Dynamic Strategies for Asset Allocation

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André F. Perold and William F. Sharpe

Most portfolios contain risky assets. Fluctuations in the values of such assets will generally cause the value of the portfolio in which they are held to change. The asset allocation of the portfolio will also change. If the risky assets increase in value, for example, the proportion of the portfolio they comprise is also likely to increase. One must decide how to rebalance the portfolio in response to such changes. *Dynamic strategies* are explicit rules for doing so.

This article examines and compares four dynamic strategies:

- buy-and-hold;
- constant mix;
- constant-proportion portfolio insurance; and
- option-based portfolio insurance.

Buy-and-hold and *constant-mix* strategies are perhaps the most familiar of the four. *Option-based portfolio insurance* strategies replicate positions that can, in principle, be obtained with options. These were the strategies first used to implement portfolio insurance programs; their popularity has, in turn, attracted much attention to the general area of dynamic strategies. *Constant-proportion portfolio insurance* is much simpler to implement than option-based portfolio insurance.[1] It is, basically, a special case of a more general set of policies (constant-proportion strategies) that also embraces the constant-mix and buy-and-hold strategies as special cases.

Different rules have different consequences in both the long term and short term. A rule preferred by one type of investor may not be preferred by another. For each strategy, we show how the portfolio performs in bull, bear and flat markets and in volatile and not-so-volatile markets. We discuss what risk tolerance an investor must have in order for a particular strategy to be the appropriate choice. To emphasize fundamentals, we focus on a choice between only two assets—stocks and bills. The concepts, however, are readily generalized to other asset classes.[2]

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PAYOFF AND EXPOSURE DIAGRAMS

This article makes extensive use of two types of diagrams. A *payoff diagram* for a given strategy relates the portfolio performance over a certain period of time to the performance of the stock market over the same period. An *exposure diagram* relates the dollars invested in stocks to total assets; it depicts the *decision rule* underlying a strategy.

Consider two extreme strategies—100 per cent bills and 100 per cent stocks. Among all possible dynamic strategies with neither borrowing nor short sales of stock, these are the minimum risk and maximum return strategies, respectively. Assume that the current level of the stock market is 100 and that the current value of total assets is $100.

Figure 1 is the payoff diagram for these two strategies. In the minimum risk (100 per cent bills) case, the value of the portfolio is unaffected by the level of the stock market. In the maximum return (100 per cent stocks) case, the value of the portfolio is related dollar for dollar to the level of the market.[3]

Figure 2 is the exposure diagram for the two strategies. In the minimum risk case, there is never any exposure to stocks; the value of stocks held is zero, no matter what the value of the portfolio may be. In the maximum return case there is 100 per cent exposure to stocks; the desired stock position equals the value of the portfolio at all times.

The exposure diagram pertaining to a given strategy also depicts the risk tolerance an investor must have in order for that strategy to be optimal for him or her.[4] In a stock–bill allocation, the optimal amount to invest in stocks is proportional to the investor's risk tolerance. For example, the minimum risk strategy is optimal for an investor who has no tolerance for risk.

BUY-AND-HOLD STRATEGIES

A buy-and-hold strategy is characterized by an initial mix (e.g., 60/40 stocks/bills) that is bought and then held. The minimum risk and maximum

