S T Q Q S S D Data: 30 • 04 • 24

TG 6 - GRUPO 5

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① $Os = \frac{A}{B} = \exp\left(\frac{-\pi \cdot \beta}{\sqrt{1-\beta^2}}\right)$

$Os = \frac{3,5}{7,7} = \exp\left(\frac{-\pi \cdot \beta}{\sqrt{1-\beta^2}}\right)$

$0,45 = \exp\left(\frac{-\pi \cdot \beta}{\sqrt{1-\beta^2}}\right)$

$\ln 0,45 = \frac{-\pi \cdot \beta}{\sqrt{1-\beta^2}}$

$(-0,7985)^2 = \left(\frac{-\pi \cdot \beta}{\sqrt{1-\beta^2}}\right)^2$

$0,6376 = \frac{\pi^2 \cdot \beta^2}{1-\beta^2}$

$0,6376 - 0,6376\,\beta^2 = \pi^2 \cdot \beta^2$

$-0,6376\,\beta^2 - \pi^2\beta^2 = -0,6376$

$-\beta^2(0,6376 + \pi^2) = -0,6376$

$\beta^2 = \frac{0,6376}{0,6376+\pi^2} = \frac{0,6376}{10,5072}$

$\beta = \sqrt{0,0606}$

$\beta = 0,2463$ psi

→ razão de declínio

$(Os)^2 = (0,45)^2 = 0,2025$

$K_p = \frac{\Delta Y}{\Delta c} = \frac{10,2-2,5}{24-15} = 0,85 \frac{mm}{psi}$

→ período de oscilação

$T = \frac{2\pi}{W} \rightarrow 3,5 = \frac{2\pi}{W} \rightarrow W = 1,795\,s$

$W = \frac{\sqrt{1-\beta^2}}{\tau_p} \rightarrow 1,795 = \frac{\sqrt{1-0,2463^2}}{\tau_p} \rightarrow \tau_p = 0,54\,s$

→ tempo de pico

$t_p = \frac{\pi \cdot \tau_p}{\sqrt{1-\beta^2}} = \frac{\pi \cdot 0,54}{\sqrt{1-0,2463^2}} \rightarrow t_p = 1,75\,s$

$\frac{\overline{Rm}(s)}{P(s)} = \frac{K}{\tau_p^2 \cdot s^2 + 2\beta\tau_p \cdot s + 1} = \frac{0,85}{(0,54)^2 s^2 + 2 \cdot 0,2463 \cdot 0,54 \cdot s + 1}$

$\frac{\overline{Rm}(s)}{P(s)} = \frac{0,85}{0,2916 \cdot s^2 + 0,2660 \cdot s + 1}$

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(2) BM para Tanque 1

$$F_{0(t)}.\rho - F_{1(t)}.\rho = \frac{dM_1}{dt} = \rho.A.\frac{dh_1}{dt}$$

$$F_{0(t)} - F_{1(t)} = A.\frac{dh_1}{dt} \Rightarrow F_{0(t)} - \beta.\sqrt{h_{1(t)} - h_{2(t)}} = A_1.\frac{dh_1}{dt}$$

EDO não linear

$$\sqrt{h_{1(t)} - h_{2(t)}} \approx \sqrt{h_{1(ss)} - h_{2(ss)}} + \frac{1}{2}\frac{(h_{1(t)} - h_{1(ss)})}{\sqrt{h_{1(ss)} - h_{2(ss)}}} - \frac{1}{2}\frac{(h_{2(t)} - h_{2(ss)})}{\sqrt{h_{1(ss)} - h_{2(ss)}}}$$

↳ linearizando:

$$F_{0(t)} - \beta\left(\sqrt{h_{1(ss)} - h_{2(ss)}} + \frac{1}{2}\frac{(h_{1(t)} - h_{1(ss)})}{\sqrt{h_{1(ss)} - h_{2(ss)}}} - \frac{1}{2}\frac{(h_{2(t)} - h_{2(ss)})}{\sqrt{h_{1(ss)} - h_{2(ss)}}}\right) = A.\frac{dh_1}{dt}$$

