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Acquiring Information through Peers†

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We develop an endogenous network formation model, in which agents form connections to acquire information. Our model features complementarity in actions as agents care not only about accuracy of their decision-making but also about the actions of other agents. In equilibrium, the information structure is a hierarchical network, and, under weakly convex cost of forming links, the equilibrium network is core-periphery. Although agents are ex ante identical, there is ex post heterogeneity in payoffs and actions. (JEL D83, D85, Z13)

Social networks permeate economic environments. A pervasive feature of networks in economics, and social sciences in general, is its core-periphery structure, a small group of densely connected agents (core) linked to a vast majority of sparsely connected agents (periphery). Core-periphery networks have been documented in distinct economic environments, from voting patterns in the US Senate, to information acquisition patterns more broadly.[1] There are two key distinctions between networks in social sciences and networks in other fields like biology or physics (e.g., a network of nervous systems or genetic networks). First, social networks are endogenous. They are a direct consequence of our choices when building relationships with one another. Second, in social networks, linkages transmit information, shaping individual decisions. For instance, congress members communicate with each other to align their votes and, in doing so, they learn about the policies in question. To formulate their own forecasting model, financial analysts choose whose models to learn from, endogenously forming a network of information transmission.

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[1] See Borgatti and Everett (2000); Galeotti and Goyal (2010); Rombach et al. (2017); Hollifield, Neklyudov, and Spatt (2017), and references therein.

We study how networks are formed and why core-periphery structures are a consistent pattern in different economic environments. To this end, we develop an endogenous network formation model in which agents acquire information through peers. In our model, agents make two decisions. First, they form their social connections, and, second, they choose an action. Each agent receives a signal about the state of the world. In addition, agents can, at a cost, form social connections to observe the signals received by other agents.

Our main result is that strategic complementarity leads to core-periphery networks as an equilibrium outcome. Specifically, when agents care about the correct choice in terms of accuracy and about coordinating their actions, we show that any strict equilibrium information structure is a *hierarchical* network. More importantly, if the marginal cost of forming connections is weakly increasing, then any strict equilibrium information structure is *core-periphery*.

A hierarchical directed network is characterized by a ladder of informational importance. Agents are sorted into tiers, and those in the top are quite influential because they have their signal observed by all other agents. Signals from agents in the second tier are observed by all members of the tiers below, and so on. This information structure determines a hierarchy of signals' importance and replicates three key features of social hierarchies.[2] First, although agents are ex ante identical and receive equally precise signals, in equilibrium, they endogenously separate themselves into tiers. Agents in the same tier exert the same influence and acquire the same information, but agents in different tiers have distinct influences over the economy. Second, the information hierarchy is self-enforcing: as more agents observe an agent's signal, the signal's influence increases, and so does the incentive to observe it. Third, an agent in a higher tier has higher equilibrium payoffs.

A core-periphery network is a particular hierarchical directed network. While a hierarchical network allows the existence of multiple social levels, a core-periphery network divides agents in two groups: the core and the periphery. Members of the periphery observe only the signals of agents in the core, and core agents observe each other's signals. As a result, the signals of core agents endogenously become public information and are common knowledge to all agents.

Formally, our model is a two-stage game. There is an unknown state of the world, and each agent receives an equally informative signal about it. In the first stage of the game, agents can observe, at a cost, any other agent's signal. The agents' interconnections implied by the decisions of observing each other's signals constitute a directed network which we define as the economy's information structure. We make two assumptions on how information propagates along the network. First, connections are not reciprocal. The benefit of connecting to an agent is that if agent i connects to agent j, then agent i observes agent j's signal, but agent j does not observe agent i's signal. Second, there is no retransmission of information. The information acquired by an agent through the network is limited to her immediate connections.

[2] See Magee and Galinsky (2008) for a definition and examples of social hierarchy. The equilibrium hierarchy of influence matches several empirical findings on information propagation, including the law of the few and how information propagates in social media. For instance, see Galeotti and Goyal (2010); Kumar, Novak, and Tomkins (2010); Goel, Watts, and Goldstein (2012); Bakshy et al. (2011).

In the second stage, each agent simultaneously chooses an action, using the information obtained through the network. An agent wants to choose an action close both to the unknown state of the world and to the average action of the economy. This beauty contest element of the payoff generates complementarity and has two immediate effects on the incentives in the network formation game. First, information is not perfectly substitutable. All agents' signals have the same precision, but some agents' signals are more informative about the average action, depending on the agent's position in the network. Second, there is strategic complementarity between agents' actions, which implies that agents rely more on signals from more influential peers.[3] The influence of an agent's signal, how correlated it is with the average action, is endogenous and measures the agent's centrality and importance in the network. Such influence goes beyond her immediate connections, even though our model does not feature retransmission of information through an agent's connections. Formally, the influence of a particular agent's signal in equilibrium is expressed recursively. It depends not only on how many agents observe her signal but also on the influence of other agents.

To characterize the network structure common to all equilibria, we define two key monotonicity properties that drive the information acquisition process. The first states that an agent would rather observe a signal that more agents observe. An individual's endogenous status at the top of the hierarchy makes her signal more observed and, thus, more influential and informative about the average action of the economy. The second monotonicity property states that such an agent has less incentives to observe signals. The intuition is that her own signal is more informative about the average action, disincentivizing the acquisition of additional information. We show that any strict Nash equilibrium of the game satisfies both properties.

An important implication of these two monotonicity properties is that every strict Nash equilibrium of this game produces a directed hierarchical network. Our analysis is more general than the information acquisition framework because our main result relies on these two monotonicity properties rather than on the specifics of our information acquisition game. Our main theorem shows that any network that satisfies both properties above is a hierarchical directed network. As a corollary, all strict Nash equilibria of our game produce a hierarchical directed network as the information structure.

We show that, if the marginal cost of observing a signal is weakly increasing in the number of observed signals, then any strict Nash equilibrium features a core-periphery network. We define a third property stating that, if an agent observes the signal of two other agents, then these two agents observe each other's signals as well. We show that, in our model, if the marginal cost of acquiring information is increasing, then any strict equilibrium satisfies this third property. Our main result (Theorem 2) shows that any network that satisfies the three properties is a core-periphery network. As a corollary, if the marginal cost of observing a signal is weakly increasing, then any strict Nash equilibrium of this game is a core-periphery network.

[3] See Bulow, Geanakoplos, and Klemperer (1985) for a definition of strategic complementarity.

The rest of the paper is organized as follows. In the next two subsections, we provide examples of information acquisition through peers and discuss our contribution to the literature. In Section I, we present and solve the model. In Section II, we characterize hierarchical and core-periphery networks and show that any equilibrium information structure has these architectures. Finally, Section III concludes.

A. *Examples*

Several real-world examples highlight the importance of acquiring information through peers in settings where agents would like to coordinate their actions. In this subsection, we describe a couple of examples in more detail.

First, we consider *sell-side analysts* forecasting an economic outcome such as earnings, GDP, inflation, unemployment, etc. Analysts would like to provide an accurate forecast, but they also have career concerns and would like to conform to the consensus in their evaluations.[4] Analysts' private signal about the state of the economy is an outcome of their own forecasting model, however they can also learn and replicate the forecasting models of other analysts. For instance, some analysts may have a more quantitative approach while others may base their forecasts on other methods. By forming connections with one another, analysts learn each others' models and approach to forecasting, effectively acquiring each others' signal about the state of the economy. Aligned with our model, analysts choose to acquire information from each other, endogenously forming an information structure.

As a second example, we consider *political party members*. A political party activist would like to support the best policy, while balancing the need for the party to display unity. Partisans have access to different sources of information about policies' impact. Although a partisan may prefer a particular source, she would rather focus on the same sources as other party members in order to coordinate and display unity. To map this example to our model, a partisan's action is her support to a particular policy and a partisan's signal is the information from her preferred source. Thus, an equilibrium information structure specifies which sources each partisan decides to follow.

