#2

$$\sqrt{\phi} = \lambda$$

1

$$\frac{d\lambda}{d\phi} = \frac{1}{2\sqrt{\phi}} = h'(\phi) \Rightarrow \frac{d\phi}{d\lambda} = \frac{1}{h'(\phi)}$$

$$\Rightarrow p_i = \frac{-h''(\phi_i)}{\{h'(\phi_i)\}^2} = -4\phi_i h''(\phi_i)$$

$$(i = 1, 2, 3)$$

$$Z = \begin{bmatrix} 1 & -1 \\ \vdots & \vdots \\ 1 & -1 \\ 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix} \quad P = \operatorname{diag}\{p_1, \ldots, p_1, p_2, \ldots, p_2, p_3, \ldots, p_3\}$$

$$Z^T P Z = m \begin{bmatrix} p_1+p_2+p_3 & p_3-p_1 \\ p_3-p_1 & p_1+p_3 \end{bmatrix}$$

$$\theta = \begin{pmatrix} \alpha \\ \Delta \end{pmatrix}$$

$$\mathrm{Var}(\hat{\theta}) = (Z^T P Z)^{-1}$$

$$= \frac{1}{m} \begin{bmatrix} p_1+p_2+p_3 & p_3-p_1 \\ p_3-p_1 & p_1+p_3 \end{bmatrix}^{-1}$$

$$\det = (p_1+p_2+p_3)(p_1+p_3) - (p_3-p_1)^2$$

$$= \cancel{p_1^2} + p_1p_3 + p_2p_1 + p_2p_3 + p_3p_1 + \cancel{p_3^2}$$

$$- \cancel{p_3^2} + 2p_1p_3 - \cancel{p_1^2} = p_1p_2 + p_2p_3 + 4p_1p_3$$

$$\boxed{\mathrm{Var}(\hat{\Delta}) = \frac{p_1+p_2+p_3}{m\{p_1p_2 + p_2p_3 + 4p_1p_3\}}}$$

3

$$\hat{V}ar_0(\hat{\Delta}) = \frac{3\hat{p}_0}{m6\hat{p}_0^2} = \frac{1}{2m\hat{p}_0}$$

$$U_\Delta = Z_\Delta^T H_\gamma^{-1}(t - \mu)$$

$$Z_\Delta^T = (-1 \ldots -1 \;\; 0 \ldots 0 \;\; 1 \ldots 1)$$

$$H_\gamma^{-1} = \mathrm{diag}\left\{ \frac{1}{h'(\phi_1)} \ldots \frac{1}{h'(\phi_1)} \; \frac{1}{h'(\phi_2)} \ldots \frac{1}{h'(\phi_2)} \; \frac{1}{h'(\phi_3)} \ldots \frac{1}{h'(\phi_3)} \right\}$$

$$Z_\Delta^T H_\gamma^{-1} = \left(-2\sqrt{\phi_1} \ldots -2\sqrt{\phi_1} \;\; 0 \ldots 0 \;\; 2\sqrt{\phi_3} \ldots 2\sqrt{\phi_3} \right)$$

$$U_\Delta = -2\sqrt{\phi_1} \sum_{j=1}^{m} \left(t_{j1} + d'(\phi_1) \right) + 2\sqrt{\phi_3} \sum_{j=1}^{m} \left(t_{j3} + d'(\phi_3) \right)$$

$$\hat{U}^0_\Delta = 2\hat{\phi}^0 m \left(\hat{f}^0_3 - \hat{f}^0_1\right)$$

$$\xi_{SR} = \left[\hat{U}^0_\Delta\right]^2 \hat{V}ar_0(\hat{\Delta})$$

$$= \frac{4\hat{\phi}^0 m^2 \left(\hat{f}^0_3 - \hat{f}^0_1\right)}{2\hat{p}_0}$$

$$\hat{p}_0 = -8\hat{\phi}^0 f''(\hat{\phi}^0)\, m$$

Sob $H_0 \Rightarrow \hat{\mu}^0 = y$

$$\xi_{SR} = -\frac{m\left(\hat{f}^0_3 - \hat{f}^0_1\right)^2}{2f''(\hat{\phi}^0)} \underset{m\to\infty}{\overset{H_0}{\sim}} \chi^2_1$$