→ EDO não linear no estado estacionário:

$$F_{0(ss)} - \beta\sqrt{h_{1(ss)} - h_{2(ss)}} = 0$$

→ EDO linearizada no estado estacionário

$$F_0(t) - F_0(ss) - \beta\sqrt{h_{1(ss)} - h_{2(ss)}} + \beta\sqrt{h_{1(ss)} - h_{2(ss)}} - \frac{\beta}{2}\frac{h_{1(t)} - h_{1(ss)}}{\sqrt{h_{1(ss)} - h_{2(ss)}}} + \frac{\beta}{2}\frac{h_{2(t)} - h_{2(ss)}}{\sqrt{h_{1(ss)} - h_{2(ss)}}} = A.\frac{dh_1}{dt}$$

→ Variável desvio

$$F_0'(t) - \frac{\beta}{2}\frac{h_1'(t)}{\sqrt{h_{1(ss)} - h_{2(ss)}}} + \frac{\beta}{2}\frac{h_2'(t)}{\sqrt{h_{1(ss)} - h_{2(ss)}}} = A\frac{h_1'}{dt}$$

→ L.P.

$$\bar{F_0}(s) - \frac{\beta}{2}\frac{\bar{h_1}(s)}{\sqrt{h_{1(ss)} - h_{2(ss)}}} + \frac{\beta}{2}\frac{\bar{h_2}(s)}{\sqrt{h_{1(ss)} - h_{2(ss)}}} = A.\bar{h_1}(s).S$$

$$A.\bar{h_1}(s).S + \frac{\beta}{2}\frac{\bar{h_1}(s)}{\sqrt{h_{1(ss)} - h_{2(ss)}}} = \bar{F_0}(s) + \frac{\beta}{2}\frac{\bar{h_2}(s)}{\sqrt{h_{1(ss)} - h_{2(ss)}}}$$

$$\bar{h_1}(s)\left(A.S + \frac{\beta}{2.\sqrt{h_{1(ss)} + h_{2(ss)}}}\right) = \bar{F_0}(s) + \frac{\beta}{2}\frac{\bar{h_2}(s)}{\sqrt{h_{1(ss)} - h_{2(ss)}}}$$

$$\bar{h_1}(s) = \frac{\bar{F_0}(s)}{A.S + \frac{\beta}{2.\sqrt{h_{1(ss)} + h_{2(ss)}}}} + \frac{\beta}{2}\frac{\bar{h_2}(s)}{\sqrt{h_{1(ss)} - h_{2(ss)}}}\left(\frac{1}{A.S + \frac{\beta}{2\sqrt{h_{1(ss)} + h_{2(ss)}}}}\right)$$

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$$\bar{h}_1(s) = \frac{\bar{F}_0(s)}{A.S + \frac{\beta}{2.\sqrt{h_{1(ss)}-h_{2(ss)}}}} + \frac{\beta.\bar{h}_2(s)}{A.S.2\sqrt{h_{1(ss)}-h_{2(ss)}} + \beta.\frac{\cancel{2\sqrt{h_{1(ss)}-h_{2(ss)}}}}{\cancel{2\sqrt{h_{1(ss)}-h_{2(ss)}}}}}$$

$$\bar{h}_1(s) = \bar{F}_0(s)\frac{\frac{2\sqrt{h_{1(ss)}+h_{2(ss)}}}{\beta}}{\frac{A.S.2\sqrt{h_{1(ss)}-h_{2(ss)}}}{\beta}+1} + \frac{1\ \cancel{\beta/\beta}.\bar{h}_2(s)}{\frac{A.S.2.\sqrt{h_{1(ss)}-h_{2(ss)}}}{\beta}+1}$$

(*) FUNÇÃO DE TRANSFERÊNCIA PARA TANQUE 1

$$\bar{h}_1(s) = \bar{F}_0(s).\frac{\overbrace{\frac{2.\sqrt{h_{1(ss)}-h_{2(ss)}}}{\beta}}^{K_{p1}}}{\underbrace{\frac{A.2.\sqrt{h_{1(ss)}-h_{2(ss)}}}{\beta}}_{\tau_{p1}}S+1} + \bar{h}_2(s).\frac{1}{S.\underbrace{\frac{A.2\sqrt{h_{1(ss)}+h_{2(ss)}}}{\beta}}_{\tau_{p1}}+1}$$

(*) B.M. tanque 2

$$F_1\rho + F_2\rho = A.\rho.\frac{dh_2}{dt} \rightarrow \beta\sqrt{h_{1(t)}-h_{2(t)}} - \beta\sqrt{h_{2(t)}} = A.\frac{dh_2}{dt}$$

