TG6 - Grupo 6

6,0

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1) Dados fornecidos pelo gráfico:

$A = 13,7 - 10,2 = 3,5$

$B = 10,2 - 2,5 = 7,7$

$T = 3,5\,s$

$\Delta y' = 10,2 - 2,5 = 7,7$

→ sobre elevação (OS)

$$OS = \frac{A}{B} = \exp\left(\frac{-\pi\,\xi}{\sqrt{1-\xi^2}}\right)$$

$$OS = \frac{3,5}{7,7} = 0,4545$$

Isolando o fator de amortecimento:

$$0,4545 = \exp\left(\frac{-\pi\,\xi}{\sqrt{1-\xi^2}}\right)$$

$$\ln(0,4545) = \frac{-\pi\,\xi}{\sqrt{1-\xi^2}}$$

$$1-\xi^2 = \frac{(-\pi)^2\,\xi^2}{(\ln(0,4545))^2}$$

$$0,6218 - 0,6218\,\xi^2 = (-\pi^2)\,\xi^2$$

$$\xi^2 = \frac{0,6218}{((-\pi)^2 + 0,6218)}$$

$$\boxed{\xi = 0,2434}$$

→ Período de oscilação

$$T = \frac{2\pi}{\omega} \iff \omega = 1{,}7952 = \frac{\sqrt{1-\xi^2}}{\tau_p}$$

Isolando a frequência de oscilação natural do sistema:

$$\tau_p = \frac{\sqrt{1-\xi^2}}{\omega}$$

$$\tau_p = \frac{\sqrt{1-0{,}2434^2}}{1{,}7952}$$

$$\boxed{\tau_p = 0{,}5403}$$

→ Ganho estatico do processo

$$K_p = \frac{\Delta y'}{\Delta c'}$$

Sabendo que $\Delta c' = 24 - 15 = 9$

$$K_p = \frac{7{,}7}{9} = 0{,}8556$$

$$\boxed{K_p = 0{,}8556}$$

→ Função de transferência

$$\boxed{\frac{\overline{R_m}(s)}{\overline{P}(s)} = \frac{0{,}8556}{0{,}2919\, s^2 + 0{,}2630 \cdot s + 1}}$$

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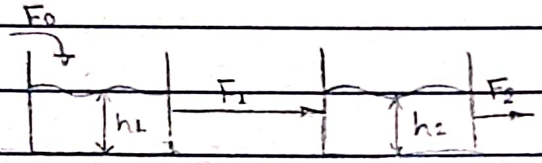
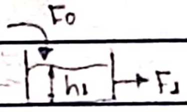
3,0/7,0

2-) 

• hipóteses: mistura completa, densidade e área de seção transversal constantes

• $F_1 = \beta.\sqrt{h_1 - h_2}$ e $F_2 = \beta.\sqrt{h_2}$

① Balanço de Massa ~ Tanque 1 

ρ cte

$\to F_0(t).\cancel{\rho} - F_1(t).\cancel{\rho} = \dfrac{dM(t)}{dt} = \dfrac{d(\rho.A.h_1(t))}{dt} = \cancel{\rho}.A.\dfrac{dh_1}{dt} \Rightarrow$

$\Rightarrow F_0(t) - F_1(t) = A.\dfrac{dh_1}{dt} \Rightarrow F_0(t) - \beta.\sqrt{h_1 - h_2} = A.\dfrac{dh_1}{dt}$ (1)

$\to$ EDO não-linear

~ Linearização por expansão de Taylor

$\to f(x,y) = f(x_{ss}, y_{ss}) + \dfrac{\partial f(x,y)}{dx}.(x - x_{ss}) + \dfrac{\partial f(x,y)}{dy}.(y - y_{ss})$

* seja uma função de duas variáveis, tal que:

