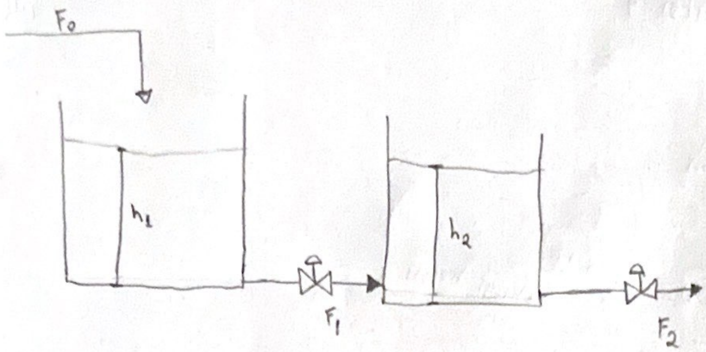
9,0

Questão 2

Esquema:



Em que

$F_1(t) = \beta\sqrt{h_1(t) - h_2(t)}$

$F_2(t) = \beta\sqrt{h_2}$

Hipóteses:

- Densidade é constante
- Área de seção Transversal dos tanques são constantes

Tanque 1

Balanço de massa:

$$\rho_0 F_0 - \rho_1 F_1 = \frac{dM}{dt} = \frac{d\rho A h}{dt} \xrightarrow{\rho \text{ é cte}} F_0 - F_1 = A\frac{dh}{dt}$$

$$F_1(t) = \beta\sqrt{h_1(t) - h_2(t)} \quad \therefore \quad F_0 - \beta\sqrt{h_1(t) - h_2(t)} = A\frac{dh}{dt} \quad (1)$$ EDO não linear :C

Linearização por expansão de Taylor

$$f(h_1; h_2) = \sqrt{h_1(t) - h_2(t)}$$

$$f(x) = f(x_0) + \left(\frac{df}{dx}\right)_{x_0} \frac{x - x_0}{1!}$$

$$\sqrt{h_1(t) h_2(t)} = \sqrt{h_{1ss} - h_{2ss}} + \frac{1}{2\sqrt{h_{1ss} - h_{2ss}}}(h_1(t) - h_{1ss}) + \frac{1}{2\sqrt{h_{1ss} - h_{2ss}}}(h_{2t} - h_{2ss}) \quad (2)$$

Substituindo (2) em (1) → EDO linearizada

$$F_0 - \beta\left(\sqrt{h_{1(ss)} - h_{2(ss)}} + \frac{1}{2\sqrt{h_{1(ss)} - h_{2(ss)}}}(h_1(t) - h_{1(ss)}) + \frac{1}{2\sqrt{h_{1(ss)} - h_{2(ss)}}}(h_{2(t)} - h_{2(ss)})\right) = A\frac{dh}{dt} \quad (3)$$

Aplica o estado estacionário na equação original:

$$F_{ss} - \beta\sqrt{h_{1ss} - h_{2ss}} = 0 \quad (4)$$

Transformando em variável desvio: (subtraindo (4) de (3))

$$F_0 - \beta\left(\sqrt{h_{1ss} - h_{2ss}} + \frac{1}{2\sqrt{h_{1ss} - h_{2ss}}}(h_1(t) - h_{1(ss)}) + \frac{1}{2\sqrt{h_{1(ss)} - h_{2ss}}}(h_2(t) - h_{2ss})\right) - F_{ss} + \beta\sqrt{h_{1ss} - h_{2ss}} = \frac{A\,dh}{dt}$$

$$F_0'(t) - \frac{\beta h_1'(t)}{2\sqrt{h_{1(ss)} - h_{2(ss)}}} + \frac{\beta h_2(t)}{2\sqrt{h_{1(ss)} - h_{2(ss)}}} = \frac{A h_1'}{dt}$$