EDO não linear

→ linearizando o segundo termo, pois o primeiro foi anteriormente:

$$\sqrt{h_{2(t)}} = \sqrt{h_{2(ss)}} + \frac{1}{2\sqrt{h_{2(ss)}}}(h_{2(t)}-h_{2(ss)})$$

substituindo as linearizações:

$$\beta\left(\sqrt{h_{1(ss)}-h_{2(ss)}} + \frac{1}{2}\frac{(h_{1(t)}-h_{1(ss)})}{\sqrt{h_{1(ss)}-h_{2(ss)}}} - \frac{1}{2}\frac{(h_{2(t)}-h_{2(ss)})}{\sqrt{h_{1(ss)}-h_{2(ss)}}}\right) - \beta\left(\sqrt{h_{2(ss)}} + \frac{1}{2}\frac{(h_{2(t)}-h_{2(ss)})}{\sqrt{h_{2(ss)}}}\right)$$

$$= A.\frac{dh_2}{dt} \rightarrow \text{EDO linear}$$

→ EDO não linear no estado estacionário

$$\beta.\sqrt{h_{1(ss)}-h_{2(ss)}} - \beta\sqrt{h_{2(ss)}} = 0$$

→ EDO linear no estado estacionário

$$\beta.\sqrt{h_{1(ss)}-h_{2(ss)}} - \beta\sqrt{h_{1(ss)}-h_{2(ss)}} + \frac{\beta(h_{1(t)}-h_{1(ss)})}{2\sqrt{h_{1(ss)}-h_{2(ss)}}} - \frac{\beta(h_{1(t)}-h_{1(ss)})}{2\sqrt{h_{1(ss)}-h_{2(ss)}}} - \beta\sqrt{h_{2(ss)}} + \beta\sqrt{h_{2(ss)}}$$

$$-\frac{\beta}{2}\frac{(h_{2(t)}-h_{2(ss)})}{\sqrt{h_{2(ss)}}} = A.\frac{dh_2}{dt}$$

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→ Variável desvio

$$\frac{\beta}{2}\frac{h_1'(t)}{\sqrt{h_{1(ss)}-h_{2(ss)}}}-\frac{\beta}{2}\frac{h_2'(t)}{\sqrt{h_{1(ss)}-h_{2(ss)}}}-\frac{\beta}{2}\frac{h_2'(t)}{\sqrt{h_{2(ss)}}}=A.\frac{dh_2'}{dt}$$

→ Aplicando T.L

$$\frac{\beta}{2}\frac{\bar{h}_{1(s)}}{\sqrt{h_{1(ss)}-h_{2(ss)}}}-\frac{\beta}{2}\frac{\bar{h}_{2(s)}}{\sqrt{h_{1(ss)}-h_{2(ss)}}}-\frac{\beta\,\bar{h}_{2(s)}}{2\sqrt{h_{2(ss)}}}=A.S.\bar{h}_{2(s)}$$

$$A.S.\bar{h}_{2(s)}+\frac{\beta}{2}\frac{\bar{h}_{2(s)}}{\sqrt{h_{1(ss)}-h_{2(ss)}}}+\frac{\beta}{2}\frac{\bar{h}_{2(s)}}{\sqrt{h_{2(ss)}}}=\frac{\beta}{2}\frac{\bar{h}_{1(s)}}{\sqrt{h_{1(ss)}-h_{2(ss)}}}$$

$$\bar{h}_{2(s)}\left(A.S+\frac{\beta}{2\sqrt{h_{1(ss)}-h_{2(ss)}}}+\frac{\beta}{2\sqrt{h_{2(ss)}}}\right)=\frac{\beta\,\bar{h}_{1(s)}}{2\sqrt{h_{1(ss)}-h_{2(ss)}}}$$

$$\bar{h}_{2(s)}.\left[A.S+\frac{\beta\left(\sqrt{h_{1(ss)}-h_{2(ss)}}\right)+\sqrt{h_{2(ss)}}}{2\sqrt{h_{1(ss)}-h_{2(ss)}}.\sqrt{h_{2(ss)}}}\right]=\frac{\beta}{2}\frac{\bar{h}_{1(s)}}{\sqrt{h_{1(ss)}-h_{2(ss)}}}$$