The information acquisition process is slightly different in these examples. In the first, analysts learn each others' forecasting models, literally acquiring information produced by other analysts. In the second, partisans acquire signals from information sources other than their preferred ones, because other partisans are acquiring them as well. In both interpretations, an agent observes a signal to obtain information regarding what her peers know. Although these are different examples of learning from peers, our framework is insightful to study their equilibrium information structures.

B. *Related Literature*

We study information acquisition from peers in a beauty contest setting.[5] A central contribution of our paper is to bridge distinct-yet-related literatures. On the

[4] The complementarity here is due to career concerns, as documented in Hong and Kubik (2003).

[5] Following Morris and Shin (2002), beauty contest has been applied to a variety of settings, for instance, Angeletos and Pavan (2004) regarding investment games; Dewan and Myatt (2008, 2012) regarding political

one hand, we add to the literature on information acquisition with strategic complementarity. This has been recently explored by Myatt and Wallace (2012, 2018) and Colombo, Femminis, and Pavan (2016). This literature considers information acquisition from sources that are outside of the economy. Closer to our paper, Hellwig and Veldkamp (2009) shows that complementarity in actions generates complementarity in information acquisition. By introducing information acquisition from peers, we highlight a new force present in information acquisition decisions: as the signal of an agent is more public, she has less incentive to obtain information, as her own signal is more informative regarding the average action. On the other hand, we add to the literature on network formation and information acquisition. Closer to our work, Galeotti and Goyal (2010) models information acquisition through social networks to explain the "law of the few," an empirical observation that individuals acquire information from a small subset of their social contacts. In their setting, information is acquired by the agents, and is considered a public good. When an agent links to another, she gains access to his information. We contribute to this literature by micro-founding the usage of information and by introducing strategic complementarities in actions. While complementarities make information sources non-substitutable, even though all agents' signals are equally informative about the state of the world, the micro-foundation disciplines the endogenous network formation.

To study information acquisition from peers, we develop a network formation model in a beauty contest setting. There is a large literature on network formation, starting from the seminal work by Jackson and Wolinsky (1996) and Bala and Goyal (2000), with different interpretations for forming a link.[6] We follow Bala and Goyal (2000) in modeling link formation as a one side individual decision, and thus we can rely on standard methods of noncooperative game theory. More recently, a large literature focuses on endogenous networks in games with local externalities, following the work of Ballester, Calvó-Armengol, and Zenou (2006) and Bramoullé and Kranton (2007). For instance, recent work include Cabrales, Calvó-Armengol, and Zenou (2011); Baetz (2015); Hiller (2017); and Kinateder and Merlino (2017, 2019). However, there are substantial differences in the models, and more importantly, in the economic forces behind the results between our work and this literature.

Let us first focus on how our model contributes to the literature on endogenous networks in games with local externalities. The literature extends the framework of interdependent linear quadratic utility functions introduced in Ballester, Calvó-Armengol, and Zenou (2006) by endogenizing the network of payoffs' interdependence. In those models, agents simultaneously choose a level of costly effort, with interdependent linear best responses. If two players are connected, their efforts impact each others' payoff according to a bilateral influence matrix, affecting the individual marginal benefit of effort. Thus, bilateral influences generate local externalities in their payoffs. If the network is endogenous, i.e., players choose their

leadership; Allen, Morris, and Shin (2006) regarding financial markets; Pavan (2016) regarding limited attention; and Hellwig (2005) and Myatt and Wallace (2015) regarding monopolistic and Cournot competition.

[6] Recent work includes Borgatti and Everett (2000); Calvó-Armengol and Zenou (2004); Goyal and Vega-Redondo (2005); Hojman and Szeidl (2008); and König, Tessone, and Zenou (2014). See Bloch and Jackson (2006), Jackson (2010), and Hellmann and Staudigl (2014) for reviews of the literature.

connections and effort simultaneously, then a player would like to connect to the players whose effort affects more her payoff, given the bilateral influence matrix. If the effects are homogeneous and positive (for instance, Baetz 2015) then a player wants to connect to players choosing higher efforts. In our model, there are local externalities, but not directly through actions. We consider a beauty contest game, in which a player's choice of action affects all other players equally, independently of whether they are connected. Thus, any local externality in our model is a result of the information acquisition decision and of the equilibrium decision of how to use that information. As an example, consider player i connected to player j, then i observe j's signal and, consequently, i's action will be influenced by j's signal. The local externality generated by i's connection relies on the equilibrium behavior of agent i. By being influenced by j's signal, agent i's action can be forecast by it, and as a result, all other players change their connectivity and forecast decisions.

Our paper also contributes to the literature on endogenous network formation by highlighting a distinct economic mechanism for why nested-split graphs may occur in equilibrium. In our setting, two equilibrium properties drive our main result of hierarchical networks as an equilibrium outcome (Properties 1 and 2). Property 1, in which agents form connections with the most connected players, precisely captures the definition of directed nested-split graphs. Similar to Hellwig and Veldkamp (2009), complementarities in actions generate complementarities in information acquisition in our model, because information is acquired from peers. This channel is captured by Property 1. The difference between Property 1 and the forces behind nested-split-graphs in the literature highlight an important contribution of our model. In our framework, a player cannot affect how attractive her signal is to other players. A player's signal is attractive if other players choose to observe it and use it when making their decisions. Due to strategic complementarity, a player that uses the signal of player j to make her decision will correlate her action with the signal of player j. Thus, if the signal of player j is observed by more agents, then it becomes more correlated with the average action than a less observed signal. This is a direct result of strategic complementarities in actions, even though actions have no local externality effects in our model. In contrast, papers about network formation typically obtain nested-split graphs by having core players attracting peripheral ones through higher effort choices. For example, in a setting of public goods, core players further invest in the public good to attract other agents (e.g., Galeotti and Goyal 2010), while in a setting of strategic complementarity, a high-tier player in a nested-split graph chooses higher costly effort to attract others to form connections with her (e.g., Baetz 2015).

While the emergence of equilibrium networks with nested layers of connections is common to our work and Galeotti and Goyal (2010), Baetz (2015), Hiller (2017), and Kinateder and Merlino (2017), Property 2 in our paper advances further the characterization of equilibrium networks.[7] Property 2 highlights that players

[7] Nested split graphs are also obtained in König, Tessone, and Zenou (2014), however network formation is not fully a strategic decision. Also, Belhaj, Bervoets, and Deroïan (2016) and Dessein, Galeotti, and Santos (2016) show that nested split graphs may be efficient in a local externalities framework and in an organizational economics setting, respectively.

may free ride on other agents' information acquisition and link-formation choices. Specifically, as more players connect to agent j, agent j has less incentive to acquire further information or to form additional links. There are two important implications of Property 2 that contributes to this literature. First, Property 2 defines who bears the cost of forming a connection. One of the equilibrium predictions of our paper is that, consistent with social hierarchies, players in the bottom layers bear the cost and connect with players in the top of the social hierarchy. In contrast, in Baetz (2015) for instance, who bears the cost of a link is defined by assumption. Second, Property 2 further limits the set of networks structures that arise in equilibrium, from the class of directed nested-split graphs to a more narrowly defined set of strict hierarchical networks (Theorem 1). Finally, our Property 3 further restricts the set of equilibrium networks to only core-periphery networks (Theorem 2). In our setting, core-periphery networks are hierarchical networks with at most two tiers of players, in which all players observe the core-players' signals. Core-periphery networks discipline the information acquisition of different players using the diminishing marginal benefit of additional information, an endogenous feature of our beauty contest model.

Finally, our theoretical predictions are consistent with several empirical findings as well. In finance, Hollifield, Neklyudov, and Spatt (2017) and Li and Schürhoff (2019) document core-periphery networks in over-the-counter markets.[8] More generally, hierarchical networks have been documented in a wide array of information acquisition situations, from product brands and fashion changes to US Senate voting decisions.[9] Focusing on online information diffusion, Kumar, Novak, and Tomkins (2010) documents the structure of Flicker and Yahoo!360°, social networks with more than five million users, and observes the existence of multiple connected components, each organized roughly as a core-periphery structure. Along similar lines, Goel, Watts, and Goldstein (2012) documents how information and behavior spread online through multiple online platforms, such as Twitter News and Videos. Instead of a viral diffusion process, long sequences of links of contagion, triggering large cascades, diffusion of information is characterized by almost all adoption (94 percent to 99 percent) happening within one degree of a seed node.