Figure 1. Payoff Diagram for Maximum Return and Minimum Risk Strategies

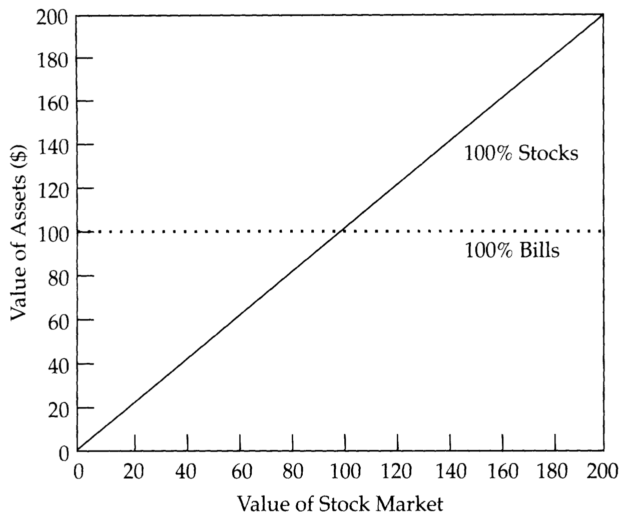
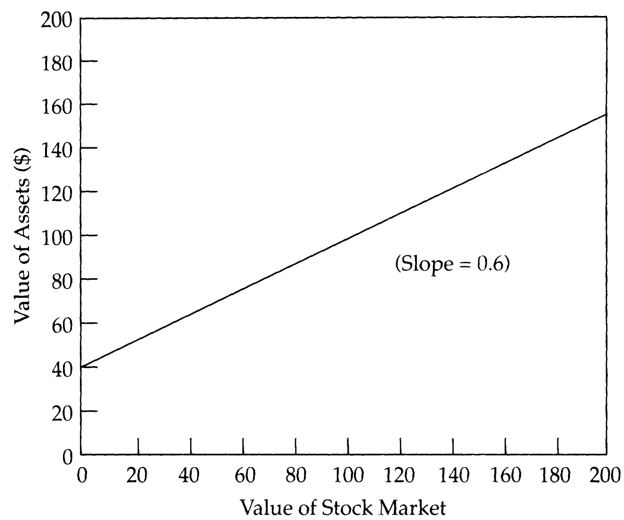


Figure 3. Payoff Diagram for 60/40 Stock/Bill Buy-and-Hold Strategy



return strategies provide examples. Buy-and-hold strategies are "do nothing" strategies: No matter what happens to relative values, no rebalancing is required. Buy-and-hold strategies are easy to analyze. They also serve as anchor points for more complex approaches.

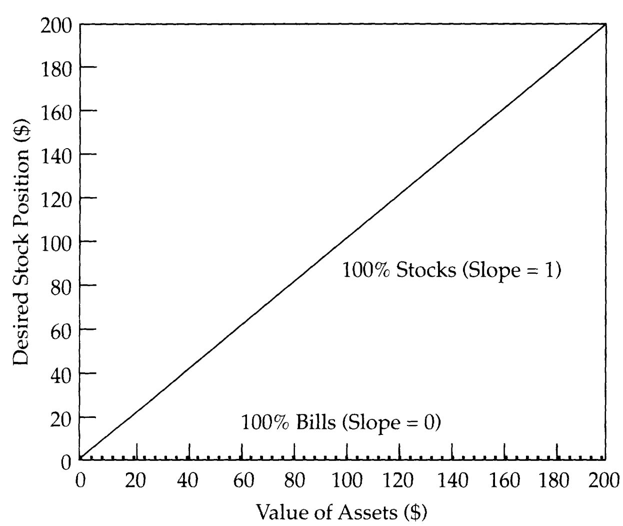
Figure 3 shows the payoff diagram for a buy-and-hold strategy with a 60/40 mix of stocks and bills. As before, we assume that the investor's portfolio is worth $100 and that the current level of the stock market is 100. The figure illustrates some general features of such strategies:

- The portfolio's value is linearly related to that of the stock market.
- Portfolio value increases as a function of stock market value, with a slope equal to the proportion in stocks in the initial mix. In the figure, every dollar of additional stock market value increases the value of the investor's portfolio by 60 cents.
- Portfolio value will never fall below the value of the initial investment in bills.
- Upside potential is unlimited.
- The greater the initial percentage invested in stocks, the better the performance of a buy-and-hold strategy when stocks outperform bills and the worse the performance of a buy-and-hold strategy when stocks underperform bills.

The payoff diagrams of other buy-and-hold strategies will differ from Figure 3 only in terms of the intercept (the point at which the line hits the vertical axis) and the slope.

Figure 4 shows the exposure diagram for a 60/40 buy-and-hold strategy. The investor's tolerance for risk becomes zero at asset levels below 60 per cent of initial wealth. The exposure diagrams of other buy-and-hold strategies all have a slope of

Figure 2. Exposure Diagram for Maximum Return and Minimum Risk Strategies



one and differ only in terms of the asset level at which the investor's tolerance for risk (hence exposure to stocks) becomes zero.