$\to f(h_1(t); h_2(t)) = \sqrt{h_1(t) - h(2)}$ ★

$\to \dfrac{\partial(\sqrt{h_1(t) - h_2(t)})}{dh_1} \rightsquigarrow (h_1(t) - h_2(t)) = U$; $\dfrac{dU}{dh_1} = 1 \Rightarrow$

$\to \sqrt{h_1(t) - h_2(t)} = (h_1(t) - h_2(t))^{1/2} = U^{1/2}$

$= \dfrac{dU^{1/2}}{dU} = \dfrac{1}{2}.U^{-1/2} = \dfrac{1}{2\sqrt{U}} = \dfrac{1}{2.\sqrt{h_1(t) - h_2(t)}}$ ★

$\to \dfrac{\partial(\sqrt{h_1(t) - h_2(t)})}{dh_2} \rightsquigarrow (h_1(t) - h_2(t)) = U$; $\dfrac{dU}{dh_2} = -1 \Rightarrow$

$= \dfrac{dU^{1/2}}{-dU} = -1.\left(\dfrac{dU^{1/2}}{dU}\right) = \dfrac{-1}{2.\sqrt{h_1(t) - h_2(t)}}$ ★

$\to \sqrt{h_1(t) - h_2(t)} \simeq \sqrt{h_{1ss} - h_{2ss}} + \dfrac{1}{2.\sqrt{h_{1ss} - h_{2ss}}}.(h_1(t) - h_{1ss}) -$

$- \dfrac{1}{2.\sqrt{h_{1ss} - h_{2ss}}}.(h_2(t) - h_{2ss})$ (2)

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~ Substituindo (2) em (1)

$$\rightarrow F_0(t) - \beta.\left(\sqrt{h_{1ss}-h_{2ss}} + \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}}.(h_1(t)-h_{1ss}) - \frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.(h_2(t)-h_{2ss})\right) = A.\frac{dh_1}{dt} \quad (3)$$

~ EDO no estado estacionário

$$\rightarrow F_{0ss} - \beta.\sqrt{h_1(t)-h_2(t)} = 0 \quad (4)$$

~ Subtraindo (4) de (3)

$$\rightarrow F_0(t) = F_{0ss} - \beta.\left(\sqrt{h_{1ss}-h_{2ss}} + \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}}.(h_1(t)-h_{1ss}) - \frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.(h_2(t)-h_{2ss})\right) + \beta.\sqrt{h_{1ss}-h_{2ss}} = A.\frac{dh_1}{dt} \Rightarrow$$

$$\Rightarrow F_0 - F_{0ss} - \beta.\left(\frac{1}{2\sqrt{h_{1ss}-h_{2ss}}}.(h_1(t)-h_{1ss}) - \frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.(h_2(t)-h_{2ss})\right) = A.\frac{dh_1}{dt} \quad (5)$$

~ Escrevendo (5) na forma de variável desvio

$$\rightarrow F_0'(t) - \beta.\left(\frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.h_1'(t) - \frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.h_2'(t)\right) = A.\frac{dh_1}{dt}$$

~ Aplicando transformada de Laplace

$$\rightarrow \mathcal{L}\{F_0'(t)\} - \mathcal{L}\left\{\beta.\frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.h_1'(t)\right\} + \mathcal{L}\left\{\beta.\frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.h_2'(t)\right\} = \mathcal{L}\left\{A.\frac{dh_1}{dt}\right\} \Rightarrow$$

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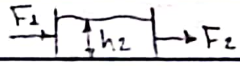
$$\Rightarrow \bar{F}_0(s) - \beta \cdot \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}} \cdot \bar{h}_1(s) + \beta \cdot \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}} \cdot \bar{h}_2 = A \cdot \bar{h}_1 \cdot S \Rightarrow$$

$$\Rightarrow \bar{h}_1 = \bar{F}_0(s) \cdot \frac{1}{A.S + \beta \cdot \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}} \cdot (\bar{h}_1(s) - \bar{h}_2(s))} \Rightarrow$$

$$\Rightarrow \bar{h}_1 = \frac{\frac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta \cdot \cancel{(\bar{h}_1(s)-\bar{h}_2(s))}}}{\frac{2.A\sqrt{h_{1ss}-h_{2ss}}}{\beta \cdot (\bar{h}_1(s)-\bar{h}_2(s))} \cdot S + 1} \cdot \bar{F}_0(s)$$

