Introduction to Causal Data Analysis and Modeling with Coincidence Analysis

Module 1.3

The General Principles of Configurational Causal Discovery

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Overview

1. Epistemic uncertainty / limitations
2. Mill's Method of Difference
3. Causal homogeneity
4. Inference to causation vs. inference to non-causation
5. Generalizing the Method of Difference
6. From the Method of Difference to CNA
7. Perfect data fit

The goal of configurational causal modeling

A	B	C	D	E
0	1	1	1	1
1	0	1	1	1
1	0	0	1	1
0	1	1	0	1
1	1	1	1	0
0	0	1	1	0
1	1	0	1	0
0	1	0	1	0
0	0	0	1	0
1	1	1	0	0
1	0	1	0	0
0	0	1	0	0
1	1	0	0	0
0	1	0	0	0
1	0	0	0	0
0	0	0	0	0

infer

$$A{*}b{*}D \; + \; a{*}B{*}C \; \leftrightarrow \; E$$

The goal of configurational causal modeling

A	C	D	E
0	1	1	1
1	1	1	1
1	0	1	1
0	1	0	1
1	1	1	0
0	1	1	0
1	0	1	0
0	0	1	0
0	0	1	0
1	1	0	0
1	1	0	0
0	1	0	0
1	0	0	0
0	0	0	0
1	0	0	0
0	0	0	0

infer

$$A{*}b{*}D \;+\; a{*}B{*}C \;\leftrightarrow\; E$$

The goal of configurational causal modeling

A	C	D	E
0	1	1	1
1	1	1	1
1	0	1	1
0	1	0	1
1	1	1	0
0	1	1	0
1	0	1	0
0	0	1	0
0	0	1	0
1	1	0	0
1	1	0	0
0	1	0	0
1	0	0	0
0	0	0	0
1	0	0	0
0	0	0	0

infer

$$A{*}b{*}D \;+\; a{*}B{*}C \;\leftrightarrow\; E$$

Problem

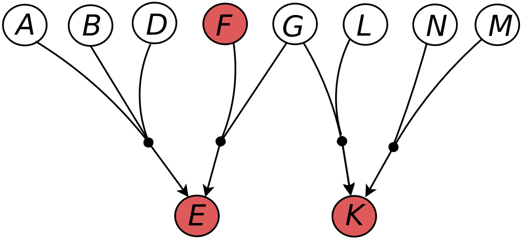
Infer the data-generating structure from data **under limited knowledge**.

How to uncover causation in light of limited knowledge?

Given that “A is a cause of E” is defined via A’s (permanent) membership in a MINUS-formula Ψ of E, it seems that in order to establish A as a cause of E, we first have to find Ψ (infer Ψ from data). But if Ψ involves many unknown factors apart from A, how can we ever find Ψ? And if we cannot find Ψ, how can we ever establish A as cause of E?

How to uncover causation in light of limited knowledge?

- **Example:**



- Suppose that F, E and K are all the known/measured factors of this causal structure.
- The complex MINUS-formula representing this causal structure is

$$(A{*}B{*}D \;+\; F{*}G \;\leftrightarrow\; E){*}(G{*}L \;+\; M{*}N \;\leftrightarrow\; K)$$

- How can we establish F as cause of E if we don't know anything about most of the relevant factors?

Mill's Method of Difference

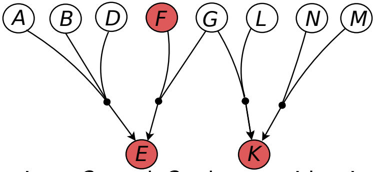
John S. Mill, *System of Logic*, book III, chapter 8:

SECOND CANON

If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance [d]in common save one[d], that one occurring only in the former; the circumstance in which alone the two instances differ, is the effect, or [e]the[e] cause, or [f]an indispensable[f] part of the cause, of the phenomenon.