I. Model

We consider an economy populated by a finite set of agents, $N = \{1, 2, \ldots, n\}$, who first choose whom to acquire information from and then choose an action. Each agent wants to choose an action close to both the true state of the economy and the average action. Next, we describe the information structure of the game and agents' payoffs.

[8] In finance, a recent literature has focused on modeling the emergence of core-periphery networks as a result of specific elements of the financial setting. For instance, Farboodi (2017) focuses on financial intermediation, Oberfield (2018) on input-output, Sambalaibat (2018) on over-the-counter markets, Akerlof and Holden (2016) on networks of investors, and Babus (2016) on interbank networks. Recent work on endogenous network formation and finance also includes Golub and Livne (2010), Erol and Vohra (2017), Erol and Lee (2018), and Sambalaibat (2018).

[9] See, for instance Galeotti and Goyal (2010) for stylized facts and a review of the literature, or Rombach et al. (2017) for a method to identify core-periphery networks and many examples of its use, in particular the voting pattern of the US Senate.

Information.—There exists a normally distributed unknown state of the economy, $\theta \sim \mathcal{N}(0,1)$. Each agent i observes one costless signal, e_i, of θ:

$$e_i = \theta + \sigma\varepsilon_i,$$

where $\varepsilon_i \overset{iid}{\sim} \mathcal{N}(0,1)$ and each ε_i is independent from θ. Besides observing her signal and the common prior, an agent i can observe, at a cost, the signal of other agents.

Agents' interconnections implied by the decisions of observing each other's signals constitute a directed network, which we define as the economy's information structure. An information structure $G = \{g_{i,j}\}_{i,j}$ is a list of ordered pairs of agents, such that if agent i observes the signal of agent j, then the pair is in the list: $g_{i,j} = 1$. Otherwise, $g_{i,j} = 0$. The links carry no intensity attachments and are nonreciprocal, in the sense that an agent observing another agent's signal does not imply the symmetric relation.

The benefit of connecting to an agent is that if i connects to j, agent i is able to observe j's signal, even if j does not observe i's signal. In addition, there is no retransmission of information: by connecting to j, agent i does not observe any other signal that j has observed, unless i observes those signals as well. Making connections is costly, and the linking party alone bears the cost.

The information set of agent i is composed of the common prior, her own signal, and the signals she has chosen to observe, which can be described by $\mathbf{I}_i = \{e_j \text{ with } j = 0,1,\ldots,i,\ldots,n, \text{ such that } g_{ij} = 1\}$. To simplify notation, we define $e_0 = 0$ as the mean of the common prior. In addition, since all agents observe their own signal and share the common prior, we consider that $g_{i,i} = g_{i,0} = 1$, for every agent $i = 1,\ldots,n$.

Figure 1 presents examples of information structures and their respective information sets. In panel A, we present a wheel network with four agents. Each agent observes one additional signal, obtaining three signals: her own, the common prior, and the additional signal being observed. Furthermore, each agent has her signal observed by only one other agent, forming a wheel. In panel B, we present a star network. Agents focus only on the first agent, as all observe the signal of agent 1. The first agent does not observe any additional signal. The first agent's information set is composed only of her own signal and the common prior, while another agent's information set is composed of his signal, the common prior and the first agent's signal.

Payoffs.—Once agent i learns from the signals she has observed, she chooses an action, a_i, to maximize her expected payoff given other agents' actions, $a_{-i} = \{a_j\}_{j=1,j\neq i}^{n}$. Agent i would like to choose an action as close as possible to the bliss action, a_i^*, which is a convex combination of the true state of the world and the average action, excluding agent i. Agent i's payoff is given by

$$\Pi(a_i, a_{-i}) = -(a_i - a_i^*)^2 - C(\mathcal{K}_i), \tag{1}$$

where $a_i^* = (1-r)\theta + r\bar{a}_{-i}$ is the bliss action, $\bar{a}_{-i} = (1/(n-1))\sum_{j=1,j\neq i}^{n} a_j$ is the average action excluding agent i's own action, $\mathcal{K}_i = \sum_{j=1,j\neq i}^{n} g_{ij}$ is the number

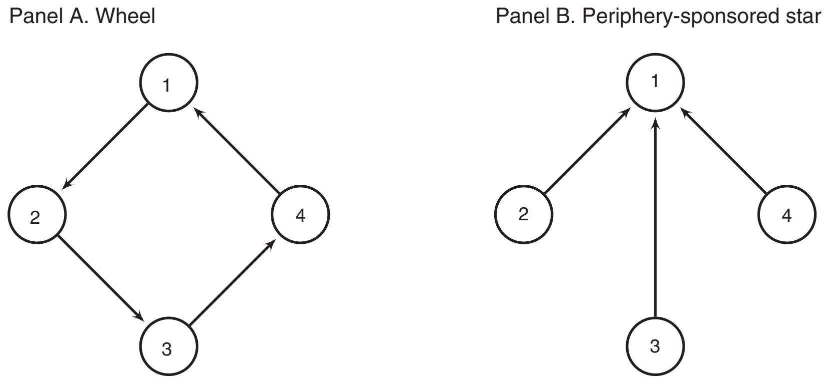

Figure 1. Examples of Networks and Implied Information Sets

Notes: In panel A, agent 1 observes agent 2's signal; thus, 1's information set is $\mathbf{I}_1 = \{e_0, e_1, e_2\}$, while for agent 4, it is $\mathbf{I}_4 = \{e_0, e_1, e_4\}$. In panel B, agent 4 observes agent 1's signal; thus, 4's information set is $\mathbf{I}_4 = \{e_0, e_1, e_4\}$, while for agent 1, it is $\mathbf{I}_1 = \{e_0, e_1\}$.

of connection formed by agent i, and $C(\cdot)$ is a link formation cost function.[10] If an agent knew both the true state and the average action, she would choose a linear combination of both. The parameter $r \in (0,1)$ captures conformity: how much an agent cares about the average action. The agent's payoff is a function of the distance between her action and the bliss action.

An agent bears a cost of acquiring information, represented by $C(\mathcal{K}_i)$, where $\mathcal{K}_i \in \{0,1,\ldots,n-1\}$ is the number of signals that agent i observes in addition to her own signal and the common prior. Throughout the paper, we assume that the cost function is increasing (Assumption 1) and, in Section IIB, we discuss how increasing marginal cost (Assumption 2) affects the equilibrium information structure of the economy.

ASSUMPTION 1: *Observing an additional agent's signal is costly:*

$$C(\mathcal{K}_i + 1) - C(\mathcal{K}_i) > 0.$$

ASSUMPTION 2: *Obtaining information has weakly increasing marginal cost:*

$$C(\mathcal{K}_i + 2) - C(\mathcal{K}_i + 1) \geq C(\mathcal{K}_i + 1) - C(\mathcal{K}_i).$$

As a solution concept, Definition 1 formalizes pure strategy Nash equilibria in our framework.

