Grupos de Movimento

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IFSC, USP

16 de junho de 2019

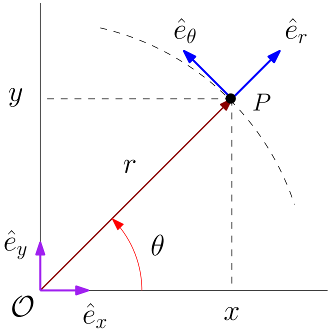
Conteúdo I

Conteúdo II

Conteúdo III

Rotação em torno de um eixo fixo

- Coords. cartesianas:
  $\mathbf{r} = x_i\,\mathbf{e}_i,\ r^2 = \mathbf{r}\cdot\mathbf{r} = x^2 + y^2$



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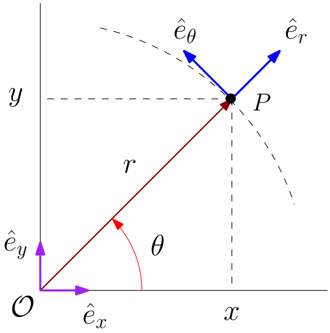

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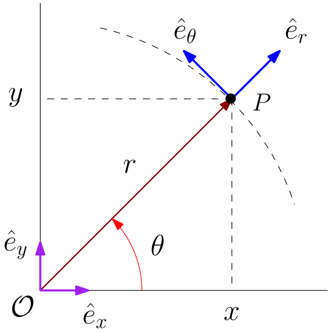

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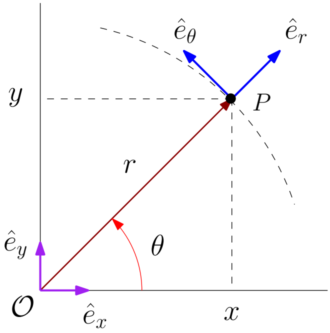

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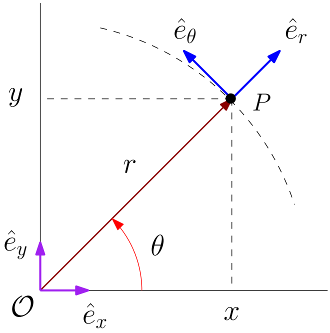

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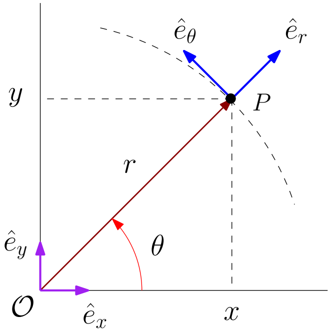

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- ds^2 não é invariante pelas transformações de Galileu–Newton:

$$x = \bar{x} + Vt, \quad y = \bar{y}, \quad z = \bar{z}, \quad t = \bar{t}$$

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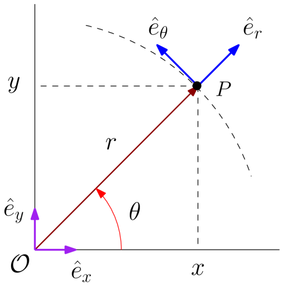

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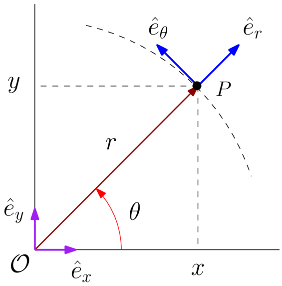

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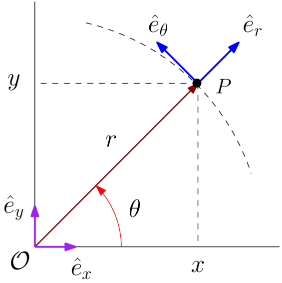

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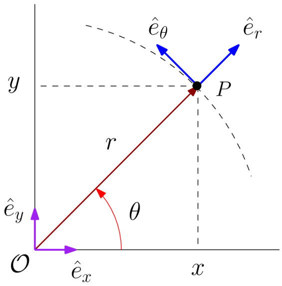

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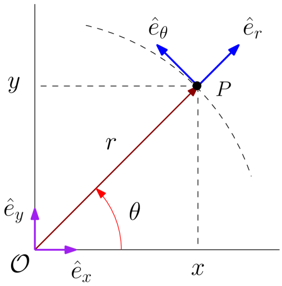

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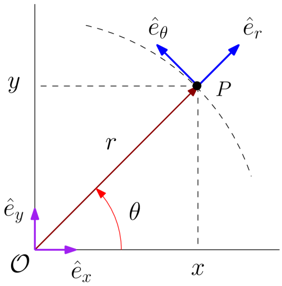

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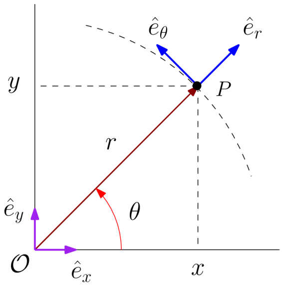

