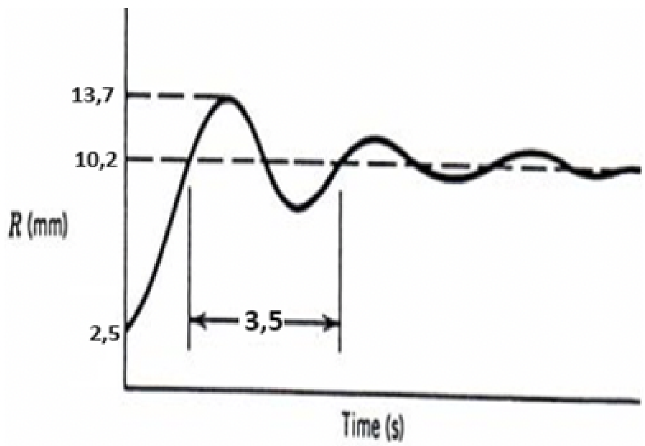
1) Uma mudança em degrau na pressão de um vaso de 15 para 24 psi resulta em uma resposta no medidor de pressão de acordo com a Figura 1. Assuma dinâmica de segunda ordem, calcule todos os parâmetros importantes e escreva a FT na forma:

$$\frac{\overline{Rm}(s)}{\overline{P}(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

Em que Rm → instrumento de medida (mm); P → pressão (psi)

Figura 1: Resposta dinâmica do sistema de medida



RESOLUÇÃO

- Sobre elevação (Overshoot):

$$OS = \frac{A}{B} = exp\left(\frac{-\pi\cdot\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$OS = \frac{A}{B} = \frac{(13,7-10,2)}{(10,2-2,5)} = 0,4545 = exp\left(\frac{-\pi\cdot\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$ln\, 0,4545 = \left(\frac{-\pi\cdot\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$\zeta = 0,2434$$

- Frequência natural de oscilação:

$$T \cdot \omega = 2\pi \qquad \omega = \frac{\sqrt{1-\zeta^2}}{\tau p}$$

$$3,5 \cdot \frac{\sqrt{1-0,2434^2}}{2\pi} = \tau$$

$$\tau = 0,5403$$

- Ganho estático do processo:

$$Kp = \frac{\Delta Rm}{\Delta P} = \frac{(10,2-2,5)}{(24-15)}$$

$$Kp = 0,8556\, mm/psi$$

- Fórmula final:

$$\frac{\overline{Rm}_{(s)}}{\overline{P}_{(s)}} = \frac{0,8556}{0,2919\cdot S^2+0,263\cdot S+1}$$

2) Encontre a função de transferência, de um sistema composto por 2 tanques em série como mostrado na Figura 2, que descreva o comportamento do sistema (h_2(t)) frente a uma perturbação na vazão de entrada (F_0 (t)). → tanque de mistura/pulmão.

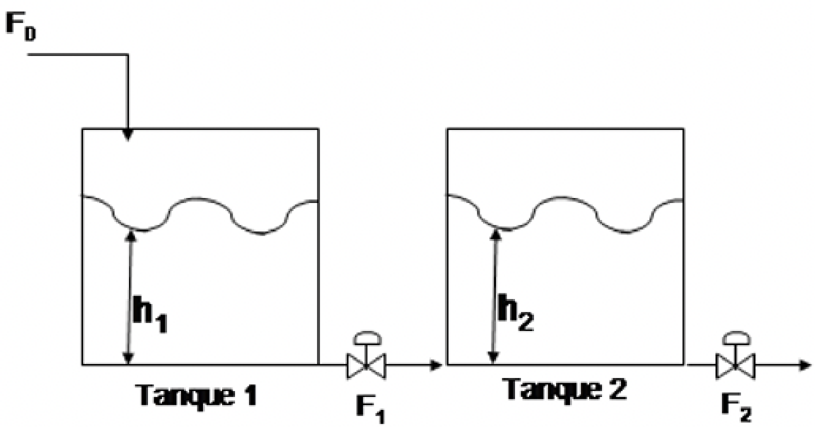


Figura 2: Tanques em série.