Version of the Method of Difference in modern terminology Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two test situations that are identical in all factors except for an exogenous (test) factor F and an outcome E. If F is set to value 1 in $\mathcal{S}_1$ and to value 0 in $\mathcal{S}_2$ and if E likewise takes value 1 in $\mathcal{S}_1$ and value 0 in $\mathcal{S}_2$, it follows that F is a non-redundant difference-maker of E in the context of $\mathcal{S}_1$ and, hence, a cause of E.

Mill's Method of Difference



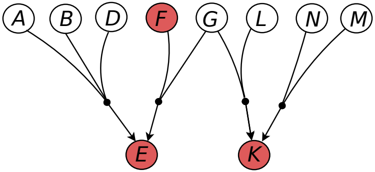
1. Given two test situations $\mathcal{S}_1$ and $\mathcal{S}_2$ that are identical except for the values on F and E.
2. Test result:

$\mathcal{S}_1$	$\mathcal{S}_2$
F=1	F=0
E=1	E=0

3. As effects do not occur without any of their causes, there must exist a cause of E=1 in $\mathcal{S}_1$.
4. As E=0 in $\mathcal{S}_2$ and as $\mathcal{S}_1$ and $\mathcal{S}_2$ are identical except for F and E, there is no unknown/unmeasured cause responsible for E=1 in $\mathcal{S}_1$.

➙ Therefore, F=1 must be a non-redundant part of at least one cause of E=1 that is operative in $\mathcal{S}_1$.

Mill's Method of Difference



- The crucial assumption in this inference is the identity (apart from F and E) of $\mathcal{S}_1$ and $\mathcal{S}_2$.
- Strictly speaking, however, there do not exist two identical test situations.
- But strict identity is not required for a causal inference under epistemic limitations.
- $\mathcal{S}_1$ and $\mathcal{S}_2$ must (only) be assumed to be **homogenous with respect to complete off-path causes** of the outcome.

Causal homogeneity

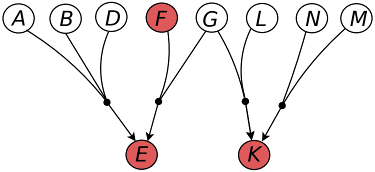
Complete off-path cause (for one test factor)

A *complete off-path* cause of an outcome E relative to an exogenous (test) factor F is a minimally sufficient condition of E that is located on a causal path to E on which F is not located.

Causal homogeneity (for two test situations)

Two test situations $\mathcal{S}_1$ and $\mathcal{S}_2$ are causally *homogeneous* relative to an outcome E and a (test) factor F iff $\mathcal{S}_1$ and $\mathcal{S}_2$ agree with respect to instantiations of complete off-path causes of E relative to F.

Causal homogeneity



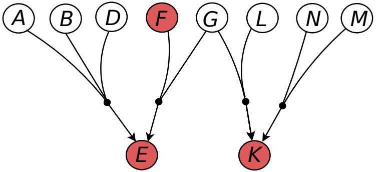
- Causal homogeneity **excludes causal errors**.
- In the above structure, there cannot exist two causally homogenous test situations of the following type:

$\mathcal{S}_1$	$\mathcal{S}_2$
F=1	F=0
K=1	K=0

Justifying causal homogeneity?

- How is it possible to establish (be certain about) causal homogeneity?
- The short answer is: **it is not possible**.
- Still, there are heuristics to render homogeneity plausible: in small-n studies via **familiarity with the cases**; in large-n studies via **randomization** or **inclusion of off-path causes** in the analysis.
- Every procedure of causal inference needs causal background assumptions.
- Background assumptions **guarantee the error-freeness** of a method's inferences, but violations of background assumptions do not automatically yield causal errors. Robustness analyses or inference tests can counterbalance homogeneity violations.