Figure 4. Exposure Diagram for 60/40 Stock/Bill Buy-and-Hold Strategy

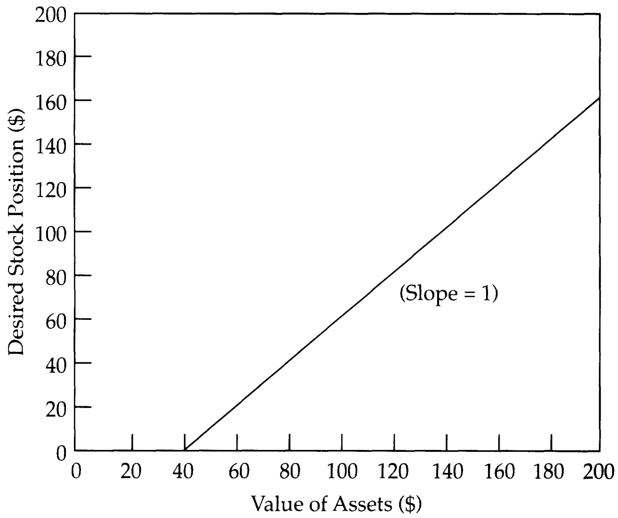
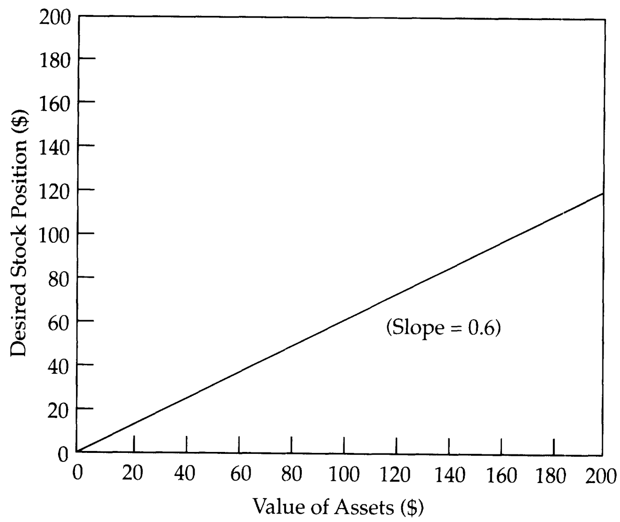


Figure 5. Exposure Diagram for 60/40 Stock/Bill Constant-Mix Strategy



CONSTANT-MIX STRATEGIES

Constant-mix strategies maintain an exposure to stocks that is a constant proportion of wealth. Figure 5 shows the exposure diagram for a 60/40 constant-mix policy. Investors who like constant-mix strategies have tolerances for risk that vary proportionately with their wealth. They will hold stocks at all wealth levels.

Constant-mix strategies are dynamic ("do something") approaches to investment decision-making. Whenever the relative values of assets change, purchases and sales are required to return to the desired mix.

Consider an investor who has put $60 in stocks and $40 in bills and wishes to maintain a 60/40 constant mix. Now assume that the stock market declines by 10 per cent (from 100 to 90). The investor's stocks are now worth $54, giving a total portfolio value of $94. At this point, the stock proportion is $54/$94, or 57.4 per cent—well below the desired 60 per cent level. To achieve the desired level, the portfolio must have 60 per cent of $94, or $56.40, in stocks. Thus the investor must purchase $2.40 ($56.40 − $54.00) of stocks, obtaining the money by selling a comparable amount of bills. Table 1 outlines the steps involved.

In general, rebalancing to a constant mix requires *the purchase of stocks as they fall in value*, where, strictly speaking, changes in value are measured in relative terms.[5]

Implementation of any dynamic strategy requires a rule concerning the conditions under which rebalancing will actually be undertaken. Typical approaches avoid transactions until either the value of the portfolio or a portion of it (e.g., stocks) has changed by at least a given percentage. For purposes of illustration, we will assume that rebalancing occurs whenever the stock market changes by 10 points (as in the preceding example).

Table 2 shows what would happen if stocks fell from 100 to 90, then from 90 to 80, and so on until they became worthless.[6] Table 3 illustrates the more pleasant case in which stocks rise from 100 to 110, then to 120, and so. In this example, and in general, rebalancing to a constant mix requires *the sale of stocks as they rise in value.*

Figure 6 uses the results from these two examples to produce a payoff diagram. For comparison, the line showing results for a 60/40 buy-and-hold strategy is also shown. In this case, the buy-and-hold strategy clearly dominates the constant-mix strategy. Whether the stock market goes up or down, the buy-and-hold investor has more money than his constant-mix companion!