- $K_{p1} = \frac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta}$
- $T_{p1} = \frac{2.A.\sqrt{h_{1ss}-h_{2ss}}}{\beta}$

(II) Balanço de Massa ~ Tanque 2



$$\to F_1(t) \cdot \cancel{\rho} - F_2(t) \cdot \cancel{\rho} = \frac{dM(t)}{dt} = \cancel{\rho} \cdot A \cdot \frac{dh_2}{dt} \Rightarrow$$

$$\Rightarrow \beta \cdot \sqrt{h_1(t) - h_2(t)} - \beta \cdot \sqrt{h_2(t)} = A \cdot \frac{dh_2}{dt} \quad (6)$$ → EDO não-linear

~ Linearização

$$\to f(x,y) = f(x_{ss}; y_{ss}) + \frac{d(x,y)}{dx} \cdot (x - x_{ss}) + \frac{d(x,y)}{dy} \cdot (y - y_{ss})$$

$$\to \sqrt{h_1(t) - h_2(t)} \approx \sqrt{h_{1ss} - h_{2ss}} + \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}} \cdot (h_1(t) - h_{1ss}) - \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}} \cdot (h_2(t) - h_{2ss}) \quad (7)$$

$$\to \sqrt{h_2(t)} \approx \sqrt{h_{2ss}} + \frac{1}{2\sqrt{h_{2ss}}} \cdot (h_2(t) - h_{2ss}) \quad (8)$$

~ Substituindo (7) e (8) em (6)

$$\to \beta \cdot \left(\sqrt{h_{1ss}-h_{2ss}} + \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}} \cdot (h_1(t) - h_{1ss}) - \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}} \cdot (h_2(t) - h_{2ss}) \right) - \beta \cdot \left(\sqrt{h_{2ss}} + \frac{1}{2\sqrt{h_{2ss}}} \cdot (h_2(t) - h_{2ss}) \right) = A \cdot dh_2/dt \quad (9)$$

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~ EDO no estado estacionário

$$\Rightarrow \beta.\sqrt{h_{1ss}-h_{2ss}} - \beta.\sqrt{h_{2ss}} = 0 \quad (10)$$

~ Subtraindo (10) de (9)

$$\Rightarrow \beta.\left(\cancel{\sqrt{h_{1ss}-h_{2ss}}} + \frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.(h_1(t)-h_{1ss}) - \frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.(h_2(t)-h_{2ss})\right) - \cancel{\beta.\sqrt{h_{1ss}-h_{2ss}}} - \beta.\left(\cancel{\sqrt{h_{2ss}}} + \frac{1}{2.\sqrt{h_{2ss}}}.(h_2(t)-h_{2ss})\right) + \cancel{\beta.\sqrt{h_{2ss}}} = A.\frac{dh_2}{dt}$$

$$\Rightarrow \beta.\left(\frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.(h_1(t)-h_{1ss}) - \frac{1}{2\sqrt{h_{1ss}-h_{2ss}}}.(h_2(t)-h_{2ss})\right) - \beta.\left(\frac{1}{2.\sqrt{h_{2ss}}}.(h_2(t)-h_{2ss})\right) = A.\frac{dh_2}{dt} \quad (11)$$

~ Escrevendo (11) na forma de variável desvio

$$\Rightarrow \beta.\left(\frac{1}{2\sqrt{h_{1ss}-h_{2ss}}}.h_1'(t) - \frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.h_2'(t)\right) - \beta.\left(\frac{1}{2\sqrt{h_{2ss}}}.h_2'(t)\right) = A.\frac{dh_2}{dt} \quad (12)$$