Considere: $F_1 = \beta\sqrt{h_1 - h_2}$ $\quad F_2 = \beta\sqrt{h_2}$ em que β é o termo associado às resistências aos fluxos volumétricos F_1 e F_2, respectivamente.

RESOLUÇÃO

Hipóteses: mistura completa, densidade constante e área de seção transversal (área) constante.

Balanço de massa pro tanque 1:

$$F_{0(t)} \cdot \rho_0 - F_{1(t)} \cdot \rho_1 = \frac{dM(t)}{dt} = \frac{d\left(\rho \cdot A \cdot h_{1(t)}\right)}{dt} = \rho \cdot A \cdot \frac{d\left(h_{1\,(t)}\right)}{dt}$$

$$F_{0(t)} - F_1 = A \cdot \frac{dh_1}{dt}$$

$$F_{0(t)} - \beta\sqrt{h_{1(t)} - h_{2(t)}} = A \cdot \frac{dh_1}{dt} \qquad (1)$$ EDO não-linear - Termo não linear, raiz quadrada.

- Linearização por expansão de Taylor:

$$\sqrt{h_{1(t)} - h_{2(t)}} \approx \sqrt{h_{1ss} - h_{2ss}} + \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{1(t)} - h_{1ss}\right) - \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right) \qquad (2)$$

- Substituindo (2) na (1):

$$F_{0(t)} - \beta \cdot \left(\sqrt{h_{1ss} - h_{2ss}} + \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{1(t)} - h_{1ss}\right) - \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right)\right) = A \cdot \frac{dh_1}{dt} \qquad (3)$$

- Estado estacionário na OG:

$$F_{0ss} - \beta\sqrt{h_{1ss} - h_{2ss}} = 0 \qquad (4)$$

- Subtraindo (4) de (3):

$$\left(F_{0(t)} - F_{0ss}\right) - \beta \cdot \left(\sqrt{h_{1ss} - h_{2ss}} + \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{1(t)} - h_{1ss}\right) - \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right)\right) + \beta\sqrt{h_{1ss} - h_{2ss}} = A \cdot \frac{dh_1}{dt}$$

$$\left(F_{0(t)} - F_{0ss}\right) - \beta \cdot \left(\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{1(t)} - h_{1ss}\right) - \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right)\right) = A \cdot \frac{dh_1}{dt} \qquad (5)$$

- Escrevendo na forma de variável desvio:

$$F'_{0(t)} - \beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h'_{1(t)} - h'_{2(t)}\right) = A \cdot \frac{dh'_1}{dt} \qquad (6)$$

- Aplicando o Teorema de Laplace:

$$L\left\{F'_{0(t)}\right\} - L\left\{\beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}} \cdot h'_{1(t)}\right\} + L\left\{\beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}} \cdot h'_{2(t)}\right\} = L\left\{A \cdot \frac{dh'_1}{dt}\right\}$$

$$\overline{F_{0(s)}} - \beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}} \cdot \overline{h_{1(s)}} + \beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}} \cdot \overline{h_{2(s)}} = A \cdot \overline{h_1} \cdot S$$

$$\overline{F_{0(s)}} + \beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}} \cdot \overline{h_{2(s)}} = \left(A \cdot S + \beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right) \cdot \overline{h_{1(s)}}$$

$$\overline{h}_{1(s)} = \frac{1}{A\cdot S+\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}} \cdot \overline{F_{0(s)}} + \frac{\frac{\beta}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}}{A\cdot S+\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}} \cdot \overline{h_{2(s)}}$$

$$\overline{h}_{1(s)} = \frac{\frac{2\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta}}{\frac{2\cdot A\sqrt{h_{1ss}-h_{2ss}}}{\beta}\cdot S+1} \cdot \overline{F_{0(s)}} + \frac{1}{\frac{2\cdot A\sqrt{h_{1ss}-h_{2ss}}}{\beta}\cdot S+1} \cdot \overline{h_{2(s)}} \qquad (7)$$

$$Kp_1 = \frac{2\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta} \qquad \tau_{p1} = \frac{2\cdot A_1\sqrt{h_{1ss}-h_{2ss}}}{\beta}$$

