9,5

6° Trabalho em Grupo – Comportamento dinâmico de sistemas de 2ª. ordem

Grupo: 4

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1) Uma mudança em degrau na pressão de um vaso de 15 para 24 psi resulta em uma resposta no medidor de pressão de acordo com a Figura 1. Assuma dinâmica de segunda ordem, calcule todos os parâmetros importantes e escreva a FT na forma:

$$\frac{\overline{Rm}(s)}{\overline{P}(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

Em que Rm → instrumento de medida (mm); P → pressão (psi)

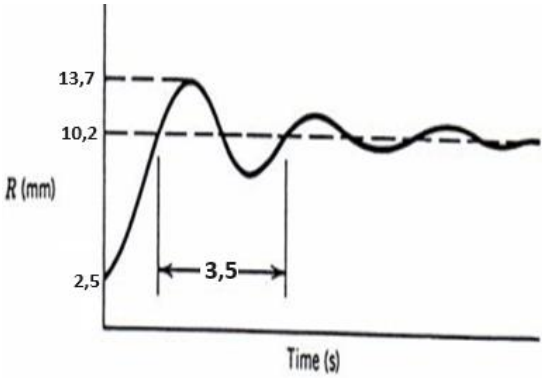

Figura 1: Resposta dinâmica do sistema de medida

2) Encontre a função de transferência, de um sistema composto por 2 tanques em série como mostrado na Figura 2, que descreva o comportamento do sistema ($h_2(t)$) frente a uma perturbação na vazão de entrada (F_0 (t)).

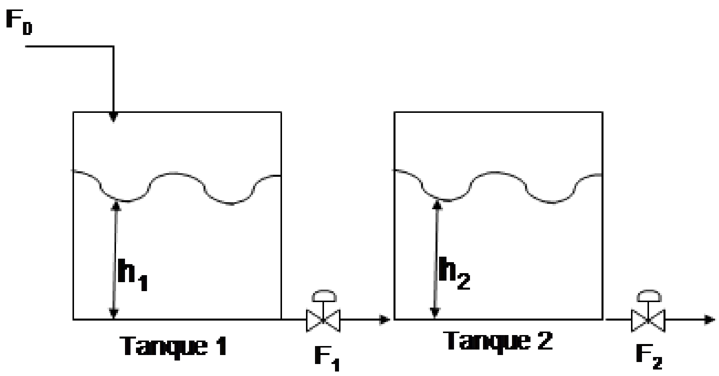

Figura 2: Tanques em série

Considere: $F_1 = \beta\sqrt{h_1 - h_2}$ $\quad F_2 = \beta\sqrt{h_2}$ em que β é o termo associado às resistências aos fluxos volumétricos F_1 e F_2, respectivamente.

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TG6 (1)

(1) $A = 3,5$ $\quad$ degrau: $24 - 15 = 9$

$B = 7,7 = \Delta y$ $\quad$ $K_p = \frac{7,7}{9} = 0,85$

$$OS = \frac{A}{B} = \frac{3,5}{7,7} = 0,45$$

$$\frac{C}{A} = (0,45)^2 \rightarrow C = (0,45)^2 \cdot 3,5$$

$$C = 0,70.$$

$$OS = \exp\left(\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}\right)$$

$$\ln 0,45 = \cancel{\ln}\left(\cancel{\exp}\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)\right)$$

$$0,8 = \frac{\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\left((0,8)\cdot\cancel{\sqrt{1-\zeta^2}}\right)^{\cancel{2}} = (\pi\zeta)^2$$

$$0,64 \cdot (1-\zeta^2) = \pi^2 \cdot \zeta^2$$

$$(0,64) - (0,64 \cdot \zeta^2) = \pi^2 \cdot \zeta^2$$

$$\frac{(0,64)}{\pi^2} - \frac{(0,64 \cdot \zeta^2)}{\pi^2} = \zeta^2$$

$$0,06484 - (0,06484 \cdot \zeta^2) = \zeta^2$$

$$0,06484 = 1,06484\, \zeta^2$$

$$\zeta^2 = 0,0609 \rightarrow \zeta = \sqrt{0,0609} = 0,2467 \quad (\zeta)$$

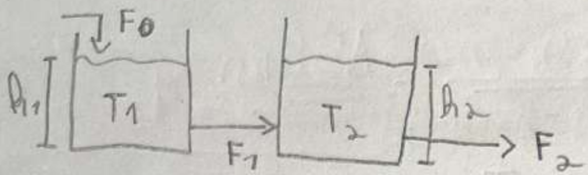
$$T = \frac{2\pi}{\omega} \rightarrow 3,5 = \frac{2\pi}{\omega} \rightarrow \omega = \frac{2\pi}{3,5} = 1,795$$