DEFINITION 1 (Equilibrium): *A pure strategy Nash equilibrium consists of n connection decisions, $g_i \in \{0,1\}^n$ for all $i \in N$, and an action rule functional, $a(g_i, g_{-i}) : \{0,1\}^{n\times n} \to \mathcal{F}$, where $\mathcal{F}$ is a set of functions whose domain is the set*

[10] Alternatively, we could have defined $a_i^* = (1-\tilde{r})\theta + \tilde{r}\bar{a}$, where $\bar{a} = (1/n)\sum_{j=1}^{n} a_j$. This leads to the same optimal action, by setting $\tilde{r} = rn/(r+n-1)$.

of possible signals' realization and codomain is the set of possible actions. The function $a_i = a(\mathbf{I}_i) = a(g_i, g_{-i})$ *specifies agent i's actions as a function of her observed signals' realizations. Given other agents' connection decisions,* g_{-i}, *and action rules,* $\{a_j\}_{j \neq i}$, *agent i's connections,* g_i, *and action rule,* a_i, *maximize agent i's expected payoff. An equilibrium is strict, as usual, if agent i's choices strictly maximize her payoff.*

We focus on Nash equilibria, agent i chooses her connections g_i and action rule a_i to maximize agent i's expected payoff, taking as given other agents' connections and action rules. Even though agents do not observe each others' signal selection, in equilibrium agents conjecture the correct network structure when choosing their actions. Following the literature,[11] our equilibrium concept imposes that agents actions are a function of their information set, i.e., $a_i = a(\mathbf{I}_i)$. This implies a symmetric action choice, as any two agents with identical information sets will necessarily take the same action. However, as we discuss at length in Section II, asymmetric equilibria can arise because agents can choose to acquire different signals. As a result, agents can have heterogeneous information sets and take distinct actions in equilibrium.

A. *Action Choice*

This game can be represented as a two-stage game. In the first stage, agents acquire information about the state of the world by observing other agents' signals, thus forming an information structure. In the final stage, each agent chooses an action, given her implied information set. In this section, we discuss the solution of the second stage. Online Appendix Section A has the detailed derivations.

The agent chooses an action in order to maximize her expected payoff. An agent does not know either the true state of the world or the average action, but uses the information available to her to choose an optimal action that solves

$$\max_{a_i} -E\left[(a_i - a_i^*)^2 + C(\mathcal{K}_i) \,|\, \mathbf{I}_i\right].$$

The first-order condition leads to the following optimal action:

$$a_i = E\left[a_i^* \,|\, \mathbf{I}_i\right] = (1-r)E\left[\theta \,|\, \mathbf{I}_i\right] + rE\left[\bar{a}_{-i} \,|\, \mathbf{I}_i\right].$$

An agent's optimal action is a linear combination of the best predictor of the true state and the best predictor of the average action. The weight given to each is determined by the parameter r, which captures how much the agent conforms to the average action.

The following proposition shows that in any equilibrium, given the resulting information structure G, agents' actions will be a unique linear combination of the realized signals they observe.[12]

[11] For example, see Hellwig and Veldkamp (2009).

[12] The restriction to linear equilibrium is a standard assumption in the literature using the normal-quadratic approach. For instance, see Angeletos and Pavan (2007), Calvò-Armengol, de Martí, and Prat (2015), and Dewan

PROPOSITION 1: *In any equilibrium with a resulting information structure G, the action rule function $a(\mathbf{I}_i)$ is unique and linear in the realized signals.*

The proof of the proposition, presented in online Appendix Section A.1, consists of guessing and verifying a linear equilibrium, and showing that it is unique. The proposition implies that an action is a linear combination of the signals in the economy, with the obvious restriction that a signal not observed must be assigned to a coefficient of zero. Formally,

$$a_i = \sum_{j=0}^{n} \lambda_{ij} e_j, \tag{2}$$

where λ_{ij} are coefficients determined in equilibrium. The coefficient λ_{ij} represents the relative influence of agent j's signal on the action of agent i. In equilibrium, $\lambda_{ij} = 0$ whenever agent i does not observe signal j.

As result, the average action of the economy is also a linear combination of the signals:

$$\bar{a} = \frac{1}{n}\sum_{i=1}^{n} a_i = \sum_{j=0}^{n} \beta_j e_j, \tag{3}$$

where $\beta_j = (1/n)\sum_{k=1}^{n} \lambda_{kj}$ is a coefficient determined in equilibrium and it represents the relative influence of agent j's signal on the average action $\bar{a}$. Furthermore, if agent i's action and the average action are linear combinations of signals, it must be that the average action of all other agents, excluding agent i, is also a linear combination of the signals:

$$\bar{a}_{-i} = \frac{1}{n-1}\sum_{s=1, s\neq i}^{n} a_s = \sum_{j=0}^{n} \beta_{-i,j} e_j, \tag{4}$$

where $\beta_{-i,j} = \big(1/(n-1)\big)\sum_{k\neq i} \lambda_{kj}$ is also determined in equilibrium and measures the relative influence of agent j's signal on the average action excluding agent i's action $(\bar{a}_{-i})$. It is important to highlight that $\beta_{-i,j}$ depends exclusively on the action of agents other than i and therefore agent i's actions have no direct impact on $\beta_{-i,j}$. We show in online Appendix Section A.1 that the linear coefficients sum to 1:

$$\sum_{j=0}^{n} \lambda_{ij} = 1, \forall i, \quad \sum_{j=0}^{n} \beta_j = 1, \quad \text{and} \quad \sum_{j=0}^{n} \beta_{-i,j} = 1. \tag{5}$$

Agent j's coefficient β_j is a network centrality measure. It indicates the influence of agent j's signal on the average action of the economy and it is given by the following recursive formulation:

$$\begin{aligned} &\left[n - \tilde{r}\left(\bar{\mathcal{K}}_j + 1\right)\right] \beta_j \\ &\quad = (1-\tilde{r})\sum_{i=1}^{n} \frac{g_{ij}}{\sigma^2 + \mathcal{K}_i + 1} + \tilde{r}\sum_{i=1}^{n} \frac{g_{ij}}{\sigma^2 + \mathcal{K}_i + 1}\left[\sum_{s=0}^{n} \beta_s (1 - g_{is})\right], \end{aligned} \tag{6}$$

and Myatt (2008). We, however, follow Hellwig and Veldkamp (2009) and show that only the unique linear action function is an equilibrium.

where $\tilde{r} = rn/(r+n-1)$, $\bar{\mathcal{K}}_j = \sum_{s=1,s\neq j}^{n} g_{sj}$ and $\mathcal{K}_i = \sum_{s=1,s\neq i}^{n} g_{is}$. The equivalent expression for the influence of the common prior, namely β_0, is given by[13]

$$(7) \quad n(1-\tilde{r})\beta_0 = (1-\tilde{r})\sum_{i=1}^{n}\frac{\sigma^2}{\sigma^2+\mathcal{K}_i+1} + \tilde{r}\sum_{i=1}^{n}\frac{\sigma^2}{\sigma^2+\mathcal{K}_i+1}\left[\sum_{s=0}^{n}\beta_s(1-g_{is})\right].$$

The influence of agent j's signal on the average action is a measure of her centrality in the network.[14] Although our model does not feature retransmission of information through an agent's connections, an agent's signal influence goes beyond her immediate connections. Influence is expressed as a recursive formula, depending on the influence of every agent. Formally, influence of agent j's signal is a linear combination of two terms, weighted by the conformity parameter, r. The first term on the right-hand side of equation (6) is the number of agents observing agent j's signal, weighted by how many signals each one of them observes. This represents the influence of agent j's signal on the posterior distribution of the true state of the economy. If more agents observe agent i's signal, then it becomes more influential as more agents use that signal to forecast the true state. The following lemma solves for influence coefficients under two limiting cases, one without coordination motives as r approaches 0 from above (i.e., $r \to 0^+$) and another with strong complementarities as r approaches 1 from below (i.e., $r \to 1^-$). These two extreme case have distinct implications for the equilibrium influence coefficients.