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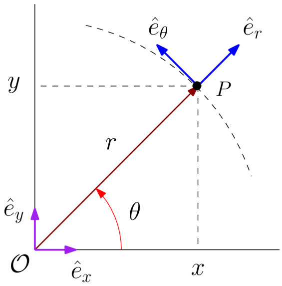

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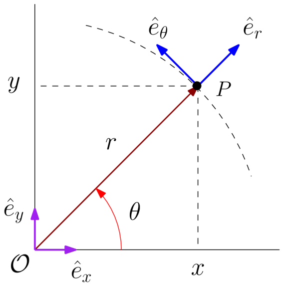

- $(\bar{\mathbf{e}}_1\ \bar{\mathbf{e}}_2) = (\mathbf{e}_1\ \mathbf{e}_2)R(\theta)$
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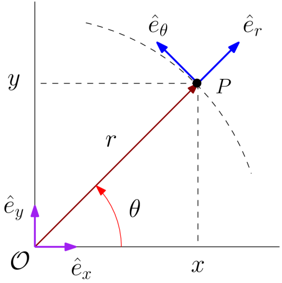

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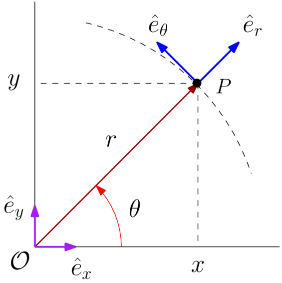

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Conteúdo I

Conteúdo II

Conteúdo III

Em torno da identidade...

- Até primeira ordem em $\Delta\theta$: $\mathrm{i} = \sqrt{-1}$

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$$R(\theta) = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} = \mathrm{e}^{-\mathrm{i}\theta J_z}, \quad J_z = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \mathrm{e}^{A} = \sum_{k=0}^{\infty} \frac{A^k}{k!}$$

- Potências: $J_z^{2k} = I, \quad J_z^{2k+1} = J_z, \quad \Lambda^k = (M^{-1} J_z M)^k = M^{-1} J_z^k M$

$$\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma_z$$

- Diagonalização de R:

$$M^{-1} R M = \mathrm{e}^{-\mathrm{i}\theta M^{-1} J_z M} = \mathrm{e}^{-\mathrm{i}\theta \Lambda} = \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\theta} & 0 \\ 0 & \mathrm{e}^{+\mathrm{i}\theta} \end{pmatrix}$$

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Conteúdo III

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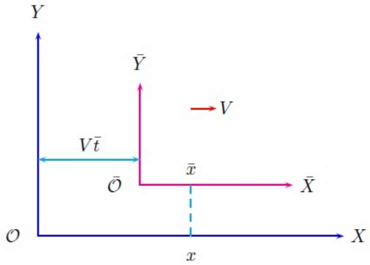

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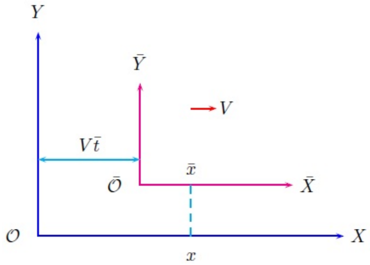

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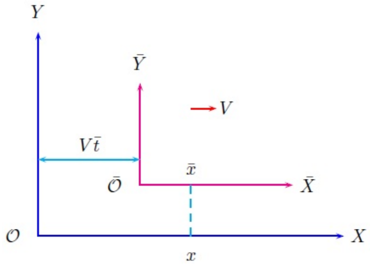

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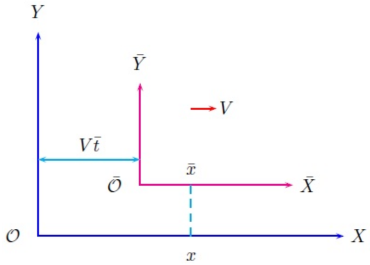

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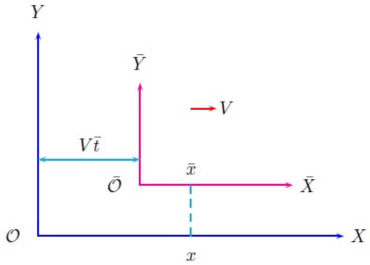

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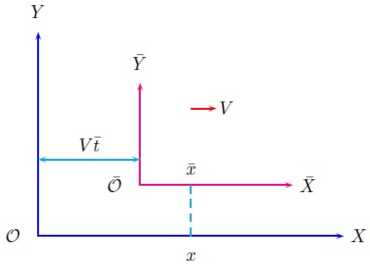

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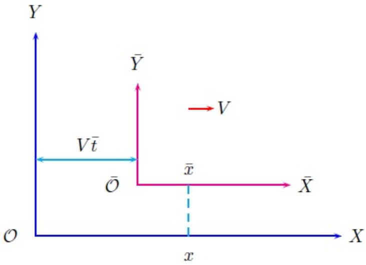