$$\omega = \frac{\sqrt{1-\zeta^2}}{\tau_p} \rightarrow 1,795 = \frac{\sqrt{1-0,0609}}{\tau_p} \rightarrow \tau_p = \frac{0,97}{1,795} = 0,54 \quad (\tau_p)$$

$$\frac{Rm(s)}{P(s)} = \frac{0,85}{0,54^2 \cdot s^2 + 2 \cdot 0,2467 \cdot 0,54 \cdot s + 1}$$

$$\frac{Rm(s)}{P(s)} =$$

Ex 2 TG 6:



$F_1 = \beta\sqrt{h_1 - h_2}$

$F_2 = \beta\sqrt{h_2}$

Hp: ∘ Área de seção transversal constante

∘ ρ (densidade) constante

Balanço de massa para tanque 1:

$$\rho F_{0(t)} - \rho F_{1(t)} = \frac{dM}{dt} = \frac{\rho\, dV}{dt} = \rho \cdot A_1 \cdot \frac{dh_1}{dt} \quad \left\{ M_{(kg)} = \rho_{\left(\frac{kg}{m^3}\right)} \cdot A_{(m^2)} \cdot h_{(m)} \right.$$

$$F_{0(t)} - F_{1(t)} = A\, d h_{1(t)} \rightarrow F_{0(t)} - \beta\sqrt{h_{1(t)} - h_{2(t)}} = A_1 \frac{dh_{1(t)}}{dt} \quad ① \text{ EDO Ñ-Lin}$$

no estado estacionário (SS):

$$0 = F_{0ss} - \beta\sqrt{h_{1ss} - h_{2ss}} \quad (2)$$

linearização:

$$\sqrt{h_{1(t)} - h_{2(t)}} \cong \sqrt{h_{1ss} - h_{2ss}} + \frac{1}{2\sqrt{h_{1ss} + h_{2ss}}}\, h_{1(t)} - h_{1ss} - \frac{1}{2\sqrt{h_{1ss} - h_{2ss}}} \cdot (h_{2(t)} - h_{2ss}) \quad (3)$$

Substituindo (3) em (1):

$$F_{0(t)} - \beta\left(\sqrt{h_{1ss} - h_{2ss}} + \frac{1\, h_{1(t)}}{2\sqrt{h_{1ss} - h_{2ss}}} - h_{1ss} - \frac{1(h_{2(t)} - h_{2ss})}{2\sqrt{h_{1ss} - h_{2ss}}}\right) = A_1 \frac{dh_{1(t)}}{dt} \quad (4)$$

Subtraindo (2) de (4):

$$(F_{0(t)} - F_{0ss}) - (\beta\sqrt{h_{1ss} - h_{2ss}}) + \frac{\beta\,(h_{1(t)} - h_{1ss})}{2\sqrt{h_{1ss} - h_{2ss}}} - \frac{\beta\,(h_{2(t)} - h_{2ss})}{2\sqrt{h_{1ss} - h_{2ss}}} + \beta\sqrt{h_{1ss} - h_{2ss}}$$
$$= A_1 \frac{dh_1}{dt}$$

$$F_{0(t)} - F_{0ss} + \frac{\beta(h_{1(t)} - h_{1ss})}{2\sqrt{h_{1ss} - h_{2ss}}} - \frac{\beta(h_{2(t)} - h_{2ss})}{2\sqrt{h_{1ss} - h_{2ss}}} = A_1 \frac{dh_1}{dt} \quad (5)$$

(5) na forma de variável desvio: $(X' = X_{(t)} - X_{ss})$

$$F'_{0(t)} + \frac{\beta\, h'_{1(t)}}{2\sqrt{h_{1ss} - h_{2ss}}} - \frac{\beta\, h'_{2(t)}}{2\sqrt{h_{1ss} - h_{2ss}}} = A_1 \frac{dh'_1}{dt} \quad (6)$$