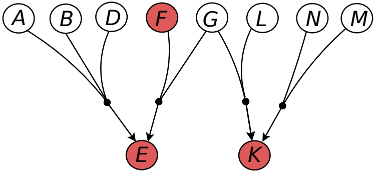
Possible difference test results



- A simple difference test can generate 4 types of results:

type 1		type 2		type 3		type 4	
$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_2$
$F=1$	$F=0$	$F=1$	$F=0$	$F=1$	$F=0$	$F=1$	$F=0$
$E=1$	$E=0$	$E=0$	$E=1$	$E=1$	$E=1$	$E=0$	$E=0$

Possible difference test results



- A simple difference test can generate 4 types of results:

type 1		type 2		type 3		type 4	
$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_1$	$\mathcal{S}_2$
$F{=}1$	$F{=}0$	$F{=}1$	$F{=}0$	$F{=}1$	$F{=}0$	$F{=}1$	$F{=}0$
$E{=}1$	$E{=}0$	$E{=}0$	$E{=}1$	$E{=}1$	$E{=}1$	$E{=}0$	$E{=}0$

➙ A type 1 result entails that F is a cause of E.

➙ A type 2 result entails that f ($\neg F$) is a cause of E.

➙ Type 3 and type 4 results do not entail anything, in particular, not causal irrelevance! Causal irrelevance is very difficult to establish.

Generalizing the difference test: the 4-field test

- A difference test allows for an inference to a gappy MINUS-formula

$$F{*}X_1 + Z_1 \leftrightarrow E \quad (1)$$

- To properly locate further factors in this rudimentary model, the design of simple difference tests must be generalized.

- A **4-field test** locates a second causal factor A in (1):

$4f_1$	F=1	F=0
A=1	E=1	E=0
A=0	E=0	E=0

- What inference is warranted by this 4f-test result?

Generalizing the difference test: the 4-field test

- A difference test allows for an inference to a gappy MINUS-formula

$$F{*}X_1 + Z_1 \leftrightarrow E \tag{1}$$

- To properly locate further factors in this rudimentary model, the design of simple difference tests must be generalized.

- A **4-field test** locates a second causal factor A in (1):

$4f_1$	F=1	F=0
A=1	E=1	E=0
A=0	E=0	E=0

- What inference is warranted by this 4f-test result?

$$A{*}F{*}X_1' + Z_1 \leftrightarrow E \tag{2}$$

Further examples of 4-field tests

Example 1

$4f_2$	F=1	F=0
A=1	E=1	E=1
A=0	E=1	E=0

Further examples of 4-field tests

Example 1

$4f_2$	F=1	F=0
A=1	E=1	E=1
A=0	E=1	E=0

$$F*X_1 \;+\; A*X_2 \;+\; Z_1 \;\leftrightarrow\; E \tag{3}$$

Further examples of 4-field tests

Example 1

$4f_2$	F=1	F=0
A=1	E=1	E=1
A=0	E=1	E=0

$$F{*}X_1 + A{*}X_2 + Z_1 \leftrightarrow E \qquad (3)$$

Example 2

$4f_3$	F=1	F=0
A=1	E=1	E=0
A=0	E=1	E=1

Further examples of 4-field tests

Example 1

$4f_2$	F=1	F=0
A=1	E=1	E=1
A=0	E=1	E=0

$$F * X_1 \;+\; A * X_2 \;+\; Z_1 \;\leftrightarrow\; E \tag{3}$$

Example 2

$4f_3$	F=1	F=0
A=1	E=1	E=0
A=0	E=1	E=1

$$F * X_1 \;+\; a * X_2 \;+\; Z_1 \;\leftrightarrow\; E \tag{4}$$

Further examples of 4-field tests

Example 1

$4f_2$	F=1	F=0
A=1	E=1	E=1
A=0	E=1	E=0

$$F{*}X_1 + A{*}X_2 + Z_1 \leftrightarrow E \tag{3}$$

Example 2

$4f_3$	F=1	F=0
A=1	E=1	E=0
A=0	E=1	E=1

$$F{*}X_1 + a{*}X_2 + Z_1 \leftrightarrow E \tag{4}$$

Example 3

$4f_4$	F=1	F=0
A=1	E=1	E=0
A=0	E=0	E=1

Further examples of 4-field tests

Example 1

$4f_2$	F=1	F=0
A=1	E=1	E=1
A=0	E=1	E=0

$$F{*}X_1 + A{*}X_2 + Z_1 \leftrightarrow E \tag{3}$$

Example 2

$4f_3$	F=1	F=0
A=1	E=1	E=0
A=0	E=1	E=1

$$F{*}X_1 + a{*}X_2 + Z_1 \leftrightarrow E \tag{4}$$

Example 3

$4f_4$	F=1	F=0
A=1	E=1	E=0
A=0	E=0	E=1

$$A{*}F{*}X_1 + a{*}f{*}X_2 + Z_1 \leftrightarrow E \tag{5}$$

Causal homogeneity generalized

Causal homogeneity can be generalized for an open number of test factors and situations:

Causal homogeneity

Test situations $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_i$ that are compared in order to investigate the causal structure behind the behavior of an outcome E relative to a set of exogenous (test) factors $\boldsymbol{F} = \{F_1, \ldots, F_n\}$ are causally *homogeneous* iff $\mathcal{S}_1$ to $\mathcal{S}_i$ agree with respect to instantiations of complete off-path causes of E relative to $\boldsymbol{F}$.

Generalizing further

- The application of the basic methodological idea behind the difference test is not restricted to the laboratory.
- As long as homogeneity of the uncontrolled causal background can be rendered plausible, value configurations of analyzed factors can simply be recorded from **observed cases/units**.
- In order to not only register the configurations but also the number of cases featuring each configuration, observational data are typically not recorded in the form of cross-tables but in the form of **lists of configurations**.

Configurational data

	F	A	E
a	1	1	1
b	1	1	1
c	0	0	1
d	0	0	1
e	0	1	0
f	0	1	0
g	1	0	0
h	1	0	0

Configurational data

	F	A	E
a	1	1	1
b	1	1	1
c	0	0	1
d	0	0	1
e	0	1	0
f	0	1	0
g	1	0	0
h	1	0	0

What MINUS-formula (causal model) follows from this table?

Configurational data

	F	A	E
a	1	1	1
b	1	1	1
c	0	0	1
d	0	0	1
e	0	1	0
f	0	1	0
g	1	0	0
h	1	0	0

What MINUS-formula (causal model) follows from this table?

$$A{*}F{*}X_1 + a{*}f{*}X_2 + Z_1 \leftrightarrow E \tag{6}$$

Or, if we treat model incompleteness implicitly:

$$A{*}F + a{*}f \leftrightarrow E \tag{7}$$

n exogenous factors

- Likewise, nothing in the logic behind the method of difference restricts its applicability to one or two exogenous (test) factors or to one endogenous factor.
- In principle, an open number of exogenous and endogenous factors can be configurationally modeled.
- There are only practical (mostly computational) constraints limiting the size and dimensionality of the data of in real-life discovery contexts.

n exogenous factors

	A	B	D	E
a	1	1	1	1
b	1	1	0	0
c	1	0	1	1
d	1	0	0	1
e	0	1	1	1
f	0	1	0	0
g	0	0	1	1
h	0	0	0	0

	A	B	C	D	E
a	1	1	1	1	1
b	1	1	1	0	0
c	1	1	0	1	1
d	1	1	0	0	1
e	1	0	1	1	1
f	1	0	1	0	0
g	1	0	0	1	1
h	1	0	0	0	0
i	0	1	1	1	0
j	0	1	1	0	0
k	0	1	0	1	1
l	0	1	0	0	1
m	0	0	1	1	0
n	0	0	1	0	0
o	0	0	0	1	0
p	0	0	0	0	0

n exogenous factors

	A	B	D	E
a	1	1	1	1
b	1	1	0	0
c	1	0	1	1
d	1	0	0	1
e	0	1	1	1
f	0	1	0	0
g	0	0	1	1
h	0	0	0	0

$$A{*}b + D \leftrightarrow E$$

	A	B	C	D	E
a	1	1	1	1	1
b	1	1	1	0	0
c	1	1	0	1	1
d	1	1	0	0	1
e	1	0	1	1	1
f	1	0	1	0	0
g	1	0	0	1	1
h	1	0	0	0	0
i	0	1	1	1	0
j	0	1	1	0	0
k	0	1	0	1	1
l	0	1	0	0	1
m	0	0	1	1	0
n	0	0	1	0	0
o	0	0	0	1	0
p	0	0	0	0	0

$$A{*}D + B{*}c \leftrightarrow E$$

Three data types

- Configurational data need not only feature binary factors but may also involve factors with more than two values or values from the interval $[0, 1]$:

	A	B	D	E
a	1	1	1	1
b	1	1	0	0
c	1	0	1	1
d	1	0	0	1
e	0	1	1	1
f	0	1	0	0
g	0	0	1	1
h	0	0	0	0

	A	B	D	E
a	2	1	1	1
b	3	1	3	0
c	1	2	1	1
d	1	0	0	1
e	0	1	2	1
f	0	1	0	0
g	3	0	1	1
h	2	2	2	0

	A	B	D	E
a	0.1	0.2	1	0.8
b	0.4	0.2	0.4	0.5
c	0.2	0.4	1	1
d	0.4	0.4	0.8	1
e	0	0.4	1	0.9
f	0	0.2	0	0.7
g	0.3	0	1	0.4
h	0.9	0.4	0	0

From the method of difference to CNA

- Configurational causal modeling using the Method of Difference quickly reaches computational limits.
- What is needed is an algorithm that mechanically identifies all MINUS-formulas that fit the analyzed data.

➙ This is the problem to be solved by CNA:

The problem

Given data δ, **algorithmically** find all MINUS-formulas that fit δ.

What does data fit mean?

A	B	D	E	$((A$	$*$	$\neg$	$B)$	$+$	$D)$	$\leftrightarrow$	E
1	1	1	1	1			1		1		1
1	1	1	0	1			1		1		0
1	1	0	1	1			1		0		1
1	1	0	0	1			1		0		0
1	0	1	1	1			0		1		1
1	0	1	0	1			0		1		0
1	0	0	1	1			0		0		1
1	0	0	0	1			0		0		0
0	1	1	1	0			1		1		1
0	1	1	0	0			1		1		0
0	1	0	1	0			1		0		1
0	1	0	0	0			1		0		0
0	0	1	1	0			0		1		1
0	0	1	0	0			0		1		0
0	0	0	1	0			0		0		1
0	0	0	0	0			0		0		0

What does data fit mean?

A	B	D	E	$((A$	$*$	$\neg$	$B)$	$+$	$D)$	$\leftrightarrow$	E
1	1	1	1	1		0	1		1		1
1	1	1	0	1		0	1		1		0
1	1	0	1	1		0	1		0		1
1	1	0	0	1		0	1		0		0
1	0	1	1	1		1	0		1		1
1	0	1	0	1		1	0		1		0
1	0	0	1	1		1	0		0		1
1	0	0	0	1		1	0		0		0
0	1	1	1	0		0	1		1		1
0	1	1	0	0		0	1		1		0
0	1	0	1	0		0	1		0		1
0	1	0	0	0		0	1		0		0
0	0	1	1	0		1	0		1		1
0	0	1	0	0		1	0		1		0
0	0	0	1	0		1	0		0		1
0	0	0	0	0		1	0		0		0

What does data fit mean?

A	B	D	E	$((A$	$*$	$\neg$	$B)$	$+$	$D)$	$\leftrightarrow$	E
1	1	1	1	1	0	0	1		1		1
1	1	1	0	1	0	0	1		1		0
1	1	0	1	1	0	0	1		0		1
1	1	0	0	1	0	0	1		0		0
1	0	1	1	1	1	1	0		1		1
1	0	1	0	1	1	1	0		1		0
1	0	0	1	1	1	1	0		0		1
1	0	0	0	1	1	1	0		0		0
0	1	1	1	0	0	0	1		1		1
0	1	1	0	0	0	0	1		1		0
0	1	0	1	0	0	0	1		0		1
0	1	0	0	0	0	0	1		0		0
0	0	1	1	0	0	1	0		1		1
0	0	1	0	0	0	1	0		1		0
0	0	0	1	0	0	1	0		0		1
0	0	0	0	0	0	1	0		0		0

What does data fit mean?