LEMMA 1: *For a given network structure, the influence coefficients β specified in equations (6) and (7) feature the following limiting results:*

$$(a) \qquad \lim_{r\to 0^+}\beta_j = \begin{cases} \frac{1}{n}\sum_{i=1}^{n}\frac{\sigma^2}{\sigma^2+\mathcal{K}_i+1} & \text{if } j = 0 \\ \frac{1}{n}\sum_{i=1}^{n} g_{ij}\frac{1}{\sigma^2+\mathcal{K}_i+1} & \text{if } j \geq 1, \end{cases}$$

and

$$(b) \qquad \lim_{r\to 1^-}\beta_j = \begin{cases} \frac{\sigma^2}{\sigma^2+\mathcal{K}_{PU}} & \text{if } j = 0 \\ \frac{1}{\sigma^2+\mathcal{K}_{PU}} & \text{if } j \geq 1, j \in PU \\ 0 & \text{if } j \geq 1, j \notin PU, \end{cases}$$

where $PU = \{j : j \geq 1, g_{ij} = 1, \forall i = 1,\ldots,n\}$ is the set of signals observed by every agent, i.e., public signals, and $\mathcal{K}_{PU}$ is the number of elements in PU.

The proof of Lemma 1 is presented in online Appendix Section A.2. The coefficients $\{\beta_i\}_{i=0}^{n}$ depend on the entire network structure according to equations (6) and (7), however Lemma 1 shows closed-formed solution for these coefficients in the limit as r approaches 0 and 1. For the case of r approaching 0, agents do not

[13] Equations (6) and (7) are derived in online Appendix Section A.1.

[14] Our measure relates more closely to the degree centrality and to eigenvector centrality, depending on the level of complementarity in the economy. For more details, Bloch, Jackson, and Tebaldi (2017) provides an excellent analysis on network centrality measures.

care for the average action (i.e., $r \to 0^+$ and thus $\tilde{r} \to 0^+$). In this case, the influence of agent j's signal becomes entirely driven by how agents rely on j's signal when updating their forecast of the state of the world according to Bayes' rule. This is captured by the first term on the right-hand side of equation (6).

When r approaches 1, agents care only about the average action, and the influence of a signal depends primarily on how public that signal is. Lemma 1 shows that, if a signal is not observed by everyone in the economy, i.e., there is at least one agent not observing it $(j \geq 1, j \notin PU)$, then its influence converges to 0. However, public signals' influence converge to a constant. Hence, if agents care more about the average action and less about state of the economy, then (in the limit) agents only consider signals that are known to everyone. Effectively, as r goes to 1 and coordination becomes the main driver of information acquisition, agents perfectly coordinate on the public information available.

B. *Link-Formation Incentives*

We now consider agents' information acquisition decisions. When an agent decides which connections to form, she has not observed any signal yet and the only information she has about the state of the world is the common prior. The first step is to compute the agent's ex ante expected payoff as a function of other agents' connections and their resulting optimal actions. Let $g_i = \left(g_{ij}\right)_{j=0}^{n}$ be agent i's connections and g_{-i} be all the other connection in the network, then, given other agents' connections, agent i's payoff from forming connections g_i is given by

$$E\left[U_i(g_i, g_{-i}) \,|\, g_{-i}\right] = -E\left[\left(E[a_i^* \,|\, \mathbf{I}_i] - a_i^*\right)^2 \,|\, g_{-i}\right] - C(\mathcal{K}_i),$$

where $U_i(g_i, g_{-i})$ is the payoff from the second stage assuming all other agents are optimizing their actions taking the network as given.

The following proposition characterizes the expected payoff of an agent as a function of other agents' link-formation decisions.

PROPOSITION 2: *The ex ante expected payoff of agent i choosing a set of signals to observe can be written as a function of her own information acquisition choices, g_i, and the influence that agents have over the average action not including agent i's action:*

$$E\left[U_i(g_i, g_{-i})\right] = -\frac{\sigma^2}{\sigma^2 + \mathcal{K}_i + 1}\left(1 - r\sum_{j=0}^{n} g_{ij}\beta_{-i,j}\right)^2 - r^2\sigma^2\sum_{j=0}^{n}(1 - g_{ij})\beta_{-i,j}^2 - C(\mathcal{K}_i).$$

The proof of the proposition is presented in online Appendix Section A.3. It consists of a long sequence of algebraic manipulation, in which the last step is a direct result of the Sherman-Morrison theorem regarding the inversion of matrix additions.

By definition, $\beta_{-i,j}$ does not depend on the action-strategy of agent i since it builds on action-strategies used by all other agents, except on agent i's, as defined in equation (4). It is important to emphasize that agents take other agents' actions as given when choosing which connections to form. Formally, this translates into agent i taking each $\beta_{-i,j}$ as given when making decisions. Therefore, the payoff formulation above implies that agent i's decision of forming connections, i.e., the choice of $\{g_{ij}\}_j$, depends only on the action-strategies of other agents, namely $\{\beta_{-i,j}\}_j$.

The payoff specification from Proposition 2 also makes the trade-off involved in making a connection explicit. By observing another agent's signal, agent i increases her payoff in two ways. First, she increases the information that she knows, thus increasing the first term. At the same time, she reduces the information that she does not know, thus increasing the second term. In addition, by observing another agent's signal, her payoff decreases by the link-formation cost.

An important feature of the payoff formulation above is concavity of the benefits of information acquisition. How much a connection adds to agent i's payoff decreases with how much information agent i already has. For instance, if all agents have the same influence on the average action excluding i's action, that is $\beta_{-i,j} = b$, $\forall j$, then a connection adds more to agent i's payoff than any subsequent one.

Another feature of the payoff formulation from Proposition 2 is monotonicity. From agent i's perspective, the payoff gain from connecting to an agent j is higher the more influential j is. Given the connections that other agents are choosing, as well as the resulting influence on the average action not including agent i's action, agent i chooses to connect to the most influential agents (agent j with highest $\beta_{-i,j}$). However, as agent i makes a decision, she is concerned about the most influential signals regarding the average action not including her own action. Hence, in principle, different agents may rank other agents' influence differently. In the next section, we show that this is not the case in equilibrium.

C. *Equilibrium Properties*

Next, we characterize two network properties, and later, in Proposition 3, we show that both hold in equilibrium. Properties 1 and 2 along with Proposition 3 formalize three main lessons regarding connection decisions. First, in equilibrium, all agents share the same ranking over signals' influence. Second, an agent's influence is monotonic in her in-degree centrality, which is the number of agents observing her signal. And, third, information acquisition is more valuable to agents whose signals are less influential. Property 1 formalizes the first and second lessons, while Property 2 formalizes the third.

PROPERTY 1: *If an agent is observing another agent's signal, she must also be observing the signal of any other agent with a higher in-degree centrality.*

$$\text{For all } i \text{ and } l \neq i, \quad g_{i,l} = 1 \Rightarrow g_{i,m} = 1, \quad \forall m : \bar{\mathcal{K}}_m \geq \bar{\mathcal{K}}_l,$$

where $\bar{\mathcal{K}}_j = \sum_{s=1,s\neq j}^{n} g_{sj}$.

Property 1 establishes a monotonicity result: if agent i is connected to agent l, then i must also be connected to any agent m whose signal is more observed than l's signal.[15] The intuition for Property 1 is that, when observing a signal, an agent obtains information regarding not only the true state of the world, but also about the average action. Even though all signals are equally informative about the state of the world, the same is not true of the average action: the more influential the source, the more informative the signal.

The fact that agents are ex ante identical does not imply that the signals are equally informative. If an agent's signal is being observed by more agents, her own signal is more informative about the average action. As a result, she has less incentive to acquire additional information than an agent whose signal is not as public. Property 2 summarizes this feature.

PROPERTY 2: *If an agent's signal is more observed than another agent's, she cannot be observing more signals:*

$$\bar{\mathcal{K}}_f \geq \bar{\mathcal{K}}_h \Rightarrow \mathcal{K}_f \leq \mathcal{K}_h \quad \forall f, h,$$

where $\bar{\mathcal{K}}_j = \sum_{s=1, s\neq j}^{n} g_{sj}$ *and* $\mathcal{K}_i = \sum_{s=1, s\neq i}^{n} g_{is}$.

Property 2 establishes a different monotonicity result: if agent f's signal is more observed than agent h's signal, then agent f must be observing fewer signals than agent h is. While Property 1 ranks signals by the number of agents observing them, Property 2 ranks agents by the number of agents observing their signals and implies that a higher-ranked agent must observe fewer signals. Properties 1 and 2 combined provide the intuition that the more observed agent f's signal is, the more it informs about the average action. For this reason, agent f has less incentive to acquire additional information, thereby observing fewer signals. The following proposition shows that both properties hold true in equilibrium.

