

## \* TG.6 - GRUPO 5

## NOMES:

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## Nº USP:

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$$① O_s = \frac{A}{B} = \exp\left(\frac{-\tilde{\eta} \cdot b}{\sqrt{1-b^2}}\right)$$

$$O_s = \frac{3,5}{7,7} = \exp\left(\frac{-\tilde{\eta} \cdot b}{\sqrt{1-b^2}}\right)$$

$$0,45 = \exp\left(\frac{-\tilde{\eta} \cdot b}{\sqrt{1-b^2}}\right)$$

$$\ln 0,45 = \frac{-\tilde{\eta} \cdot b}{\sqrt{1-b^2}}$$

$$(-0,7985)^2 = \left(\frac{-\tilde{\eta} \cdot b}{\sqrt{1-b^2}}\right)^2$$

$$0,6376 = \frac{\tilde{\eta}^2 \cdot b^2}{1-b^2}$$

$$0,6376 - 0,6376 b^2 = \tilde{\eta}^2 \cdot b^2$$

$$-0,6376 b^2 - \tilde{\eta}^2 b^2 = -0,6376$$

$$-b^2(0,6376 + \tilde{\eta}^2) = -0,6376$$

$$b^2 = \frac{0,6376}{0,6376 + \tilde{\eta}^2} = \frac{0,6376}{10,5072}$$

$$b = \sqrt{0,0606}$$

$$b = 0,2463 \text{ px}$$

→ razão de declínio

$$(O_s)^2 = (0,45)^2 = 0,2025$$

$$k_p = \frac{\Delta Y}{\Delta c} = \frac{10,2 - 2,5}{24 - 15} = 0,85 \frac{\text{mm}}{\text{px}}$$

→ período de oscilação

$$T = \frac{2\tilde{\eta}}{W} \rightarrow 3,5 = \frac{2\tilde{\eta}}{W} \rightarrow W = 1,795 \text{ us}$$

$$W = \frac{\sqrt{1-b^2}}{\zeta_p} \rightarrow 1,795 = \frac{\sqrt{1-0,2463^2}}{\zeta_p} \rightarrow \zeta_p = 0,54 \text{ us}$$

→ tempo de pico

$$t_p = \frac{\tilde{\eta} \cdot \zeta_p}{\sqrt{1-b^2}} = \frac{\tilde{\eta} \cdot 0,54}{\sqrt{1-0,2463^2}} \rightarrow t_p = 1,75 \text{ us}$$

$$① \frac{\bar{R}_m(s)}{P(s)} = \frac{k}{\zeta_p^2 s^2 + 2\zeta_p \zeta_p s + 1} = \frac{0,85}{(0,54)^2 s^2 + 2 \cdot 0,2463 \cdot 0,54 \cdot s + 1}$$

$$\frac{\bar{R}_m(s)}{P(s)} = \frac{0,85}{0,2916 s^2 + 0,2660 s + 1}$$

2) BM para Torque 1.

$$F_o(t) \cdot \rho - F_i(t) \cdot \rho = \frac{dM}{dt} = \rho \cdot A \cdot \frac{dh}{dt}$$

$$F_o(t) - F_i(t) = A \cdot \frac{dh}{dt} \Rightarrow F_o(t) - \beta \cdot \sqrt{h_1(t) - h_2(t)} = A \cdot \frac{dh}{dt}$$

↓  
EDO não linear

$$\sqrt{h_1(t) - h_2(t)} \approx \sqrt{h_1(ss) - h_2(ss)} + \frac{1}{2} \frac{(h_1(t) - h_1(ss))}{\sqrt{h_1(ss) - h_2(ss)}} - \frac{1}{2} \frac{(h_2(t) - h_2(ss))}{\sqrt{h_1(ss) - h_2(ss)}}$$

↳ linearizando:

$$F_o(t) - \beta \left( \sqrt{h_1(ss) - h_2(ss)} + \frac{1}{2} \frac{(h_1(t) - h_1(ss))}{\sqrt{h_1(ss) - h_2(ss)}} - \frac{1}{2} \frac{(h_2(t) - h_2(ss))}{\sqrt{h_1(ss) - h_2(ss)}} \right) = A \cdot \frac{dh}{dt}$$