Aplicando Transformada de Laplace em (6):

$$\bar F_{D(s)} + \frac{\beta \bar h_{1(s)}}{2\sqrt{h_{1ss}-h_{2ss}}} - \frac{\beta \bar h_{2(s)}}{2\sqrt{h_{1ss}-h_{2ss}}} = A_1 s \bar h_{1(s)}$$

$$A_1 s \bar h_{1(s)} + \frac{\beta \bar h_{1(s)}}{2\sqrt{h_{1ss}-h_{2ss}}} = \bar F_{D(s)} + \frac{\beta \bar h_{2(s)}}{2\sqrt{h_{1ss}-h_{2ss}}}$$

$$\bar h_{1(s)} = \frac{\bar F_{D(s)}}{A_1 s + \frac{\beta}{2\sqrt{h_{1ss}-h_{2ss}}}} + \frac{\beta \bar h_{2(s)} / 2\sqrt{h_{1ss}-h_{2ss}}}{A_1 s + \frac{\beta}{2\sqrt{h_{1ss}-h_{2ss}}}} \cdot \left(\frac{2\sqrt{\;}}{\beta}\right)$$

$$\bar h_{1(s)} = \frac{\frac{2\bar F_{D(s)}\sqrt{h_{1ss}-h_{2ss}}}{\beta}}{\frac{2A_1 s\sqrt{h_{1ss}-h_{2ss}}}{\beta}+1} + \frac{\frac{\beta \bar h_{2(s)}}{2\sqrt{h_{1ss}-h_{2ss}}}\cdot\frac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta}}{\frac{2A_1 s\sqrt{h_{1ss}-h_{2ss}}}{\beta}+1}$$

$$\bar h_{1(s)} = \frac{\overbrace{\frac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta}}^{K_{P_1}}}{\underbrace{\frac{2A_1 s\sqrt{h_{1ss}-h_{2ss}}}{\beta}}_{\tau_{P_1}}+1}\bar F_{D(s)} + \frac{1}{\frac{2A_1\sqrt{h_{1ss}-h_{2ss}}}{\beta}s+1}\bar h_{2(s)} \quad (7)$$

Balanço de massa para o tanque 2:

$$\rho F_1 - \rho F_2 = \frac{dM}{dt} = \rho\frac{dV}{dt} = \rho A_2 \frac{dh_2}{dt}$$

$$A_2\frac{dh_2}{dt} = \beta\sqrt{h_{1(t)}-h_{2(t)}} - \beta\sqrt{h_{2(t)}} \quad (8) \text{ EDO ÑL}$$

no SS:

$$0 = \beta\sqrt{h_{1ss}-h_{2ss}} - \beta\sqrt{h_{2ss}} \quad (9)$$

linearizando a EDO:

$$F_1 \to \sqrt{h_{1(t)}-h_{2(t)}} = \sqrt{h_{1ss}-h_{2ss}} + \frac{1\,(h_{1(t)}-h_{1ss})}{2\sqrt{h_{1ss}-h_{2ss}}} - \frac{1\,(h_{2(t)}-h_{2ss})}{2\sqrt{h_{1ss}-h_{2ss}}}$$

$$F_2 \to \sqrt{h_{2(t)}} = \sqrt{h_{2ss}} + \frac{1}{2\sqrt{h_{2ss}}}(h_{2(t)}-h_{2ss})$$

Substituindo na EDO Ñ Linear:

$$A_2\frac{dh_2}{dt} = \beta\left(\sqrt{h_{1ss}-h_{2ss}} + \frac{(h_{1(t)}-h_{1ss})}{2\sqrt{h_{1ss}-h_{2ss}}} - \frac{(h_{2(t)}-h_{2ss})}{2\sqrt{h_{1ss}-h_{2ss}}}\right) - \beta\left(\sqrt{h_{2ss}} + \frac{(h_{2(t)}-h_{2ss})}{2\sqrt{h_{2ss}}}\right) \quad (10) \text{ EDO linearizada}$$

Subtraindo (9) de (10):

$$A_2\frac{dh_2}{dt} = \cancel{\beta\sqrt{h_{1ss}-h_{2ss}}} - \cancel{\beta\sqrt{h_{1ss}-h_{2ss}}} + \beta\frac{(h_{1(t)}-h_{1ss})}{2\sqrt{h_{1ss}-h_{2ss}}} - \beta\frac{(h_{2(t)}-h_{2ss})}{2\sqrt{h_{1ss}-h_{2ss}}} - \cancel{\beta\sqrt{h_{2ss}}} + \cancel{\beta\sqrt{h_{2ss}}} - \beta\frac{(h_{2(t)}-h_{2ss})}{2\sqrt{h_{2ss}}}$$

$$A_2\frac{dh_2}{dt} = \beta\frac{(h_{1(t)}-h_{1ss})}{2\sqrt{h_{1ss}-h_{2ss}}} - \beta\frac{(h_{2(t)}-h_{2ss})}{2\sqrt{h_{1ss}-h_{2ss}}} - \beta\frac{(h_{2(t)}-h_{2ss})}{2\sqrt{h_{2ss}}} \quad (11)$$