<u>Balanço de massa pro tanque 2:</u>

$$F_{1(t)}\cdot\rho_1 - F_{2(t)}\cdot\rho_2 = \frac{dM(t)}{dt} = \frac{d\left(\rho\cdot A\cdot h_{2(t)}\right)}{dt} = \rho\cdot A\cdot\frac{d\left(h_{2\,(t)}\right)}{dt}$$

$$F_{1(t)} - F_{2(t)} = A\cdot\frac{dh_{2\,(t)}}{dt}$$

$$\beta\sqrt{h_{1(t)} - h_{2(t)}} - \beta\sqrt{h_{2(t)}} = A\cdot\frac{dh_{2\,(t)}}{dt} \qquad (8)\ \ \text{EDO não-linear}$$

- Linearização

$$\sqrt{h_{1(t)} - h_{2(t)}} \approx \sqrt{h_{1ss} - h_{2ss}} + \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{1(t)} - h_{1ss}\right) - \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right) \qquad (9)$$

$$\sqrt{h_{2(t)}} \approx \sqrt{h_{2ss}} + \frac{1}{2\sqrt{h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right) \qquad (10)$$

- Substituindo (9) e (10) na (8):

$$\beta\cdot\left(\sqrt{h_{1ss} - h_{2ss}} + \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{1(t)} - h_{1ss}\right) - \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right)\right) - \beta\cdot\left(\sqrt{h_{2ss}} + \frac{1}{2\sqrt{h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right)\right) = A\cdot\frac{dh_{2(t)}}{dt}$$

(11)

- Estado estacionário:

$$\beta\sqrt{h_{1ss} - h_{2ss}} - \beta\sqrt{h_{2ss}} = 0 \qquad (12)$$

- Subtraindo (12) da (11):

$$\beta\cdot\left(\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{1(t)} - h_{1ss}\right) - \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right)\right) - \beta\cdot\left(\frac{1}{2\sqrt{h_{2ss}}}\left(h_{2(t)} - h_{2ss}\right)\right) = A\cdot\frac{dh_{2(t)}}{dt} \qquad (13)$$

- Escrevendo na forma de variável desvio:

$$\beta\cdot\left(\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\cdot h'_{1(t)} - \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\cdot h'_{2(t)}\right) - \beta\cdot\left(\frac{1}{2\sqrt{h_{2ss}}}\cdot h'_{2(t)}\right) = A_2\cdot\frac{dh_{2(t)}}{dt} \qquad (14)$$

- Aplicando o Teorema de Laplace:

$$\beta\cdot\left(\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\cdot\bar{h}_{1(s)} - \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\cdot\bar{h}_{2(s)}\right) - \beta\cdot\left(\frac{1}{2\sqrt{h_{2ss}}}\cdot\bar{h}_{2(s)}\right) = A_2\cdot\bar{h}_{2(s)}\cdot S \qquad (15)$$

$$\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\cdot\bar{h}_{1(s)} - \beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\cdot\bar{h}_{2(s)} - \beta\cdot\frac{1}{2\sqrt{h_{2ss}}}\cdot\bar{h}_{2(s)} = A_2\cdot\bar{h}_{2(s)}\cdot S$$

$$\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\cdot\bar{h}_{1(s)} = A\cdot\bar{h}_{2(s)}\cdot S + \beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\cdot\bar{h}_{2(s)} + \beta\cdot\frac{1}{2\sqrt{h_{2ss}}}\cdot\bar{h}_{2(s)}$$

$$\bar{h}_{2(s)} = \frac{\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}{\left(A_1 S + \beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}} + \beta\cdot\frac{1}{2\sqrt{h_{2ss}}}\right)}\cdot\bar{h}_{1(s)}$$

$$\overline{h}_{2(s)} = \frac{\left(\beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}{\left(A\cdot S+\beta\cdot\frac{\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}}{2\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)} \cdot \overline{h}_{1(s)}$$

$$\overline{h}_{2(s)} = \frac{\left(\beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}{\left(\frac{2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\cdot S+1\right)} \cdot \overline{h}_{1(s)} \qquad (16)$$