→ EDO não linear no estado estacionário:

$$F_o(ss) - \beta \sqrt{h_1(ss) - h_2(ss)} = 0$$

→ EDO linearizada no estado estacionário

$$F_o(t) - F_o(ss) - \beta \sqrt{h_1(ss) - h_2(ss)} + \beta \sqrt{h_1(ss) - h_2(ss)} - \beta \frac{h_1(t) - h_1(ss)}{2 \sqrt{h_1(ss) - h_2(ss)}} + \beta \frac{h_2(t) - h_2(ss)}{2 \sqrt{h_1(ss) - h_2(ss)}} = A \cdot \frac{dh}{dt}$$

→ Variável desvio

$$F_o'(t) - \beta \frac{h_1'(t)}{2 \sqrt{h_1(ss) - h_2(ss)}} + \beta \frac{h_2'(t)}{2 \sqrt{h_1(ss) - h_2(ss)}} = A \frac{h_1'}{dt}$$

→ L.P.

$$\bar{F}_o(s) - \beta \frac{\bar{h}_1(s)}{2 \sqrt{h_1(ss) - h_2(ss)}} + \beta \frac{\bar{h}_2(s)}{2 \sqrt{h_1(ss) - h_2(ss)}} = A \cdot \bar{h}_1(s) \cdot s$$

$$A \cdot \bar{h}_1(s) \cdot s + \beta \frac{\bar{h}_1(s)}{2 \sqrt{h_1(ss) - h_2(ss)}} = \bar{F}_o(s) + \beta \frac{\bar{h}_2(s)}{2 \sqrt{h_1(ss) - h_2(ss)}}$$

$$\bar{h}_1(s) \left( A \cdot s + \frac{\beta}{2 \sqrt{h_1(ss) - h_2(ss)}} \right) = \bar{F}_o(s) + \beta \frac{\bar{h}_2(s)}{2 \sqrt{h_1(ss) - h_2(ss)}}$$

$$\bar{h}_1(s) = \frac{\bar{F}_o(s)}{A \cdot s + \frac{\beta}{2 \sqrt{h_1(ss) - h_2(ss)}}} + \beta \frac{\bar{h}_2(s)}{2 \sqrt{h_1(ss) - h_2(ss)}} \left( \frac{1}{A \cdot s + \frac{\beta}{2 \sqrt{h_1(ss) - h_2(ss)}}} \right)$$



$$\bar{h}_1(s) = \frac{\bar{F}_0(s)}{A \cdot s + \frac{\beta}{2 \cdot \sqrt{h_1(ss) - h_2(ss)}}} + \frac{\beta \cdot \bar{h}_2(s)}{A \cdot s \cdot 2 \sqrt{h_1(ss) - h_2(ss)} + \beta \cdot \frac{2 \sqrt{h_1(ss) - h_2(ss)}}{2 \sqrt{h_1(ss) - h_2(ss)}}$$

$$\bar{h}_1(s) = \bar{F}_0(s) \frac{\frac{2 \sqrt{h_1(ss) - h_2(ss)}}{\beta}}{A \cdot s \cdot 2 \sqrt{h_1(ss) - h_2(ss)} + 1} + \frac{\beta/\beta \cdot \bar{h}_2(s)}{A \cdot s \cdot 2 \sqrt{h_1(ss) - h_2(ss)} + 1}$$

\* FUNÇÃO DE TRANSFERÊNCIA PARA TANQUE 1.

$$\bar{h}_1(s) = \bar{F}_0(s) \cdot \frac{\frac{2 \sqrt{h_1(ss) - h_2(ss)}}{\beta}}{\frac{A \cdot 2 \sqrt{h_1(ss) - h_2(ss)}}{\beta} \cdot s + 1} + \bar{h}_2(s) \cdot \frac{1}{\frac{A \cdot 2 \sqrt{h_1(ss) - h_2(ss)}}{\beta} \cdot s + 1}$$

$\nearrow K_{p1}$   
 $\searrow \tau_{p1}$

\* B.M. tanque 2.