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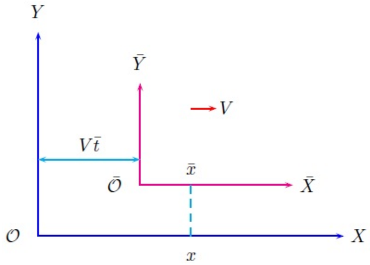

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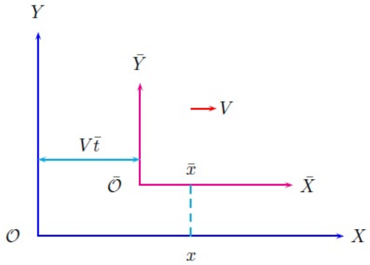

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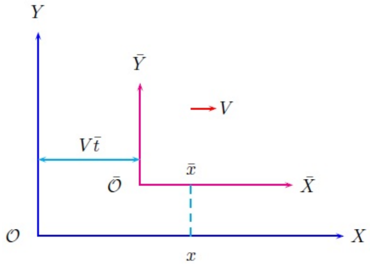

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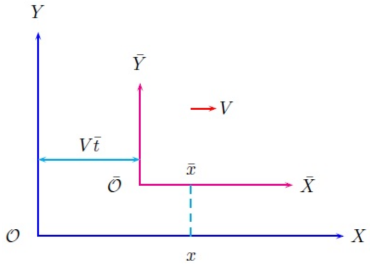

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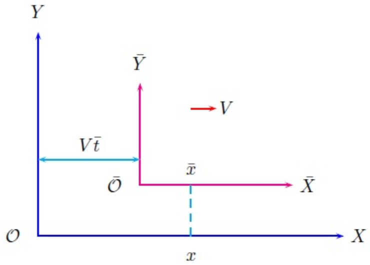

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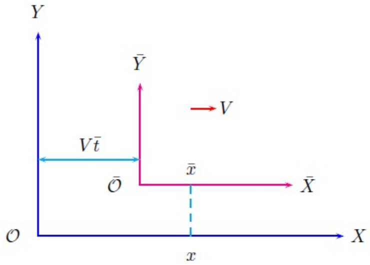

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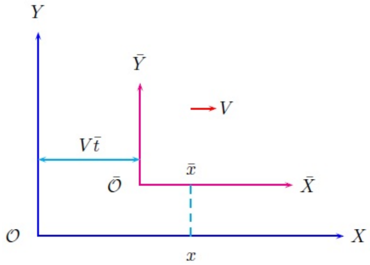

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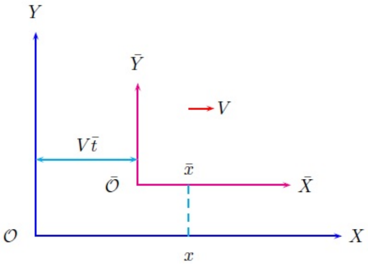

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Conteúdo I

Conteúdo II

Conteúdo III

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- $\boxed{W = \dfrac{U + V}{1 + \dfrac{UV}{c^2}}}$ **A velocidade da luz é um limite superior.**

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Conteúdo I

Conteúdo II

Conteúdo III

Grupo das translações.

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$$\big\langle p_x\big|p'_x\big\rangle = 2\pi\delta(p_x - p'_x), \quad \frac{1}{2\pi}\int_{-\infty}^{+\infty} dp_x\,\big|p_x\big\rangle\big\langle p_x\big| = \mathbb{I}.$$

- Ação do grupo:

$$T_{p_x}(x)\big|p_x\big\rangle = \mathrm{e}^{-\mathrm{i}\,xP_x}\big|p_x\big\rangle = \mathrm{e}^{-\mathrm{i}\,xp_x}\big|p_x\big\rangle$$

$$p_x T_{p_x}(x)\big|p_x\big\rangle = \mathrm{e}^{-\mathrm{i}\,xp_x}p_x\big|p_x\big\rangle = \mathrm{i}\,\frac{\partial}{\partial x}T_{p_x}(x)\big|p_x\big\rangle$$

- p_x é uma etiqueta (real) para cada irrep.

Conteúdo I

Conteúdo II

Conteúdo III

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Rodar e transladar = grupo E_2

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$$\vec{r}' = g(\alpha, \vec{a})\vec{r} \implies x_i' = a_i + \sum_{j=1}^{2} R_{ij}(\alpha) x_j$$

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Conteúdo I

Conteúdo II

Conteúdo III

Representações:

- No grupo: $\vec{r}' = g(\alpha, \vec{a})\vec{r}$

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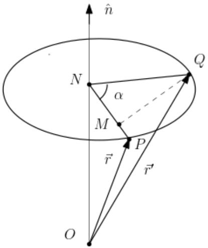
Conteúdo I

Conteúdo II

Conteúdo III

Parametrização:



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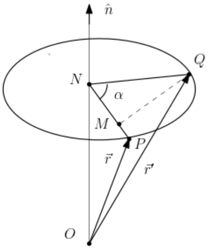