PROPOSITION 3: *Given Assumption 1, any strict Nash equilibrium of the game above satisfies Properties 1 and 2.*

The proof is presented in online Appendix Section B.1. The main contribution of the proposition is to restrict the information structures that can occur in equilibrium. For instance, neither the wheel nor the bilateral star in Figure 2 occurs in equilibrium. The wheel does not satisfy Property 1, while the bilateral star does not satisfy Property 2. The wheel violates Property 1 since $g_{1,2} = 1$, but $g_{1,3} = 0$ although $\bar{\mathcal{K}}_2 = \bar{\mathcal{K}}_3$. The bilateral star violates Property 2 since $\mathcal{K}_1 > \mathcal{K}_2$ although $\bar{\mathcal{K}}_1 > \bar{\mathcal{K}}_2$.

The intuition of Property 1 holding in equilibrium relies on players' incentives to form connections. After accounting for equilibrium behavior of agents, the importance of a signal is summarized by its agent's in-degree centrality. In other words, in-degree centrality, namely $\bar{\mathcal{K}}$'s, is a sufficient statistic that agents take into account when forming connections. As more agents observe a signal, the more important that signal is in equilibrium. For Property 2, the intuition is different. As more

[15] Given our direct network setup, Property 1 coincides with the definition of nested-split graphs. See Definition 2 in Hiller (2017).

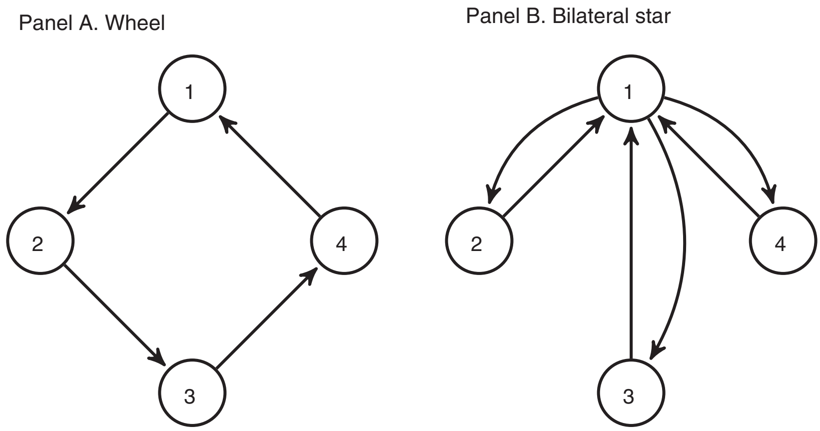

Figure 2. Property 1 and Property 2 Restrict Equilibrium Information Structures

Notes: In panel A, the wheel information structure is not an equilibrium because it violates Property 1. In panel B, the bilateral star information structure is not an equilibrium because it violates Property 2.

agents observe a given agent's signal, say agent i's signal, that signal becomes more influential in equilibrium. As a result, agent i endogenously becomes endowed with a more informative signal, which diminishes her incentive to acquire additional information. Thus, as more agents observe a signal from an agent, fewer is the number of signals that agent acquires in equilibrium.

Properties 1 and 2 hold true only in equilibrium. In online Appendix Section B.2, we provide an example highlighting how neither of the properties is an immediate implication of our setup. The fact that j's signal being observed by more agents implies that it is more informative about the average action is not a mechanical feature of agents' influence, but an equilibrium result as well.

II. Equilibrium

In this section, we characterize equilibrium information structures. In Subsection IIA, we show that Properties 1 and 2 are sufficient to characterize all information structures in equilibrium as hierarchical networks. Subsection IIB discusses sufficient conditions on the link formation cost function to have only core-periphery networks in equilibrium, a particular case of a hierarchical network.

A. *Hierarchical Network*

Proposition 3 is a first step in characterizing the set of equilibria, by showing that every equilibrium satisfies Properties 1 and 2. On the one hand, Property 1 suggests that agents are organized in a hierarchy of influence, ranked by their in-degree centrality. On the other hand, Property 2 suggests that a more influential agent has less incentive to acquire information. The following definition of hierarchical networks captures these intuitions.

DEFINITION 2: *A network is a hierarchical directed network if, and only if, there exists a partition on the set of agents*, $A_{s\in\{1,2,\ldots,N\}}$, *such that*

(*i*) $i \in A_s$ *if, and only if*, $\forall j \notin \cup_{k=1}^{s} A_k$, $g_{j,i} = 1$;

(*ii*) *if there is* $i \in A_s$ *and* $j \notin \cup_{k=1}^{s-1} A_k$, *such that* $g_{i,j} = 1$, *then* $g_{l,m} = 1$ $\forall l,m \in A_s$; *and*

(*iii*) *if there is* $i \in A_s$ *and* $j \notin \cup_{k=1}^{s} A_k$, *such that* $g_{i,j} = 1$, *then* $j \in A_{s+1}$ *and for any* $l \in A_s, g_{l,j} = 1$. *Furthermore*, $g_{l,m} = 0$ *for any other* $m \notin \cup_{k=1}^{s} A_k$.

A network is a hierarchical directed network if agents can be partitioned into tiers, such that the signal of any agent of a certain tier is observed by all agents in tiers below hers. Formally, item (i) specifies that an agent i is a member of the first tier if her signal is observed by all agents who are not in the first tier. An agent is in the second tier if her signal is observed by all agents who are not in the first or second tier. By induction, it specifies that an agent i is in tier s if her signal is observed by all agents who are not in a tier above hers. Furthermore, within a certain tier, either all agents are observing the signals of all other agents in that tier or no one is. Item (ii) specifies that if agent i observes the signal of a member of her own tier s or of any tier below hers, then everyone in her tier observes each other's signals. Finally, item (iii) specifies that if an agent i observes the signal of an agent in a tier below hers, then there will be only one such signal, and all other members of her tier also observe that same signal, as well. Intuitively both items (ii) and (iii) specify that agents in a tier are "equal." All members of the same tier equally observe and are equally observed by members of their own tier. Either everyone looks at everyone's signal or no one look at no one's signal. Furthermore, they observe the same set of signals outside their own tier.

To help clarify the definition, we present a couple of examples in Figure 3. The wheel network in panel A is not hierarchical since, if all agents were in the same tier, then agents would not observe all or none of the signals within that tier, while if the agents were in different tiers, not all agents from a level below would be looking to all agents in the upper tiers. On the other hand, the network depicted in panel B satisfies the hierarchical definition with the partition that places agents 1 and 2 in one group, agent 3 in a second group, and the other agents in a third group.

Next, we show one of our main results specifying conditions for the emergence of hierarchical directed networks.

THEOREM 1: *Any directed network satisfying Property 1 and Property 2 is a hierarchical directed network.*

The proof is by induction on the tiers of the network and is detailed in online Appendix Section B.3. The theorem is more general than our benchmark model. It relies on how Properties 1 and 2 restrict the network formation process, instead of on other specific modeling assumptions. Theorem 1 shows that any network satisfying both properties is hierarchical. For the purpose of our game, Theorem 1 along with Proposition 3 imply that all strict Nash equilibria feature a hierarchical directed

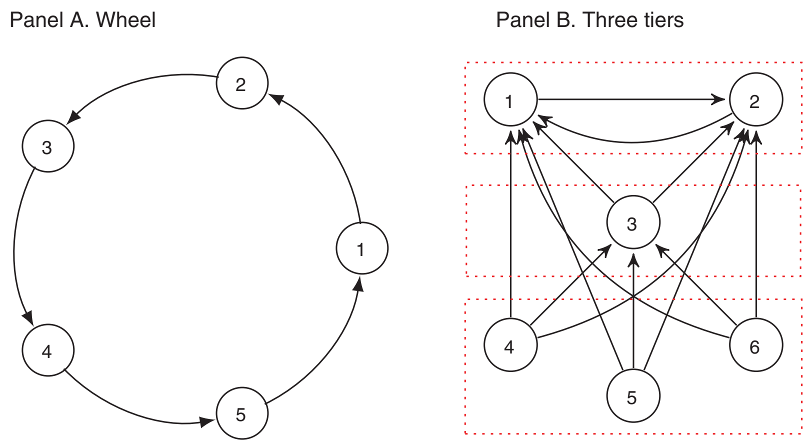

Figure 3. Examples of Networks: One Not Hierarchical and One Hierarchical

Notes: In panel A, the wheel is not a hierarchical directed network. In panel B, the three-tier network is a hierarchical directed network.

network. This result characterizes the set of equilibria, and we summarize it in the following corollary.