$$F_1 P + F_2 P = A \cdot P \cdot \frac{dh_2}{dt} \rightarrow \beta \sqrt{h_1(t) - h_2(t)} - \beta \sqrt{h_2(t)} = A \cdot \frac{dh_2}{dt}$$

↓ EDO não linear

→ linearizando o segundo termo, pois o primeiro foi anteriormente:

$$\sqrt{h_2(t)} = \sqrt{h_2(ss)} + \frac{1}{2 \sqrt{h_2(ss)}} (h_2(t) - h_2(ss))$$

substituindo na linearização:

$$\beta \left( \sqrt{h_1(ss) - h_2(ss)} + \frac{1}{2 \sqrt{h_1(ss) - h_2(ss)}} (h_1(t) - h_1(ss)) - \frac{1}{2 \sqrt{h_1(ss) - h_2(ss)}} (h_2(t) - h_2(ss)) \right) - \beta \left( \sqrt{h_2(ss)} + \frac{1}{2 \sqrt{h_2(ss)}} (h_2(t) - h_2(ss)) \right) = A \cdot \frac{dh_2}{dt} \rightarrow \text{EDO linear}$$

→ EDO não linear no estado estacionário.

$$\beta \sqrt{h_1(ss) - h_2(ss)} - \beta \sqrt{h_2(ss)} = 0$$

→ EDO linear no estado estacionário

$$\beta \sqrt{h_1(ss) - h_2(ss)} - \beta \sqrt{h_1(ss) - h_2(ss)} + \frac{\beta (h_1(t) - h_1(ss))}{2 \sqrt{h_1(ss) - h_2(ss)}} - \frac{\beta (h_2(t) - h_2(ss))}{2 \sqrt{h_1(ss) - h_2(ss)}} - \beta \sqrt{h_2(ss)} + \beta \sqrt{h_2(ss)}$$

$$- \frac{\beta}{2} \frac{(h_2(t) - h_2(ss))}{\sqrt{h_2(ss)}} = A \cdot \frac{dh_2}{dt}$$



→ Variáveis desvio

$$\frac{\beta}{2} \frac{\bar{h}_1'(t)}{\sqrt{h_1(ss) - h_2(ss)}} - \frac{\beta}{2} \frac{\bar{h}_2'(t)}{\sqrt{h_1(ss) - h_2(ss)}} - \frac{\beta}{2} \frac{\bar{h}_2'(t)}{\sqrt{h_2(ss)}} = A \cdot \frac{d\bar{h}_2}{dt}$$

→ Aplicando T.L

$$\frac{\beta}{2} \frac{\bar{h}_1(s)}{\sqrt{h_1(ss) - h_2(ss)}} - \frac{\beta}{2} \frac{\bar{h}_2(s)}{\sqrt{h_1(ss) - h_2(ss)}} - \frac{\beta}{2} \frac{\bar{h}_2(s)}{\sqrt{h_2(ss)}} = A \cdot s \cdot \bar{h}_2(s)$$

$$A \cdot s \cdot \bar{h}_2(s) + \frac{\beta}{2} \frac{\bar{h}_2(s)}{\sqrt{h_1(ss) - h_2(ss)}} + \frac{\beta}{2} \frac{\bar{h}_2(s)}{\sqrt{h_2(ss)}} = \frac{\beta}{2} \frac{\bar{h}_1(s)}{\sqrt{h_1(ss) - h_2(ss)}}$$

$$\bar{h}_2(s) \left( A \cdot s + \frac{\beta}{2\sqrt{h_1(ss) - h_2(ss)}} + \frac{\beta}{2\sqrt{h_2(ss)}} \right) = \frac{\beta}{2} \frac{\bar{h}_1(s)}{\sqrt{h_1(ss) - h_2(ss)}}$$

$$\bar{h}_2(s) \left[ A \cdot s + \frac{\beta (\sqrt{h_1(ss) - h_2(ss)} + \sqrt{h_2(ss)})}{2\sqrt{h_1(ss) - h_2(ss)} \cdot \sqrt{h_2(ss)}} \right] = \frac{\beta}{2} \frac{\bar{h}_1(s)}{\sqrt{h_1(ss) - h_2(ss)}}$$