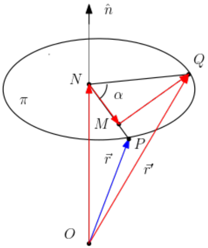

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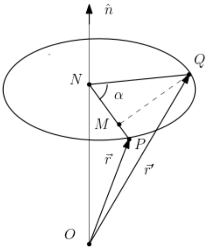

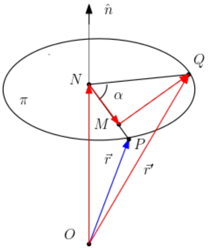

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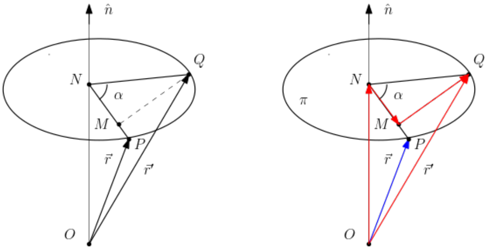

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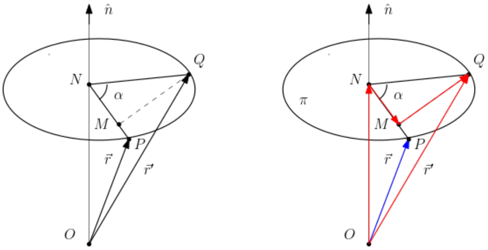

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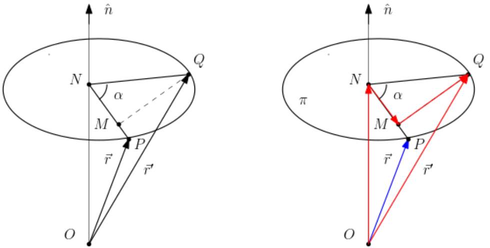

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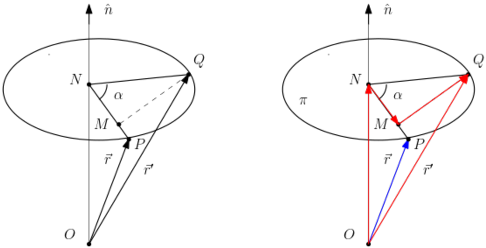

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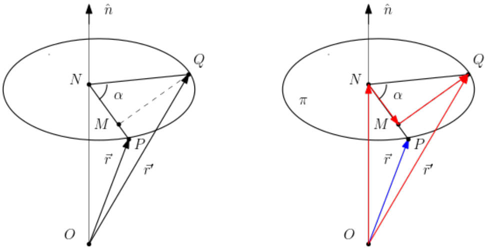

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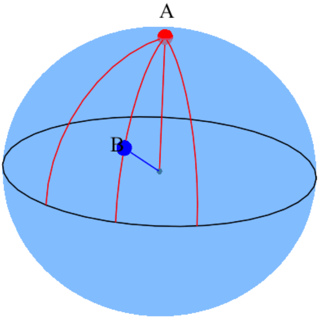

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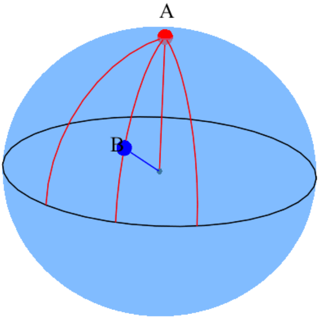

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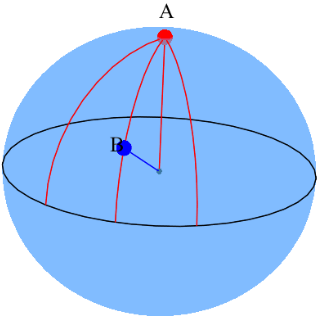

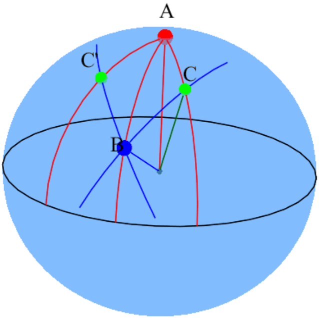

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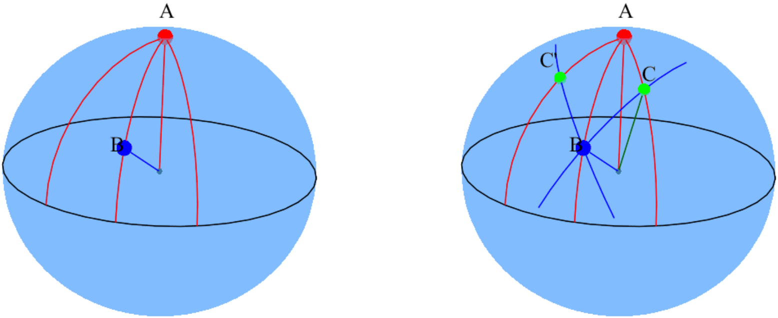