Em variável desvio: $x' = x_{(t)} - x_s$

$$A_2\frac{dh_2}{dt} = \beta\frac{h'_{1(t)}}{2\sqrt{h_{1ss}-h_{2ss}}} - \beta\frac{h'_{2(t)}}{2\sqrt{h_{1ss}-h_{2ss}}} - \beta\frac{h'_{2(t)}}{2\sqrt{h_{2ss}}} \quad (12)$$

Aplicando Transformada de Laplace em (12):

$$A_2 s\,\bar{h}_{2(s)} = \beta\frac{\bar{h}_{1(s)}}{2\sqrt{h_{1ss}-h_{2ss}}} - \beta\frac{\bar{h}_{2(s)}}{2\sqrt{h_{1ss}-h_{2ss}}} - \beta\frac{\bar{h}_{2(s)}}{2\sqrt{h_{2ss}}}$$

$$\bar{h}_{2(s)}\left(A_2 s + \frac{\beta}{2\sqrt{h_{1ss}-h_{2ss}}} + \frac{\beta}{2\sqrt{h_{2ss}}}\right) = \beta\frac{\bar{h}_{1(s)}}{2\sqrt{h_{1ss}-h_{2ss}}}$$

$$\bar{h}_{2(s)} = \frac{\dfrac{\sqrt{h_{2ss}}}{\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}}}{\dfrac{2A_2\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\sqrt{h_{2ss}}+\beta\sqrt{h_{1ss}-h_{2ss}}}s+1}\,\bar{h}_{1(s)} \quad (13)$$

Substituindo $\bar{h}_{1(s)}$ em $\bar{h}_{2(s)}$:

$$\bar{h}_{2(s)} = \frac{\dfrac{2\sqrt{h_{2ss}}\sqrt{h_{1ss}-h_{2ss}}}{\beta\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}\bar{F}_{D(s)}}{\left(\dfrac{2A_2\sqrt{h_{2ss}}\sqrt{h_{1ss}-h_{2ss}}}{\beta\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}s+1\right)\cdot\left(\dfrac{2A\sqrt{h_{1ss}-h_{2ss}}}{\beta}\cdot s+1\right)} + \frac{\left(\dfrac{\sqrt{h_{2ss}}}{\sqrt{h_{1ss}-h_{2ss}}+\sqrt{h_{2ss}}}\right)\bar{h}_{2(s)}}{\left(\dfrac{2A_2\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}s+1\right)\cdot\left(\dfrac{2A\sqrt{h_{1ss}-h_{2ss}}}{\beta}s+1\right)}$$

$$\bar{h}_{2(s)} = \frac{\dfrac{\sqrt{h_{2ss}}}{\sqrt{h_{1ss}-h_{2ss}}+\sqrt{h_{2ss}}}}{\dfrac{2A_2\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}s+1}\cdot\left[\frac{\dfrac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta}}{\dfrac{2A\sqrt{h_{1ss}-h_{2ss}}}{\beta}s+1}\bar{F}_{D(s)} + \frac{1}{\dfrac{2A\sqrt{h_{1ss}-h_{2ss}}}{\beta}s+1}\bar{h}_{2(s)}\right] \leftarrow (14)$$

Pois:

$$\bar{h}_{2(s)} = \frac{K_{p2}}{\tau_{p2}s+1}\cdot\left[\frac{K_{p1}}{\tau_{p1}s+1}\bar{F}_{D(s)} + \frac{1}{\tau_{p1}s+1}\bar{h}_{2(s)}\right]$$

$$K_{p1} = \frac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta}$$

$$\tau_{p1} = \frac{2A\sqrt{h_{1ss}-h_{2ss}}}{\beta}$$

$$K_{p2} = \frac{\sqrt{h_{2ss}}}{\sqrt{h_{1ss}-h_{2ss}}+\sqrt{h_{2ss}}}$$

$$\tau_{p2} = \frac{2A_2\sqrt{h_{2ss}}\cdot\sqrt{h_{1ss}-h_{2ss}}}{\beta\left(\sqrt{h_{2ss}}+\sqrt{h_{1ss}-h_{2ss}}\right)}$$