$$\bar{h}_2(s) = \frac{\beta}{2} \frac{\bar{h}_1(s)}{\sqrt{h_1(ss) - h_2(ss)}} \cdot \frac{1}{A \cdot s + \frac{\beta (\sqrt{h_1(ss) - h_2(ss)} + \sqrt{h_2(ss)})}{2\sqrt{h_1(ss) - h_2(ss)} \cdot \sqrt{h_2(ss)}}$$

$$\bar{h}_2(s) = \frac{\beta}{2} \frac{\bar{h}_1(s)}{A \cdot s \sqrt{h_1(ss) - h_2(ss)} + \frac{\beta (\sqrt{h_1(ss) - h_2(ss)} + \sqrt{h_2(ss)})}{\sqrt{h_2(ss)}}$$

$$\bar{h}_2(s) = \frac{\beta \cdot \bar{h}_1(s) \cdot \sqrt{h_2(ss)}}{\beta (\sqrt{h_1(ss) - h_2(ss)} + \sqrt{h_2(ss)})} \cdot \frac{1}{\left( \frac{2 \cdot A \cdot s \cdot \sqrt{h_1(ss) - h_2(ss)} \cdot \sqrt{h_2(ss)}}{\beta (\sqrt{h_1(ss) - h_2(ss)} + \sqrt{h_2(ss)})} \right) + 1}$$

$$\bar{h}_2(s) = \frac{\bar{h}_1(s) \cdot \sqrt{h_2(ss)}}{\left( \frac{2 \cdot A \cdot s \cdot \sqrt{h_1(ss) - h_2(ss)} \cdot \sqrt{h_2(ss)}}{\beta (\sqrt{h_1(ss) - h_2(ss)} + \sqrt{h_2(ss)})} \right) + 1}$$

→  $K_{Pa}$

↓  $G_{Pa}$

$$\bar{h}_2(s) = \bar{h}_1(s) \cdot \frac{K_{Pa}}{G_{Pa} \cdot s + 1}$$

$$\bar{h}_2(s) = \left( \bar{F}_1(s) \cdot \frac{K_{Pa}}{G_{Pa} \cdot s + 1} + \bar{h}_2(s) \cdot \frac{1}{G_{Pa} \cdot s + 1} \right) \cdot \frac{K_{Pa}}{G_{Pa} \cdot s + 1}$$

$$\bar{h}_2(s) = \left( \frac{\bar{h}_2(s)}{\tau_{p1} \cdot s + 1} \right) \left( \frac{K_{p2}}{\tau_{p2} \cdot s + 1} \right) = \bar{F}_0(s) \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)}$$

$$\bar{h}_2(s) \left( 1 - \frac{K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)} \right) = \bar{F}_0(s) \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)}$$

$$\bar{h}_2(s) \left( \frac{(\tau_{p1} \cdot \tau_{p2} \cdot s^2) + (\tau_{p1} \cdot s + \tau_{p2} \cdot s) + 1 - K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)} \right) = \bar{F}_0(s) \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)}$$

$$\bar{h}_2(s) = \frac{\bar{F}_0(s) \cdot K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)} \cdot \frac{1}{(\tau_{p1} \cdot \tau_{p2} \cdot s^2) + (\tau_{p1} \cdot s + \tau_{p2} \cdot s) + 1 - K_{p2}}$$

$$\bar{h}_2(s) = \frac{\bar{F}_0(s) \cdot K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot \tau_{p2} \cdot s^2) + (\tau_{p1} \cdot s + \tau_{p2} \cdot s) + 1 - K_{p2}}$$

$$\bar{h}_2(s) = \frac{\frac{K_{p1} \cdot K_{p2}}{1 - K_{p2}}}{\frac{\tau_{p1} \tau_{p2} s^2}{1 - K_{p2}} + \frac{(\tau_{p1} + \tau_{p2}) \cdot s}{1 - K_{p2}} + 1} \cdot \bar{F}_0(s)$$