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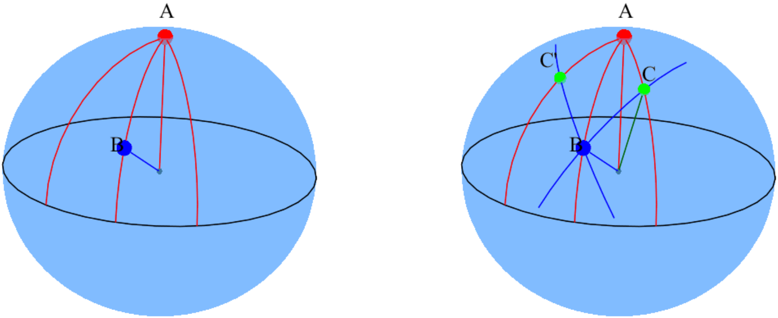

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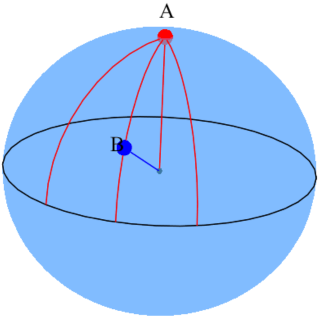

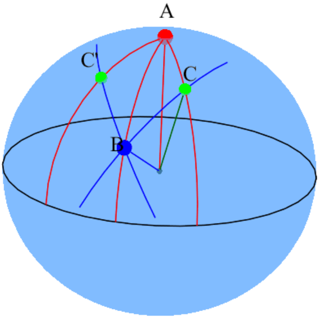

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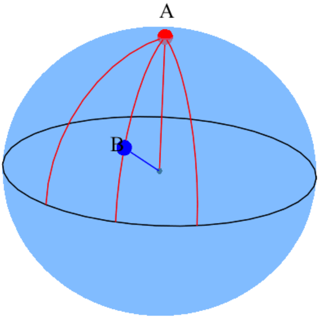

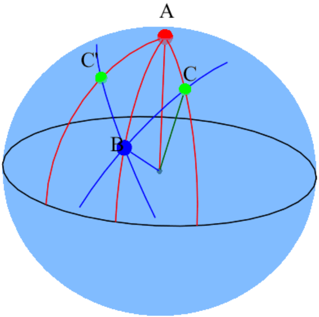

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Conteúdo I

Conteúdo II

Conteúdo III

Geradores:

- $R(\vec{\alpha}) = \mathrm{e}^{\alpha_1 L_1 + \alpha_2 L_2 + \alpha_3 L_3}, \quad L_k = \left.\frac{\partial R(\vec{\alpha})}{\partial \alpha_k}\right|_{\vec{\alpha}=0}, \quad \vec{\alpha} = \alpha \hat{n}, \quad (L_i)_{jk} = -\varepsilon_{ijk}$

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$$L_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad L_3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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Conteúdo I

Conteúdo II

Conteúdo III

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Conteúdo I

Conteúdo II

Conteúdo III

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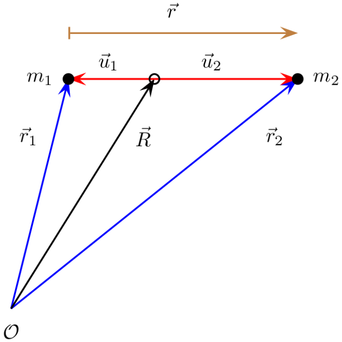
  $$\vec{R} = \frac{1}{M}\int_V dm\,\vec{r},\ M = \int_V dm$$



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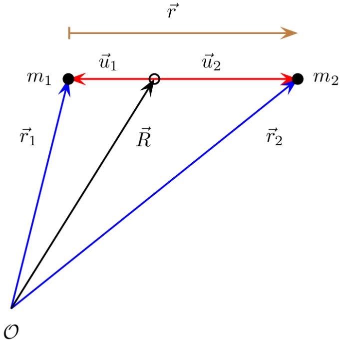

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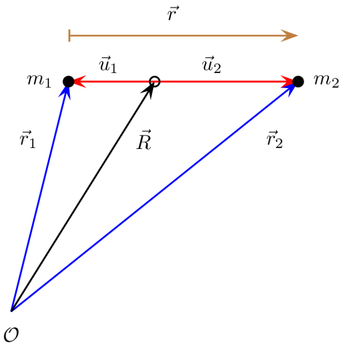

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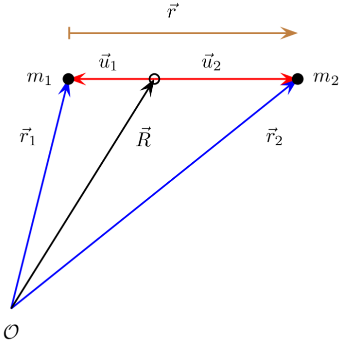