Aplicando a Transformada de Laplace

$$\bar{F}_0(s) - \frac{\beta\,\bar{h}_1(s)}{2\sqrt{h_1(ss)-h_2(ss)}} + \frac{\beta\,\bar{h}_2(s)}{2\sqrt{h_1(ss)-h_2(ss)}} = A\bar{h}_1(s)\cdot s$$

$$A\bar{h}_1(s) + \frac{\beta\,\bar{h}_1(s)}{2\sqrt{h_1(ss)-h_2(ss)}} = \bar{F}_0(s) + \frac{\beta\,\bar{h}_2(s)}{2\sqrt{h_1(ss)-h_2(ss)}}$$

$$\bar{h}_1(s) = \frac{F_0(s)}{As + \frac{\beta}{2\sqrt{h_1(ss)-h_2(ss)}}} + \frac{\beta\,\bar{h}_2(s)}{2\sqrt{h_1(ss)-h_2(ss)}}\cdot\left(\frac{1}{As + \frac{\beta}{2\sqrt{h_1(ss)-h_2(ss)}}}\right)$$

$$h_1(s) = \frac{\bar{F}_0}{As + \frac{\beta}{2\sqrt{h_{1ss}-h_{2ss}}}} + \frac{\beta\,\bar{h}_2(s)}{As\,2\sqrt{h_1(ss)-h_2(ss)} + \beta\,\frac{2\sqrt{h_{1ss}-h_{2ss}}}{2\sqrt{h_{1ss}-h_{2ss}}}}$$

$$\bar{h}_1(s) = \bar{F}_0(s)\cdot\frac{\frac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta}}{\frac{2As\sqrt{h_{1ss}-h_{2ss}}}{\beta}+1} + \frac{\beta/\beta\cdot\bar{h}_2(s)}{\frac{2As\sqrt{h_{1ss}-h_{2ss}}+1}{\beta}}$$

Encontra-se então a FT para o tanque 1:

$$\bar{h}_1(s) = \bar{F}_0(s)\cdot\frac{\frac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta}}{\frac{2A\sqrt{h_{1ss}-h_{2ss}}}{\beta}\cdot s+1} + \bar{h}_2(s)\cdot\frac{1}{\frac{2A\sqrt{h_{1ss}-h_{2ss}}}{\beta}\,s+1}$$

onde $\frac{2\sqrt{h_{1ss}-h_{2ss}}}{\beta}$ é K_{p1} e $\frac{2A\sqrt{h_{1ss}-h_{2ss}}}{\beta}$ é τ_{p1}

Tanque 2

Balanço de massa:

$$\rho_1 F_1 + \rho_2 F_2 = \frac{d A \rho h_2}{dt} \rightarrow F_1 + F_2 = A \frac{dh_2}{dt}$$

Substituindo F_1 e F_2:

$$\beta \sqrt{h_1(t) - h_2(t)} - \beta \sqrt{h_2(t)} = A \frac{dh_2}{dt} \quad (5)$$ EDO não linear :(

linearizando $\sqrt{h_2(t)}$ por serie de Taylor:

$$\sqrt{h_2(t)} = h_{2(ss)} + \frac{1}{2\sqrt{h_{2ss}}} (h_2(t) - h_{2ss}) \quad (6)$$

Substituindo (6) e (2) em (5):

$$\beta \left(\sqrt{h_{1ss} - h_{2ss}} + \frac{1 (h_1(t) - h_{1ss})}{2\sqrt{h_{1ss} - h_{2ss}}} - \frac{1 (h_2(t) - h_{2ss})}{2\sqrt{h_{1ss} - h_{2ss}}} \right) = A \frac{dh_2}{dt} \quad (7)$$ EDO Linearizada :)

Aplicando estado estacionário em (5):

$$\beta \sqrt{h_{1ss} - h_{2ss}} - \beta \sqrt{h_{2ss}} = 0 \quad (8)$$

Subtraindo (8) de (7):

$$\beta \sqrt{h_{1ss} - h_{2ss}} - \beta \sqrt{h_{1ss} - h_{2ss}} + \frac{\beta h_1(t) - h_{1ss}}{2\sqrt{h_{1ss} - h_{2ss}}} - \frac{\beta h_2(t) - h_{2ss}}{2\sqrt{h_{1ss} - h_{2ss}}} - \beta \sqrt{h_{2ss}} + \beta \sqrt{h_{2ss}} - \frac{\beta h_2(t) - h_{2ss}}{2\sqrt{h_{2ss}}} = A \frac{dh_2}{dt}$$