Why, then, would anyone want to adopt a constant-mix strategy? To find the answer, we must consider other ways in which the stock market might move.

Table 1. Rebalancing to a Constant Mix

Case	Stock Market	Value of Stock	Value of Bills	Value of Assets	Percentage in Stocks
Initial	100	60.00	40.00	100.00	60.0%
After change	90	54.00	40.00	94.00	57.4
After rebalancing	90	56.40	37.60	94.00	60.0

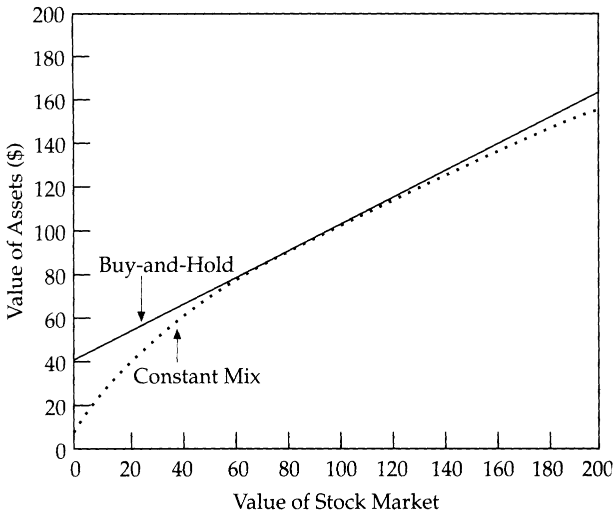
Table 2. Rebalancing to a Constant Mix when Stock Value Falls

Case	Stock Market	Value of Stock	Value of Bills	Value of Assets	Percentage in Stocks
Initial	100	60.00	40.00	100.00	60.0%
After change	90	54.00	40.00	94.00	57.4
After rebalancing	90	56.40	37.60	94.00	60.0
After change	80	50.13	37.60	87.73	57.1
After rebalancing	80	52.64	35.09	87.73	60.0
After change	70	46.06	35.09	81.15	56.8
After rebalancing	70	48.69	32.46	81.15	60.0
After change	60	41.74	32.46	74.20	56.3
After rebalancing	60	44.52	29.68	74.20	60.0
After change	50	37.10	29.68	66.78	55.6
After rebalancing	50	40.07	26.71	66.78	60.0
After change	40	32.05	26.71	58.76	54.5
After rebalancing	40	35.26	23.51	58.76	60.0
After change	30	26.44	23.51	49.95	52.9
After rebalancing	30	29.97	19.98	49.95	60.0
After change	20	19.98	19.98	39.96	50.0
After rebalancing	20	23.98	15.98	39.96	60.0
After change	10	11.99	15.98	27.97	42.9
After rebalancing	10	16.78	11.19	27.97	60.0
After change	0	0.00	11.19	11.19	0.0
After rebalancing	0	6.71	4.48	11.19	60.0

Table 3. Rebalancing to a Constant Mix when Stock Value Rises

Case	Stock Market	Value of Stock	Value of Bills	Value of Assets	Percentage in Stocks
Initial	100	60.00	40.00	100.00	60.0%
After change	110	66.00	40.00	106.00	62.3
After rebalancing	110	63.60	42.40	106.00	60.0
After change	120	69.38	42.40	111.78	62.1
After rebalancing	120	67.07	44.71	111.78	60.0
After change	130	72.66	44.71	117.37	61.9
After rebalancing	130	70.42	46.95	117.37	60.0
After change	140	75.84	46.95	122.79	61.8
After rebalancing	140	73.67	49.12	122.79	60.0
After change	150	78.94	49.12	128.05	61.6
After rebalancing	150	76.83	51.22	128.05	60.0
After change	160	81.95	51.22	133.17	61.5
After rebalancing	160	79.90	53.27	133.17	60.0
After change	170	84.90	53.27	138.17	61.4
After rebalancing	170	82.90	55.27	138.17	60.0
After change	180	87.78	55.27	143.04	61.4
After rebalancing	180	85.83	57.22	143.04	60.0
After change	190	90.59	57.22	147.81	61.3
After rebalancing	190	88.69	59.12	147.81	60.0
After change	200	93.35	59.12	152.48	61.2
After rebalancing	200	91.49	60.99	152.48	60.0