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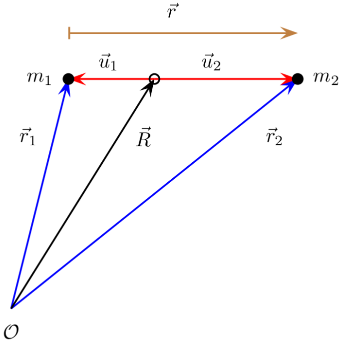

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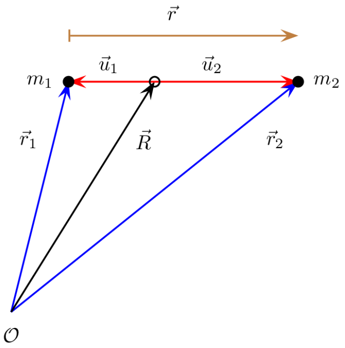

$$E_c = \sum_{i=1}^{2}\frac{1}{2}m_i\|\dot{\vec{r}}_i\|^2 = \frac{1}{2}M\|\dot{\vec{R}}\|^2 + \frac{1}{2}\sum_{i=1}^{2}m_i\|\dot{\vec{u}}_i\|^2 = \frac{1}{2}M\|\dot{\vec{R}}\|^2 + \frac{1}{2}\mu\|\dot{\vec{r}}\|^2$$

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Conteúdo I

Conteúdo II

Conteúdo III

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Conteúdo I

Conteúdo II

Conteúdo III

Poisson-Lie:

- $\{u, v\} = \sum_{k=1}^{3} \left(\frac{\partial u}{\partial x_k} \frac{\partial v}{\partial p_k} - \frac{\partial v}{\partial x_k} \frac{\partial u}{\partial p_k} \right), \quad \{u, v\} = -\{v, u\}, \quad \{x_i, p_j\} = \delta_{ij}$

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Conteúdo I

Conteúdo II

Conteúdo III

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Conteúdo III

$so(4) = so(3) \oplus so(3)$

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Conteúdo I

Conteúdo II

Conteúdo III

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- Definição: $\boxed{[L_i, A_j] = \mathrm{i}\,\hbar\epsilon_{ijk}A_k}$, $\quad \mathbf{L} = \mathbf{r}\times\mathbf{p}, \quad \mathbf{A}$ qualquer
- Lema 1: $[L_i, A_i] = 0 \implies \boxed{\mathbf{L}\cdot\mathbf{A} - \mathbf{A}\cdot\mathbf{L} = 0}$
- Teorema 1: $\boxed{\mathbf{L}\times\mathbf{A} + \mathbf{A}\times\mathbf{L} = 2\,\mathrm{i}\,\hbar\mathbf{A}}$
  $(\mathbf{L}\times\mathbf{A})_i + (\mathbf{A}\times\mathbf{L})_i = \epsilon_{ilm}L_lA_m + \epsilon_{ijk}A_jL_k =$
  $\epsilon_{ilm}(A_mL_l + \mathrm{i}\,\hbar\epsilon_{lma}A_a) + \epsilon_{ijk}A_jL_k = \mathrm{i}\,\hbar\epsilon_{alm}\epsilon_{ilm}A_a = 2\,\mathrm{i}\,\hbar\delta_{ia}A_a = 2\,\mathrm{i}\,\hbar A_i$
- Lema 2: $\boxed{\mathbf{L}\times\mathbf{L} = \mathrm{i}\,\hbar\mathbf{L}} \quad \mathbf{p}\times\mathbf{p} = 0 \quad \mathbf{r}\times\mathbf{r} = 0$
- Teorema 2: $\boxed{\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}} \quad A_i\epsilon_{ijk}B_jC_k = \epsilon_{ijk}A_iB_jC_k$
- Lema 3: $\boxed{\mathbf{r}\cdot\mathbf{L} = \mathbf{p}\cdot\mathbf{L} = 0}$

Propriedades I:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}$, $\quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}$, $\quad [x_i, p_j] = \mathrm{i}\,\hbar\delta_{ij}$
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  $(\mathbf{L}\times\mathbf{A})_i + (\mathbf{A}\times\mathbf{L})_i = \epsilon_{ilm}L_lA_m + \epsilon_{ijk}A_jL_k =$
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- Lema 2: $\boxed{\mathbf{L}\times\mathbf{L} = \mathrm{i}\,\hbar\mathbf{L}}$ $\quad \mathbf{p}\times\mathbf{p} = 0 \quad \mathbf{r}\times\mathbf{r} = 0$
- Teorema 2: $\boxed{\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}}$ $\quad A_i\epsilon_{ijk}B_jC_k = \epsilon_{ijk}A_iB_jC_k$
- Lema 3: $\boxed{\mathbf{r}\cdot\mathbf{L} = \mathbf{p}\cdot\mathbf{L} = 0}$ $\quad \mathbf{r}\cdot\mathbf{L} = \mathbf{r}\cdot(\mathbf{r}\times\mathbf{p}) = (\mathbf{r}\times\mathbf{r})\cdot\mathbf{p} = 0$