COROLLARY 1: *Given Assumption 1, the network resulting from any strict Nash equilibrium is a hierarchical directed network.*

A hierarchical information structure resembles what has been commonly defined as a social hierarchy: an implicit or explicit rank of individuals with respect to a certain social dimension.[16] In our model, the number of people observing a signal is the social dimension that ranks agents in the social hierarchy. Furthermore, this implies a ranking regarding the influence that each agent's signal has over the average action, with influential agents being at the top of the social hierarchy.

In equilibrium, agents endogenously sort themselves into tiers. Members of a group acquire the same information and have the same influence on the average action. Although the equilibrium may be asymmetric, information acquisition strategies are symmetric within each tier. Furthermore, the hierarchy of influence is self-enforcing. If more agents observe the same signal, then this signal has more influence on the average action and becomes more attractive to be observed.

Another feature of our equilibrium is that, although agents are ex ante identical, agents are ordered by their payoffs.[17] Those at the top of the hierarchy must have a higher payoff than agents in the lower tiers. Notice that a member of a higher tier can always mimic someone in a lower tier. Hence, by revealed preference, it must be that agents in higher tiers have higher payoffs in equilibrium, as well. Interestingly,

[16] For a formal definition, see Magee and Galinsky (2008).

[17] This contrasts the results in Garicano (2000), in which ex ante identical agents differentiate themselves through costly information acquisition and occupy different positions in a social hierarchy, but all receive the same payoff.

agents in lower tiers observe at least the same number of signals as those in higher tiers, and, as a result, they have more information. However, lower-tier agents bear a higher cost to have more information than those at the top, which outweighs the payoff gains from having more information.

B. *Core-Periphery Network*

In the previous subsection, we showed that any equilibrium information structure is hierarchical. Next, we further restrict the set of information structures that occur in equilibrium. We show that if the marginal cost of link formation is positive and weakly increasing (Assumptions 1 and 2), then any equilibrium information structure is core-periphery (Theorem 2 and Corollary 2).

PROPERTY 3: *If an agent i observes two distinct agents' signals, say k and j, then both agents k and j observe the signal of each other. Formally,*

$$g_{i,j} = 1 \quad \textit{and} \quad g_{i,k} = 1 \Rightarrow g_{k,j} = 1$$

for any distinct agents, i, j, and k.

Property 3 establishes that if agent i observes both agents j's and k's signals, then agent k must observe j's signal. Symmetrically, the property also implies that agent j observes k's signal. Thus, there is at most one signal observed by agent i but not observed by agent j: agent i's signal.

PROPOSITION 4: *Given Assumptions 1 and 2, any strict Nash equilibrium of the game described above satisfies Properties 1, 2, and 3.*

The intuition behind Proposition 4 is a revealed preference argument. If, in equilibrium, an agent i observes the signals from both agents j and k, then agent i is willing to pay the cost of forming these links. Because networks are hierarchical in equilibrium (Corollary 1), agent i formed at least the same connections as agent k. Thus, by the concavity of the payoff function, the benefit of an additional signal for agent i is lower than for agent k. At the same time, Assumption 2 implies that agent i's marginal cost of forming a link is greater than or equal to agent k's. Therefore, by revealed preference, if agent i is willing to observe agent j's signal, then it must be that agent k is willing to observe j's signal as well because agent k faces higher benefit and lower cost if compared to agent i. The formal proof of the proposition is in online Appendix Section B.4.

The three-tier hierarchical network presented in panel B of Figure 3 violates Property 3. Agent 5 observes both agents 1's and 3's signals, but agent 1 does not observe agent 3's signal. If agent 1 chooses not to observe agent 3's signal, it must be that the benefit did not outweigh the cost. However, for agent 5, the benefit of observing agent 3's signal is smaller, as she observes more signals, and the cost is larger, given Assumption 2. Thus, agent 5 should find it optimal to not observe agent 3's signal. This example shows that a weakly increasing marginal cost function helps to further restrict the set of possible equilibrium networks. Theorem 2 and

Corollary 2 show that if the cost function is strictly increasing with weakly increasing marginal costs, then any equilibrium information structure is core-periphery. Before discussing this result, we define directed core-periphery networks.

DEFINITION 3: *A hierarchical directed network is core-periphery if, and only if, the set of agents can be partitioned into two, the core* A_1 *and the periphery* A_2*, such that*

(*i*) *for any* j *in* A_1, $g_{ij} = 1$ *for any* $i = 1, \ldots, n$; *and*

(*ii*) *for any distinct* s *and* r *in* A_2, $g_{sr} = 0$.

A core-periphery network is a hierarchical directed network, in which agents are partitioned into, at most, two groups: the core and the periphery. Furthermore, all agents observe the signals of agents in the core; their signals are effectively public information. Agents in the periphery observe only signals from agents in the core. Also, note that both the empty and full networks are core-periphery. In the empty network, A_1 is empty and all agents are in A_2, while in the full network, A_2 is empty and all agents are in A_1.

To illustrate the core-periphery definition, Figure 4 provides four examples with six agents. The networks in panels A and B are both core-periphery. The network in panel C is not hierarchical and, thus, not core-periphery, while the one in panel D is hierarchical but not core-periphery. The network in panel C is not hierarchical because it violates condition (iii) of Definition 2. The network in panel D is hierarchical but not core-periphery since agents 1 and 2 do not observe each other's signals, violating condition (i) of Definition 3.

THEOREM 2: *Any directed network satisfying Properties 1, 2, and 3 is a core-periphery directed network.*

The proof is in online Appendix Section B.5. Similar to Theorem 1, Theorem 2 is more general than the model presented. Theorem 2 shows that any network satisfying the three properties is core-periphery. For the purpose of our game, the theorem implies that all strict Nash equilibria feature a core-periphery directed network when the marginal cost of acquiring information is weakly increasing, as captured in Corollary 2.

COROLLARY 2: *Given Assumptions 1 and 2, the network resulting from any strict Nash equilibrium is a core-periphery directed network.*

Corollary 2 characterizes the set of equilibrium information structures. If the marginal cost of acquiring information is increasing, then all strict Nash equilibria have a core-periphery information structure. The corollary also disciplines how public each agent's signal is in equilibrium, as well as each agent's payoff. Specifically, the signal of any core agent is effectively public because all agents observe all core agents' signals, and the signals of core agents influence the average more than those from the periphery. Hence, we can interpret any equilibrium

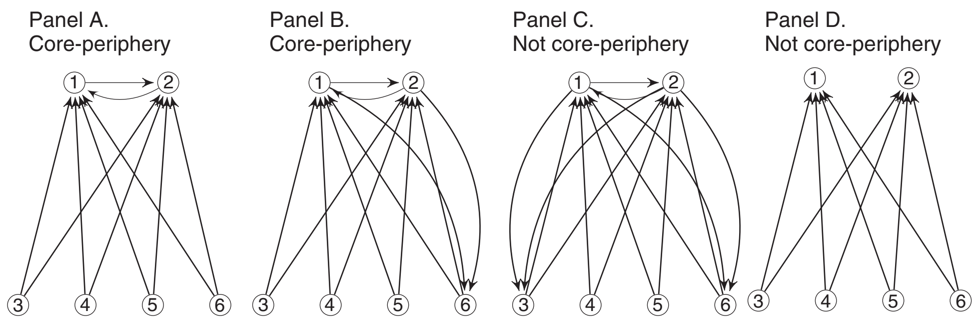

Figure 4. Different Networks: Only the First Two Are Core-Periphery Networks

as a rational ritual in which the signals of the core become endogenously common knowledge.