Passando para variável desvio:

$$\frac{\beta h_1'(t)}{2\sqrt{h_{1ss} - h_{2ss}}} - \frac{\beta h_2(t)}{2\sqrt{h_{1ss} - h_{2ss}}} - \frac{\beta h_2'(t)}{2\sqrt{h_{2ss}}} = \frac{A dh_2'}{dt}$$

aplicando a Transformada de Laplace

$$\frac{\beta \bar{h}_{1s}}{2\sqrt{h_{1ss} - h_{2ss}}} - \frac{\beta \bar{h}_2(s)}{2\sqrt{h_{1ss} - h_{2ss}}} - \frac{\beta \bar{h}_{2s}}{2\sqrt{h_{2ss}}} = A s \bar{h}_{2s}$$

$$A s \bar{h}_{2s} + \frac{\beta \bar{h}_{2s}}{2\sqrt{h_{1ss} - h_{2ss}}} + \frac{\beta \bar{h}_{2s}}{2\sqrt{h_{2ss}}} = \frac{\beta \bar{h}_{1s}}{2\sqrt{h_{1ss} - h_{2ss}}}$$

$$\bar{h}_{2s} \left(A s + \frac{\beta}{2\sqrt{h_{1ss} - h_{2ss}}} + \frac{\beta}{2\sqrt{h_{2ss}}} \right) = \frac{\beta \bar{h}_{1s}}{2\sqrt{h_{1ss} - h_{2ss}}}$$

$$\bar{h}_{2s} \left(A s + \frac{\beta (\sqrt{h_{1ss} - h_{2ss}}) + \sqrt{h_{2ss}}}{2\sqrt{h_{1ss} - h_{2ss}} \cdot \sqrt{h_{2ss}}} \right) = \frac{\beta \bar{h}_{1s}}{2\sqrt{h_{1ss} - h_{2ss}}}$$

$$\bar{h}_{2(s)} = \frac{\beta \bar{h}_{1s}}{2\sqrt{h_{1ss} - h_{2ss}}} \cdot \frac{1}{A s + \frac{\beta (\sqrt{h_{1ss} - h_{2ss}})}{2\sqrt{h_{1ss} - h_{2ss}}}} + \frac{\sqrt{h_{2ss}}}{\sqrt{h_{2ss}}}$$

$$\bar{h}_2(s) = \frac{B\bar{h}_1(s)}{2AS\sqrt{h_{1ss}-h_{2ss}} + \dfrac{B\left(\sqrt{h_{1ss}-h_{2ss}}+\sqrt{h_{2ss}}\right)}{\sqrt{h_{2ss}}}} \quad \text{multiplicando por} \quad \frac{\dfrac{\sqrt{h_{2ss}}}{B\sqrt{h_{1ss}-h_{2ss}}+\sqrt{h_{2ss}}}}{\dfrac{\sqrt{h_{2ss}}}{B\sqrt{h_{1ss}-h_{2ss}}+h_{2ss}}} :$$

$$\bar{h}_{2s} = \frac{\underbrace{\bar{h}_{1s} \cdot \dfrac{\sqrt{h_{2ss}}}{\sqrt{h_{1ss}-h_{2ss}}+\sqrt{h_{2ss}}}}_{K_{p2}}}{\underbrace{\dfrac{2A\sqrt{h_{1ss}-h_{2ss}} \cdot \sqrt{h_{2ss}}}{B\left(\sqrt{h_{1ss}-h_{2ss}}+\sqrt{h_{2ss}}\right)}}_{\tau_{p2}} s + 1}$$

Entendendo o comportamento de F_o em função de h_o:

$$\bar{h}_2(s) = \bar{h}_{1s} \cdot \frac{K_{p2}}{\tau_{p2} \cdot s + 1}$$

$$\bar{h}_2(s) = \left(\bar{F}_{os} \cdot \frac{K_{p1}}{\tau_{p1} \cdot s + 1} + \bar{h}_{2s} \cdot \frac{1}{\tau_{p1} s + 1}\right) \frac{K_{p2}}{\tau_{p2} \cdot s + 1}$$

$$h_2(s) - \left(\frac{\bar{h}_{2s}}{\tau_{p1} s + 1}\right)\left(\frac{\cdot K_{p2}}{\tau_{p2} \cdot s + 1}\right) = \bar{F}_{os} \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)}$$

$$\bar{h}_2(s)\left(1 - \frac{K_{p2}}{(\tau_{p1} s + 1)(\tau_{p2} s + 1)}\right) = \bar{F}_{os} \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1) \cdot (\tau_{p2} \cdot s + 1)}$$