Propriedades I:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}, \quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}, \quad [x_i, p_j] = i\hbar\delta_{ij}$
- Definição: $\boxed{[L_i, A_j] = i\hbar\epsilon_{ijk}A_k}, \quad \mathbf{L} = \mathbf{r}\times\mathbf{p}, \quad \mathbf{A}$ qualquer
- Lema 1: $[L_i, A_i] = 0 \implies \boxed{\mathbf{L}\cdot\mathbf{A} - \mathbf{A}\cdot\mathbf{L} = 0}$
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  $\epsilon_{ilm}(A_mL_l + i\hbar\epsilon_{lma}A_a) + \epsilon_{ijk}A_jL_k = i\hbar\epsilon_{alm}\epsilon_{ilm}A_a = 2i\hbar\delta_{ia}A_a = 2i\hbar A_i$
- Lema 2: $\boxed{\mathbf{L}\times\mathbf{L} = i\hbar\mathbf{L}} \quad \mathbf{p}\times\mathbf{p} = 0 \quad \mathbf{r}\times\mathbf{r} = 0$
- Teorema 2: $\boxed{\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}} \quad A_i\epsilon_{ijk}B_jC_k = \epsilon_{ijk}A_iB_jC_k$
- Lema 3: $\boxed{\mathbf{r}\cdot\mathbf{L} = \mathbf{p}\cdot\mathbf{L} = 0} \quad \mathbf{r}\cdot\mathbf{L} = \mathbf{r}\cdot(\mathbf{r}\times\mathbf{p}) = (\mathbf{r}\times\mathbf{r})\cdot\mathbf{p} = 0$

Propriedades I:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}$, $\quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}$, $\quad [x_i, p_j] = \mathrm{i}\,\hbar\delta_{ij}$
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- Teorema 2: $\boxed{\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}}$ $\quad A_i\epsilon_{ijk}B_jC_k = \epsilon_{ijk}A_iB_jC_k$
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Propriedades II:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}, \quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}, \quad [x_i, p_j] = \mathrm{i}\,\hbar\delta_{ij}$

Propriedades II:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}, \quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}, \quad [x_i, p_j] = \mathrm{i}\,\hbar\delta_{ij}$
- $[L_i, A_k] = \mathrm{i}\,\hbar\epsilon_{ika}A_a, \quad [L_i, B_l] = \mathrm{i}\,\hbar\epsilon_{ilb}B_b$

Propriedades II:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}, \quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}, \quad [x_i, p_j] = \mathrm{i}\,\hbar\delta_{ij}$
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- Teorema 3: $\boxed{[L_i, (\mathbf{A}\times\mathbf{B})_j] = \mathrm{i}\,\hbar\epsilon_{ijk}(\mathbf{A}\times\mathbf{B})_k}$

Propriedades II:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}, \quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}, \quad [x_i, p_j] = \mathrm{i}\hbar\delta_{ij}$
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  $[L_i, (\mathbf{A}\times\mathbf{B})_j] = [L_i, \epsilon_{jkl}A_kB_l] = \epsilon_{jkl}([L_i, A_k]B_l + A_k[L_i, B_l]) =$

Propriedades II:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}, \quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}, \quad [x_i, p_j] = \mathrm{i}\hbar\delta_{ij}$
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  $\epsilon_{jkl}(\mathrm{i}\hbar\epsilon_{ika}A_aB_l + \mathrm{i}\hbar\epsilon_{ilb}A_kB_b) = \mathrm{i}\hbar(\epsilon_{jkl}\epsilon_{ika}A_aB_l + \epsilon_{jkl}\epsilon_{ilb}A_kB_b) =$

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  $\mathrm{i}\hbar(A_iB_j - A_jB_i) = \mathrm{i}\hbar\epsilon_{ijk}A_iB_j = \mathrm{i}\hbar\epsilon_{ijk}(\mathbf{A}\times\mathbf{B})_k$

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- Teorema 4: $\boxed{[L_i, x_j] = \mathrm{i}\hbar\epsilon_{ijk}x_k}$

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  $\mathrm{i}\hbar(A_iB_j - A_jB_i) = \mathrm{i}\hbar\epsilon_{ijk}A_iB_j = \mathrm{i}\hbar\epsilon_{ijk}(\mathbf{A}\times\mathbf{B})_k$
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- Teorema 3: $\boxed{[L_i, (\mathbf{A}\times\mathbf{B})_j] = i\hbar\epsilon_{ijk}(\mathbf{A}\times\mathbf{B})_k}$
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Propriedades II:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}$, $\quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}$, $\quad [x_i, p_j] = \mathrm{i}\hbar\delta_{ij}$
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- Teorema 4: $\boxed{[L_i, x_j] = \mathrm{i}\hbar\epsilon_{ijk}x_k}$ $\quad \boxed{[L_i, p_j] = \mathrm{i}\hbar\epsilon_{ijk}p_k}$
  $[L_i, x_j] = \epsilon_{ikl}[x_kp_l, x_j] = \epsilon_{ikl}x_k[p_l, x_j] = -\mathrm{i}\hbar\delta_{jl}\epsilon_{ikl}x_k = \mathrm{i}\hbar\epsilon_{ijk}x_k$