A	B	D	E	$(($A$	$*$	$\neg$	$B)$	$+$	$D)$	$\leftrightarrow$	E
1	1	1	1	1	0	0	1	1	1		1
1	1	1	0	1	0	0	1	1	1		0
1	1	0	1	1	0	0	1	0	0		1
1	1	0	0	1	0	0	1	0	0		0
1	0	1	1	1	1	1	0	1	1		1
1	0	1	0	1	1	1	0	1	1		0
1	0	0	1	1	1	1	0	1	0		1
1	0	0	0	1	1	1	0	1	0		0
0	1	1	1	0	0	0	1	1	1		1
0	1	1	0	0	0	0	1	1	1		0
0	1	0	1	0	0	0	1	0	0		1
0	1	0	0	0	0	0	1	0	0		0
0	0	1	1	0	0	1	0	1	1		1
0	0	1	0	0	0	1	0	1	1		0
0	0	0	1	0	0	1	0	0	0		1
0	0	0	0	0	0	1	0	0	0		0

What does data fit mean?

A	B	D	E	$((A$	$*$	$\neg$	$B)$	$+$	$D)$	$\leftrightarrow$	E
1	1	1	1	1	0	0	1	1	1	1	1
1	1	1	0	1	0	0	1	1	1	0	0
1	1	0	1	1	0	0	1	0	0	0	1
1	1	0	0	1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	0	1	1	1	1
1	0	1	0	1	1	1	0	1	1	0	0
1	0	0	1	1	1	1	0	1	0	1	1
1	0	0	0	1	1	1	0	1	0	0	0
0	1	1	1	0	0	0	1	1	1	1	1
0	1	1	0	0	0	0	1	1	1	0	0
0	1	0	1	0	0	0	1	0	0	0	1
0	1	0	0	0	0	0	1	0	0	1	0
0	0	1	1	0	0	1	0	1	1	1	1
0	0	1	0	0	0	1	0	1	1	0	0
0	0	0	1	0	0	1	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	1	0

What does data fit mean?

A	B	D	E	$((A$	$*$	$\neg$	$B)$	$+$	$D)$	$\leftrightarrow$	E
1	1	1	1	1	0	0	1	1	1	1	1
1	1	1	0	1	0	0	1	1	1	0	0
1	1	0	1	1	0	0	1	0	0	0	1
1	1	0	0	1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	0	1	1	1	1
1	0	1	0	1	1	1	0	1	1	0	0
1	0	0	1	1	1	1	0	1	0	1	1
1	0	0	0	1	1	1	0	1	0	0	0
0	1	1	1	0	0	0	1	1	1	1	1
0	1	1	0	0	0	0	1	1	1	0	0
0	1	0	1	0	0	0	1	0	0	0	1
0	1	0	0	0	0	0	1	0	0	1	0
0	0	1	1	0	0	1	0	1	1	1	1
0	0	1	0	0	0	1	0	1	1	0	0
0	0	0	1	0	0	1	0	0	0	0	1
0	0	0	0	0	0	1	0	0	0	1	0

	A	B	D	E
a	1	1	1	1
b	1	1	0	0
c	1	0	1	1
d	1	0	0	1
e	0	1	1	1
f	0	1	0	0
g	0	0	1	1
h	0	0	0	0

What does data fit mean?

Perfect data fit

A MINUS-formula Ψ fits data δ perfectly iff Ψ is true in exactly those cases recorded in δ.

Non-perfect data fit

A MINUS-formula Ψ fits a data set δ to degree χ iff the configurations in which Ψ is true and the cases recorded in δ overlap to degree χ.

Difference-making pairs

There is tight connection between causation and difference-making: for every factor value in a MINUS-formula there exists a **difference-making pair** of rows in ideal data.

	A	B	D	E
a	1	1	1	1
b	1	1	0	0
c	1	0	1	1
d	1	0	0	1
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$$A * b + D \leftrightarrow E$$

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$$A*b + D \leftrightarrow E$$

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