$\times \quad \frac{2\sqrt{h_{2ss}}\sqrt{h_{1ss}-h_{2ss}}}{\beta\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}$

$$Kp_2 = \left(\beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right) \qquad \tau_{p2} = \frac{2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}$$

- Relacionando (7) com a (16):

$$\overline{h}_{2(s)} = \frac{Kp_2}{\left(\tau_{p2}\cdot S+1\right)} \cdot \left[\frac{Kp_1}{\tau_{p1}\cdot S+1} \cdot \overline{F}_{0(s)} + \frac{1}{\tau_{p1}\cdot S+1} \cdot \overline{h}_{2(s)}\right]$$

$$\overline{h}_{2(s)} - \frac{Kp_2}{\left(\tau_{p2}\cdot S+1\right)} \cdot \frac{1}{\left(\tau_{p1}\cdot S+1\right)} \cdot \overline{h}_{2(s)} = \frac{Kp_2}{\left(\tau_{p2}\cdot S+1\right)} \cdot \frac{Kp_1}{\left(\tau_{p1}\cdot S+1\right)} \cdot \overline{F}_{0(s)}$$

$$\overline{h}_{2(s)} \cdot \left[1 - \frac{Kp_2}{\tau_{p1}\cdot\tau_{p2}\cdot S^2+\left(\tau_{p2}+\tau_{p1}\right)\cdot S+1}\right] = \frac{Kp_1\cdot Kp_2}{\tau_{p1}\cdot\tau_{p2}\cdot S^2+\left(\tau_{p2}+\tau_{p1}\right)\cdot S+1} \cdot \overline{F}_{0(s)}$$

$$\overline{h}_{2(s)} \cdot \left[\frac{\left(\tau_{p1}\cdot\tau_{p2}\cdot S^2+\left(\tau_{p2}+\tau_{p1}\right)\cdot S+1\right)-Kp_2}{\tau_{p1}\cdot\tau_{p2}\cdot S^2+\left(\tau_{p2}+\tau_{p1}\right)S+1}\right] = \frac{Kp_1\cdot Kp_2}{\tau_{p1}\cdot\tau_{p2}\cdot S^2+\left(\tau_{p2}+\tau_{p1}\right)\cdot S+1} \cdot \overline{F}_{0(s)}$$

$$\overline{h}_{2(s)} = \frac{Kp_1\cdot Kp_2}{\left(\tau_{p1}\cdot\tau_{p2}\cdot S^2+\left(\tau_{p2}+\tau_{p1}\right)\cdot S+1\right)-Kp_2} \cdot \overline{F}_{0(s)}$$

$$\overline{h}_{2(s)} = \frac{Kp_1\cdot Kp_2}{\left(\frac{\tau_{p1}\cdot\tau_{p2}}{1-Kp_2}\right)\cdot S^2+\left(\frac{\tau_{p1}+\tau_{p2}}{1-Kp_2}\right)\cdot S+1} \cdot \overline{F}_{0(s)}$$

$$Kp_1 \cdot Kp_2 = \left(\frac{2\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta}\right)\cdot\left(\beta \cdot \frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right) = 1$$

$$\frac{\tau_{p1}\cdot\tau_{p2}}{1-Kp_2} = \frac{\left(\frac{2\cdot A\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta}\right)\cdot\left(\frac{2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)} = \frac{\frac{4\cdot A^2\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^2\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}$$

$$\frac{\tau_{p1}+\tau_{p2}}{1-Kp_2} = \frac{\left(\frac{2\cdot A\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta}\right)+\left(\frac{2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)} = \frac{\left(\frac{2\cdot A\cdot\sqrt{h_{1ss}-h_{2ss}}\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)+2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}$$

$$\frac{\tau_{p1}+\tau_{p2}}{1-Kp_2} = \frac{\left(\frac{2\cdot A\cdot\sqrt{h_{1ss}-h_{2ss}}\cdot\sqrt{h_{2ss}}+2\cdot A\cdot\left(h_{1ss}-h_{2ss}\right)+2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)} = \frac{\left(\frac{2\cdot A\cdot\left(\sqrt{h_{1ss}-h_{2ss}}\cdot\sqrt{h_{2ss}}+\left(h_{1ss}-h_{2ss}\right)+\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}\right)}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}$$