Propriedades II:

- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}, \quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}, \quad [x_i, p_j] = i\hbar\delta_{ij}$
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- Lema 4: $\boxed{[L_i, L_j] = \mathrm{i}\hbar\epsilon_{ijk}L_k} \quad \mathbf{L} = \mathbf{r}\times\mathbf{p}$
- Lema 5: $\boxed{[L_i, (\mathbf{p}\times\mathbf{L})_j] = \mathrm{i}\hbar\epsilon_{ijk}(\mathbf{p}\times\mathbf{L})_k}$

Propriedades II:

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- Utilidades: $\epsilon_{ika}\epsilon_{jla} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}, \quad \epsilon_{iml}\epsilon_{jml} = 2\delta_{ij}, \quad [x_i, p_j] = \mathrm{i}\hbar\delta_{ij}$
- $[L_i, A_k] = \mathrm{i}\hbar\epsilon_{ika}A_a, \quad [L_i, B_l] = \mathrm{i}\hbar\epsilon_{ilb}B_b$
- Teorema 3: $\boxed{[L_i, (\mathbf{A}\times\mathbf{B})_j] = \mathrm{i}\hbar\epsilon_{ijk}(\mathbf{A}\times\mathbf{B})_k}$

  $[L_i, (\mathbf{A}\times\mathbf{B})_j] = [L_i, \epsilon_{jkl}A_kB_l] = \epsilon_{jkl}([L_i, A_k]B_l + A_k[L_i, B_l]) =$
  $\epsilon_{jkl}(\mathrm{i}\hbar\epsilon_{ika}A_aB_l + \mathrm{i}\hbar\epsilon_{ilb}A_kB_b) = \mathrm{i}\hbar(\epsilon_{jkl}\epsilon_{ika}A_aB_l + \epsilon_{jkl}\epsilon_{ilb}A_kB_b) =$
  $\mathrm{i}\hbar(A_iB_j - A_jB_i) = \mathrm{i}\hbar\epsilon_{ijk}A_iB_j = \mathrm{i}\hbar\epsilon_{ijk}(\mathbf{A}\times\mathbf{B})_k$
- Teorema 4: $\boxed{[L_i, x_j] = \mathrm{i}\hbar\epsilon_{ijk}x_k}$ $\boxed{[L_i, p_j] = \mathrm{i}\hbar\epsilon_{ijk}p_k}$

  $[L_i, x_j] = \epsilon_{ikl}[x_kp_l, x_j] = \epsilon_{ikl}x_k[p_l, x_j] = -\mathrm{i}\hbar\delta_{jl}\epsilon_{ikl}x_k = \mathrm{i}\hbar\epsilon_{ijk}x_k$
- Lema 4: $\boxed{[L_i, L_j] = \mathrm{i}\hbar\epsilon_{ijk}L_k}$ $\quad \mathbf{L} = \mathbf{r}\times\mathbf{p}$
- Lema 5: $\boxed{[L_i, (\mathbf{p}\times\mathbf{L})_j] = \mathrm{i}\hbar\epsilon_{ijk}(\mathbf{p}\times\mathbf{L})_k}$

Conteúdo I

Conteúdo II

Conteúdo III

Rotações euclidianas:

- Geradores: $L_i = \left.\frac{dR_i}{d\theta_i}\right|_{\theta_i=0}, \ L_i^T = -L_i, \quad J_i = \mathrm{i}\, L_i, \ J_i^\dagger = J_i$

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$$R_1(\theta_1) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta_1 & -\operatorname{sen}\theta_1 \\ 0 & 0 & \operatorname{sen}\theta_1 & \cos\theta_1 \end{pmatrix}, \quad L_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

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Rotações hiperbólicas:

- $K_i = \left.\frac{d\Lambda_i}{d\psi_i}\right|_{\psi_i=0}, \; K_i^T = K_i, \quad ds^2 = c^2dt^2 - d\vec{r}\cdot d\vec{r}, \; \beta_i = \frac{V_i}{c}$

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Conteúdo I

Conteúdo II

Conteúdo III

Álgebra $so(1,3)$:

- $\boxed{[L_i, L_j] = \epsilon_{ijk} L_k}$ $\boxed{[K_i, K_j] = -\epsilon_{ijk} L_k}$ $\boxed{[L_i, K_j] = \epsilon_{ijk} K_k}$

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- $N_{i\pm} = \frac{1}{2}(L_i \pm K_i)$: $[N_{i\pm}, N_{j\pm}] = \pm\epsilon_{ijk} K_k \quad [N_{i\pm}, N_{j\mp}] = \epsilon_{ijk} L_k$

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$$J_\pm = \mathrm{i}\,(L_1 \pm \mathrm{i}\,L_2),\ J_3 = \mathrm{i}\,L_3, \quad M_\pm = \mathrm{i}\,(K_1 \pm \mathrm{i}\,K_2),\ M_3 = \mathrm{i}\,M_3$$

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