~ Aplicando transformada de Laplace

$$\Rightarrow \beta.\left(\frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.\bar{h}_1(s) - \frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.\bar{h}_2(s)\right) - \beta.\left(\frac{1}{2.\sqrt{h_{2ss}}}.\bar{h}_2(s)\right) = A.\bar{h}_2(s).S \Rightarrow \beta.\frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.\bar{h}_1(s) = A.\bar{h}_2(s).S + \beta.\frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}.\bar{h}_2(s) + \beta.\frac{1}{2\sqrt{h_{2ss}}}.\bar{h}_2(s) \Rightarrow$$

$$\Rightarrow \bar{h}_1(s) = \bar{h}_2(s).\frac{A.S + \beta.\frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}} + \beta.\frac{1}{2\sqrt{h_{2ss}}}}{\beta.\frac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}} \Rightarrow$$

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$$\Rightarrow \frac{\dfrac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\bar{h}_1(s)-\bar{h}_2(s))}}{\dfrac{2.A.\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\bar{h}_1(s)-\bar{h}_2(s))}.S+1}.\bar{F}_0(s) = \bar{h}_2(s).\frac{A.S+\beta.\dfrac{1}{2\sqrt{h_{1ss}-h_{2ss}}}+\beta.\dfrac{1}{2\sqrt{h_{2ss}}}}{\beta.\dfrac{1}{2\sqrt{h_{1ss}-h_{2ss}}}} \Rightarrow$$

$$\Rightarrow \bar{h}_2(s) = \bar{F}_0(s).\frac{\dfrac{2.\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\bar{h}_1(s)-\bar{h}_2(s))}}{\dfrac{2.A.\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\bar{h}_1(s)-\bar{h}_2(s))}.S+1}.\frac{\beta.\dfrac{1}{2.\sqrt{h_{1ss}-h_{2ss}}}}{A.S+\beta.\dfrac{1}{2\sqrt{h_{1ss}-h_{2ss}}}+\beta.\dfrac{1}{2\sqrt{h_{2ss}}}} \Rightarrow$$

$$\Rightarrow \bar{h}_2(s) = \bar{F}_0(s).\frac{\dfrac{2.\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\bar{h}_1(s)-\bar{h}_2(s))}}{\dfrac{2.A.\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\bar{h}_1(s)-\bar{h}_2(s))}.S+1}.\frac{\dfrac{\sqrt{h_{2ss}}}{\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}}}{\dfrac{2.A.\sqrt{h_{2ss}}.\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}})}.S+1} \Rightarrow$$

$$\Rightarrow \bar{h}_2(s) = \bar{F}_0(s).\frac{\dfrac{2.\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\bar{h}_1(s)-\bar{h}_2(s))}.\dfrac{\sqrt{h_{2ss}}}{\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}}}{\dfrac{4.A^2.(h_{1ss}-h_{2ss}).\sqrt{h_{2ss}}}{\beta^2.(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}).(\bar{h}_1(s)-\bar{h}_2(s))}.S^2+\left(\dfrac{2.A.\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\bar{h}_1(s)-\bar{h}_2(s))}+\dfrac{2.A.\sqrt{h_{2ss}}.\sqrt{h_{1ss}-h_{2ss}}}{\beta.(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}})}\right).S+1}$$

~ Aplicando Laplace inversa

(19) $\dfrac{1}{\tau\sqrt{1-\zeta}}.e^{-\zeta.t/\tau}.\text{sen}(\sqrt{1-\zeta^2}.t/\tau)$

$\tau^2.s^2+2.\zeta^2.\tau s+1$

* sabemos que é necessário aplicar Laplace Inversa para chegar em $h_2(t)$ pedida, no entanto não sabemos como fazer isso.

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