$$\bar{h}_{2s} = \left(\frac{\tau_{p1} \tau_{p2} \cdot s^2 + (\tau_{p1} s + \tau_{p2} \cdot s) + 1 - K_{p2}}{(\tau_{p1} s + 1)(\tau_{p2} \cdot s + 1)}\right) = \bar{F}_{os} \cdot \frac{K_{p1} \cdot K_{p2}}{(\tau_{p1} s + 1)(\tau_{p2} \cdot s + 1)}$$

$$\bar{h}_2(s) = \frac{\bar{F}_{os} \cdot K_{p1} \cdot K_{p2}}{(\tau_{p1} \cdot s + 1)(\tau_{p2} \cdot s + 1)} \cdot \frac{1}{\left(\dfrac{\tau_{p1} \tau_{p2} s^2 + (\tau_{p1} s + \tau_{p2} \cdot s) + 1 - K_{p2}}{(\tau_{p1} s + 1)(\tau_{p2} \cdot s + 1)}\right)}$$

$$\bar{h}_{2s} = \frac{F_{os} \cdot K_{p1} \cdot K_{p2}}{\tau_{p1} \tau_{p2} s^2 + (\tau_{p1} s + \tau_{p2} s) + 1 - K_{p2}}$$

$$\bar{h}_{2s} = \frac{\dfrac{K_{p1} \cdot K_{p2}}{1 - K_{p2}}}{\dfrac{\tau_{p1} \tau_{p2} s^2}{1 - K_{p2}} + \dfrac{(\tau_{p1} + \tau_{p2}) s}{1 - K_{p2}} + 1} \cdot \bar{F}_{os} \quad //$$

Questão ①

TG - 6

2,0/3,0

$$\frac{A}{B} = \exp\left(\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}\right) \Rightarrow \frac{3,5}{10,2} = 0,343$$

$A = 13,7 - 10,2 = 3,5$

$\cancel{B = 10,2}$

$$\ln\frac{A}{B} = \frac{-\pi \zeta}{\sqrt{1-\zeta^2}} \Rightarrow \ln(0,343) = \frac{-\pi \zeta}{\sqrt{1-\zeta^2}}$$

$$-1,070^2 \cdot \sqrt{1-\zeta^2}^{\,2} = (-\pi \cdot \zeta)^2$$

$$-1,1441 + 1,1441\zeta^2 = -\pi^2 \cdot \zeta^2$$

$$\frac{-1,1441 + 1,1441\zeta^2}{\zeta^2} = -\pi^2$$

$$-\frac{1,1441}{\zeta^2} + 1,1441 + \pi^2 = 0$$

$$1,1441 + \pi^2 = \frac{1,1441}{\zeta^2}$$

$$\frac{1}{1,1441 + \pi^2} = \frac{\zeta^2}{1,1441}$$

$$\sqrt{\frac{1,1441}{1,1441+\pi^2}} = \zeta^2$$

$$\sqrt{\frac{1,1441}{1,1441+\pi^2}} = \zeta$$

$\boxed{\zeta = 0,3183}$

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$T = 3{,}5$

$T = \dfrac{2\pi}{\omega} \Rightarrow 3{,}5 = \dfrac{2\pi}{\omega}$

$\omega = 1{,}7952$

$\omega = \dfrac{\sqrt{1-\xi^2}}{\tau_p}$

$1{,}7852 = \dfrac{\sqrt{1-(0{,}3183)^2}}{\tau_p}$

$\tau_p = 0{,}5281$

$K_p = \dfrac{\Delta y'}{\Delta c'} = \dfrac{10{,}2 - 2{,}5}{24-15}$

$K_p = \dfrac{\Delta y'}{\Delta c'} = \dfrac{10{,}2 - 2{,}5}{24-15}$

$K_p = \dfrac{7{,}7}{9}$ $\quad K_p = 0{,}856$

$\dfrac{\overline{R_m(s)}}{\overline{P}(s)} = \dfrac{0{,}856}{0{,}5281^2 s^2 + 2 \cdot 0{,}1681 s + 1} =$

$\dfrac{\overline{R_m(s)}}{\overline{P}(s)} = \dfrac{0{,}856}{0{,}53^2 s^2 + 0{,}336 s + 1}$