#1) $Y_i \overset{iid}{\sim} NI(\mu, \phi)$

$H_0: \phi = 1$

$H_1: \phi \neq 1 \quad (i=1,\dots,n)$

$$f(y;\mu,\phi) = \sqrt{\frac{\phi}{2\pi y^3}} \exp\left\{-\frac{\phi(y-\mu)^2}{2\mu^2 y}\right\}$$

$$L(\mu,\phi) = \frac{n}{2}\log(\phi) - \sum_{j=1}^{n}\log(2\pi y^3)$$

$$-\frac{\phi}{2\mu}\sum_{j=1}^{n}(y_j-\mu)^2/y_j$$

$$\boxed{\hat{\mu} = \hat{\mu}^0 = \bar{y}}$$

$$\xi_{RV} = 2\{L(\hat{\mu},\hat{\phi}) - L(\hat{\mu},1)\}$$

$$= m\log(\hat{\phi}) - \frac{\hat{\phi}}{\bar{y}^2}\sum_{j=1}^{m}\frac{(y_j-\bar{y})^2}{y_j} + \frac{1}{\bar{y}^2}\sum_{i=1}^{m}\frac{(y_i-\bar{y})^2}{y_i}$$

$$\hat{\phi} = \frac{m}{D(y;\hat{\mu})}$$

$$D(y;\hat{\mu}) = \sum_{j=1}^{m}\frac{(y_j-\bar{y})^2}{y_j\bar{y}^2}$$

$$\Rightarrow \frac{m}{\hat{\phi}} = \sum_{j=1}^{m}\frac{(y_j-\bar{y})^2}{y_j\bar{y}}$$

Então

$$\xi_{RV} = m\log(\hat{\phi}) - \frac{\hat{\phi}\, m}{\hat{\phi}} + \frac{m}{\hat{\phi}}$$

$$= n\log(\hat{\phi}) - n + \frac{n}{\hat{\phi}}$$

$$\xi_{RV} = n(\hat{\phi}^{-1} - 1) + n\log(\hat{\phi}) \underset{n\to\infty}{\overset{H_0}{\sim}} \chi^2_1$$