$$\bar{h}_{2(s)}=\frac{\beta}{2}\frac{\bar{h}_{1(s)}}{\sqrt{h_{1(ss)}-h_{2(ss)}}}\cdot\frac{1}{A.S+\dfrac{\beta\left(\sqrt{h_{1(ss)}-h_{2(ss)}}\right)+\sqrt{h_{2(ss)}}}{2\sqrt{h_{1(ss)}-h_{2(ss)}}.\sqrt{h_{2(ss)}}}}$$

$$\bar{h}_{2(s)}=\frac{\beta\,\bar{h}_{1(s)}}{2\,A.S\sqrt{h_{1(ss)}-h_{2(ss)}}+\dfrac{\beta\left(\sqrt{h_{1(ss)}-h_{2(ss)}}\right)+\sqrt{h_{2(ss)}}}{\sqrt{h_{2(ss)}}}}$$

$$\bar{h}_{2(s)}=\frac{\beta.\bar{h}_{1(s)}.\dfrac{\sqrt{h_{2(ss)}}}{\beta\left(\sqrt{h_{1(ss)}-h_{2(ss)}}+\sqrt{h_{2(ss)}}\right)}}{\left(\dfrac{2.A.S\sqrt{h_{1(ss)}-h_{2(ss)}}.\sqrt{h_{2(ss)}}}{\beta\left(\sqrt{h_{1(ss)}-h_{2(ss)}}+\sqrt{h_{2(ss)}}\right)}\right)+1}$$

$$\bar{h}_{2(s)}=\frac{\bar{h}_{1(s)}\overbrace{\dfrac{\sqrt{h_{2(ss)}}}{\sqrt{h_{1(ss)}-h_{2(ss)}}+\sqrt{h_{2(ss)}}}}^{K_{p2}}}{\underbrace{\dfrac{2.A.\sqrt{h_{1(ss)}-h_{2(ss)}}.\sqrt{h_{2(ss)}}}{\beta\left(\sqrt{h_{1(ss)}-h_{2(ss)}}+\sqrt{h_{2(ss)}}\right)}}_{\tau_{p2}}.S+1}$$

$$\bar{h}_{2(s)}=\bar{h}_{1(s)}.\frac{K_{p2}}{\tau_{p2}.S+1}$$

$$\bar{h}_{2(s)}=\left(\bar{F}_{0(s)}.\frac{K_{p1}}{\tau_{p1}.S+1}+\bar{h}_{2(s)}.\frac{1}{\tau_{p1}.S+1}\right).\frac{K_{p2}}{\tau_{p2}.S+1}$$

$$\bar{h}_{2(s)} - \left(\frac{\bar{h}_{2(s)}}{\tau_{p1} \cdot s + 1}\right)\left(\frac{K_{p2}}{\tau_{p2} \cdot s + 1}\right) = \bar{F}_{0(s)} \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1) \cdot (\tau_{p2} \cdot s + 1)}$$

$$\bar{h}_{2(s)} \left(1 - \frac{K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)}\right) = \bar{F}_{0(s)} \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)}$$

$$\bar{h}_{2(s)} \left(\frac{(\tau_{p1} \cdot \tau_{p2} \cdot s^2) + (\tau_{p1} \cdot s + \tau_{p2} \cdot s) + 1 - K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)}\right) = \bar{F}_{0(s)} \cdot \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)}$$

$$\bar{h}_{2(s)} = \frac{\bar{F}_{0(s)} \cdot K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)} \cdot \frac{1}{\dfrac{(\tau_{p1} \cdot \tau_{p2} \cdot s^2) + (\tau_{p1} \cdot s + \tau_{p2} \cdot s) + 1 - K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)}}$$

$$\bar{h}_{2(s)} = \frac{\bar{F}_{0(s)} \cdot K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot \tau_{p2} \cdot s^2) + (\tau_{p1} \cdot s + \tau_{p2} \cdot s) + 1 - K_{p2}}$$

$$\bar{h}_{2(s)} = \frac{\dfrac{K_{p1} \cdot K_{p2}}{1 - K_{p2}}}{\dfrac{\tau_{p1} \tau_{p2} s^2}{1 - K_{p2}} + \dfrac{(\tau_{p1} + \tau_{p2}) \cdot s}{1 - K_{p2}} + 1} \cdot \bar{F}_{0(s)}$$