The fact that some signals are more public than others translates into different expected payoffs for any given equilibrium network. Agents in the core could mimic any peripheral agent's information acquisition by observing the same set of signals at the same cost. However, agents in the core choose not to observe at least a signal from the periphery. Thus, by revealed preference, core agents ought to have an expected payoff at least as high as peripheral agents have. Although core agents have higher expected payoffs, peripheral agents have more information because they observe at least the same number of signals as core agents do.

To further characterize the core-periphery equilibria, notice that there are two types of core-periphery networks. In the first type, agents in the core observe only other core agents' signals, e.g., network in panel A of Figure 4. Let us refer to this type of network as simple core-periphery network. In the second type of core-periphery network, agents in the core observe not only other core agents' signals, but also one signal from a peripheral agent, e.g., network in panel B of Figure 4. Let us call such a peripheral agent the key peripheral agent. The key peripheral agent is part of the periphery and not part of the core because, although core players observe her signal, other peripheral agents do not observe her signal. Let us refer to the second type as core-periphery observing down network. Next, we fully characterize the set of equilibria when the conformity parameter, r, goes to either 0 or 1.

PROPOSITION 5: *For a given cost function satisfying Assumptions 1 and 2*:

(a) *As r approaches* 0, *there is a unique equilibrium which is a core-periphery observing down network;*

(b) *As r approaches* 1, *only simple core-periphery networks are equilibrium. Moreover*, *the empty network is always an equilibrium and there exist $n_c^* \in \{0, 1, \ldots, n\}$ such that every simple core periphery network with $n_c \leq n_c^*$ core players is an equilibrium.*

The proof is in online Appendix Section B.6. As r approaches 0, there is a unique equilibrium network structure. For r sufficiently close to 0, the payoff benefit of coordinating is approximately 0. As a result, the force behind Property 2 weakens. Although different agents have their signals observed by different sets of players, their benefit of acquiring information become almost identical. Hence, all agents effectively face the same optimization problem: choosing how many signals to acquire. Thus, they all choose to acquire the same number of signals. The equilibrium information structure is unique and given by the core-periphery observing down network.

As r approaches 1, the influence coefficients, β, of nonpublic signals converge to 0 (Lemma 1). Thus, agents do not want to observe a nonpublic signal and only simple core-periphery networks hold in equilibrium. The empty network is always an equilibrium because of the strong coordination incentives that make no agent willing to observe a nonpublic signal. Furthermore, starting from an equilibrium core-periphery network, if we decrease the number of core players, then the influence coefficients of core players increases (Lemma 1), which, according to Proposition 2, leads to higher marginal benefit of keeping a connection. Hence, if a simple core-periphery network with n_c core players is an equilibrium, then any simple core-periphery network with less than n_c core players is also an equilibrium for r close enough to 1. The maximum number of core players possible, say n_c^*, is given by the incentive constraint that a periphery player does not want to break a link with a core player.

In the next section, we discuss an example in which Assumption 2 does not hold and a three-tier hierarchical network emerges in equilibrium.

C. *Example of Three-Tier Equilibrium*

In Section IIA, we showed that every strict Nash equilibrium results in hierarchical directed network as information structure whenever Assumption 1 holds (Corollary 1). In Section IIB, we add Assumption 2 and further restrict equilibrium networks to core-periphery (Corollary 2). To highlight the possibility of multi-tier hierarchical networks, we show in this section an example of an economy with a three-tier network as an equilibrium outcome.

Let us consider an economy with 5 agents, $\sigma = 1$, and $r = 0.5$. We assume the following nonconvex cost function for $C(\mathcal{K}_i)$: $C(0) = 0$, $C(1) = 0.14$, $C(2) = 0.20$, and $C(x) = 1$ for every $x \geq 3$. Notice that this example violates Assumption 2, which means that we cannot apply Theorem 2 and Corollary 2. We numerically solve for all equilibria, and there are four different information structures that can be equilibrium. All four equilibrium networks are displayed in Figure 5, and this example features an equilibrium that does not have a core-periphery structure. We have in panels A, B, and D different hierarchical core-periphery networks, while in panel C we have a hierarchical directed network that is not core-periphery. The network in panel C is a three-tier hierarchical network and holds in equilibrium.

In the three-tier hierarchical network displayed in panel C of Figure 5, agent 1 constitutes the first tier, agent 2 the second, and the third tier consists of agents 3, 4, and 5. This network structure holds in equilibrium because the cost function is nonconvex. More importantly, the additional cost of observing two agents' signals instead of only one is relatively low. Hence, third-tier agents are willing

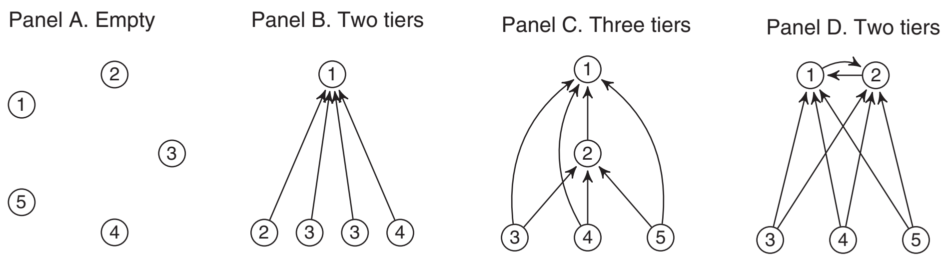

Figure 5. Example of Equilibrium with a Three-Tier Hierarchical Network

Notes: This figure plots all equilibria from an economy with five agents $(n = 5)$, $\sigma = 1$, and $r = 0.5$. We assume the following nonconvex cost function for $C(\mathcal{K}_i)$: $C(0) = 0$, $C(1) = 0.14$, $C(2) = 0.20$, and $C(x) = 1$ for every $x \geq 3$. We numerically solve for all equilibria, and there are four different information structures in equilibrium. All four equilibria are displayed in panels A, B, C, and D.

to observe two additional signals, one from agent 1 and another from agent 2. In equilibrium, agent 2's signal is more informative about the average action than are signals from the third tier. This difference in signals' informativeness makes agent 2 (second tier) willing to observe only the signal from the first tier because she already observes her own signal. Assumption 2 prevents a scenario like this one from happening. If Assumption 2 holds, then agents in the third tier would find it expensive to observe agent 2's signal, and a three-tier hierarchical network would not be an equilibrium.

III. Final Remarks

In this paper, we characterize all equilibrium information structures of an information acquisition game with complementarity in actions. We show that all strict Nash equilibria present a hierarchical directed information structure, which is characterized by the existence of different tiers of informational importance. An individual that belongs to the top tier is more influential, as members of all the other tiers observe her signal. A second tier individual's signal is observed by all members of tiers below hers, and so on. A hierarchical directed network implies multiple social levels, ranked by their influence over economic outcomes.

Our first result (Theorem 1) is more general than the information acquisition model presented in this paper, as it depends on Properties 1 and 2. Property 1 states that an agent ranks other agents' signals by how public they are: that is, according to their in-degree centrality in the network. Property 2 states that a more central agent benefits less from observing another agent's signal. We show that any strict Nash equilibrium of a network formation game satisfying Properties 1 and 2 is a hierarchical directed network.

A particular case of a hierarchical network is core-periphery. In this case, agents are partitioned into two groups: the core and the periphery. Members of the core are more influential, with their signals being effectively public information in equilibrium. We show that if the marginal cost of observing an additional signal is weakly increasing, then any equilibrium information structure is core-periphery (Theorem 2).

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