$$\overline{h}_{2(s)} = \frac{1}{\left(\frac{\frac{4\cdot A^2\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^2\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}\right)\cdot S^2+\left(\frac{\left(\frac{2\cdot A\cdot\left(\sqrt{h_{1ss}-h_{2ss}}\cdot\sqrt{h_{2ss}}+\left(h_{1ss}-h_{2ss}\right)+\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}\right)}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}\right)\cdot S+1}\cdot\overline{F}_{0(s)} \quad (17)$$

- Aplicando Laplace Inversa para colocar a FT em função do tempo:

19. $\frac{1}{\tau\sqrt{1-\zeta^2}}e^{-\zeta t/\tau}\sin\left(\sqrt{1-\zeta^2}\,t/\tau\right)$ $\qquad$ $\frac{1}{\tau^2s^2+2\zeta\tau s+1}$

$(0 \leq |\zeta| < 1)$

$$\tau^2 = \left(\frac{\tau_{p1}\cdot\tau_{p2}}{1-Kp_2}\right) \;->\; \tau = \sqrt{\frac{\tau_{p1}\cdot\tau_{p2}}{1-Kp_2}} = \sqrt{\frac{\frac{4\cdot A^2\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^2\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}}$$

$$2\cdot\varsigma\cdot\tau = \frac{\tau_{p1}+\tau_{p2}}{1-Kp_2} = 2\cdot\varsigma\cdot\sqrt{\frac{\tau_{p1}\cdot\tau_{p2}}{1-Kp_2}}$$

$$\varsigma = \frac{\tau_{p1}+\tau_{p2}}{2\cdot\left(1-Kp_2\right)}\cdot\sqrt{\frac{1-Kp_2}{\tau_{p1}\cdot\tau_{p2}}} = \frac{\left(\frac{2\cdot A\cdot\sqrt{h_{1ss}-h_{2ss}}\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)+2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{2\cdot\left(1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)\right)}\cdot\sqrt{\frac{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}{\frac{4\cdot A^2\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^2\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}}$$

$$\tau\cdot\sqrt{1-\varsigma^2} = \sqrt{\frac{\frac{4\cdot A^2\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^2\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}}\cdot\sqrt{1-\left(\frac{\left(\frac{2\cdot A\cdot\sqrt{h_{1ss}-h_{2ss}}\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)+2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{2\cdot\left(1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)\right)}\cdot\sqrt{\frac{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}{\frac{4\cdot A^2\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^2\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}}\right)^2}$$

$$e^{-\frac{\varsigma\cdot t}{\tau}} = e^{-\frac{\frac{\left(\frac{2\cdot A\cdot\sqrt{h_{1ss}-h_{2ss}}\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)+2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{2\cdot\left(1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)\right)}\cdot\sqrt{\frac{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}{\frac{4\cdot A^2\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^2\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}}}{\sqrt{\frac{\frac{4\cdot A^2\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^2\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}}}\cdot t}$$

$$sen\left(\sqrt{1-\varsigma^{2}}\cdot\frac{t}{\tau}\right)=sen\left(\sqrt{1-\left(\frac{\left(\frac{2\cdot A\cdot\sqrt{h_{1ss}-h_{2ss}}\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)+2\cdot A\cdot\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\right)}{2\cdot\left(1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)\right)}\cdot\sqrt{\frac{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}{\frac{4\cdot A^{2}\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^{2}\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}}\right)^{2}}\cdot\frac{t}{\sqrt{\frac{\frac{4\cdot A^{2}\cdot\left(h_{1ss}-h_{2ss}\right)\cdot\sqrt{h_{2ss}}}{\beta^{2}\cdot\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}}{1-\left(\beta\cdot\frac{1}{2\cdot\sqrt{h_{1ss}-h_{2ss}}}\right)}}}\right)$$