Figure 6. Payoff Diagram for 60/40 Constant-Mix Strategy



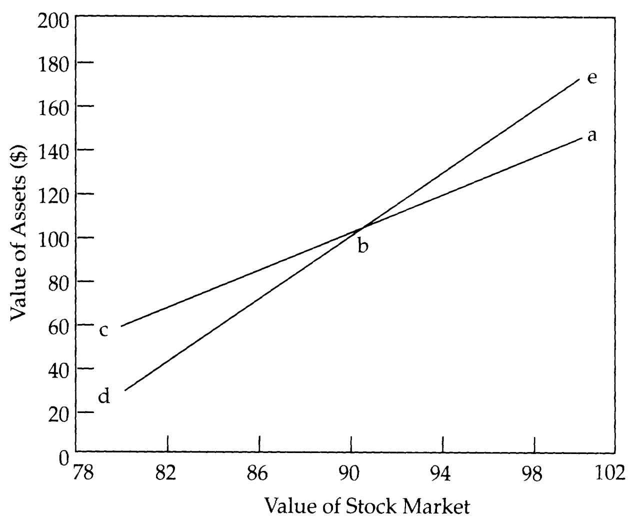
Effects of Volatility

In our previous examples, once the stock market started moving it kept moving in the same direction. In such a world the choice of an investment strategy is simple indeed. But the real world is not so simple; the stock market is perfectly capable of reversing itself. And such reversals favor constant-mix strategies over buy-and-hold approaches.

Consider a case in which stocks fall from 100 to 90, then recover to 100. The market is *flat*, in the sense that it ends up where it started; in between, however, it *oscillates* back and forth. In such a case, someone following a buy-and-hold strategy will end up with exactly the same wealth he had at the beginning. Not so the constant-mix investor. Table 4 gives his results, obtained by following the rules from the previous examples.

The constant-mix investor makes money ($0.27). Figure 7 shows why. When the stock market falls from 100 to 90, the value of the investor's assets falls to $94. In the figure, this is shown by the line from point a to point b. (The number of shares of stock held in the portfolio determines the slope of this line.) For the buy-and-hold investor, further moves in the stock market will have proportionately similar effects. Thus, if the market falls to 80, the buy-and-hold investor's assets will fall to point c; if the market rises back to 100, this investor's assets will rise back to 100 (point a). A buy-and-hold investor simply travels up and down a single straight line in the payoff diagram.

Figure 7. Payoff Diagram for 60/40 Constant-Mix and Buy-and-Hold Strategies



This is not so for the constant-mix investor. Every rebalancing changes the number of shares of stock he holds, hence the slope of the line along which he will next travel in the payoff diagram. After a fall from point a to point b, he purchases more shares of stock. This increases the slope of the line. Thus, in Figure 7, a further fall in the market to 80 will place the constant-mix investor at point d—below that of his buy-and-hold friend. But a subsequent rise in the market to 100 will place the constant-mix investor at point e—above

Table 4. Constant-Mix Results with Market Volatility

Case	Stock Market	Value of Stock	Value of Bills	Value of Assets	Percentage in Stocks
Initial	100	60.00	40.00	100.00	60.0%
After change	90	54.00	40.00	94.00	57.4
After rebalancing	90	56.40	37.60	94.00	60.0
After change	100	62.67	37.60	100.27	62.5
After rebalancing	100	60.16	40.11	100.27	60.0

that of the buy-and-hold investor. And, of course, after each such change, the constant-mix investor will rebalance, creating a new line for the next step of the journey.

Who will win overall? The answer depends on the pattern of market moves.

If the market moves from 100 to 90 and then back to 100, the constant-mix investor will end up ahead. In general, a strategy that buys stocks as they fall and sells as they rise will capitalize on reversals. The marginal purchase decisions will turn out to be good ones, as will the marginal sell decisions. A constant-mix strategy will thus outperform a comparable buy-and-hold strategy in a flat (but oscillating) market precisely because it trades in a way that exploits reversals. Greater volatility (i.e., more and/or larger reversals) will accentuate this effect.

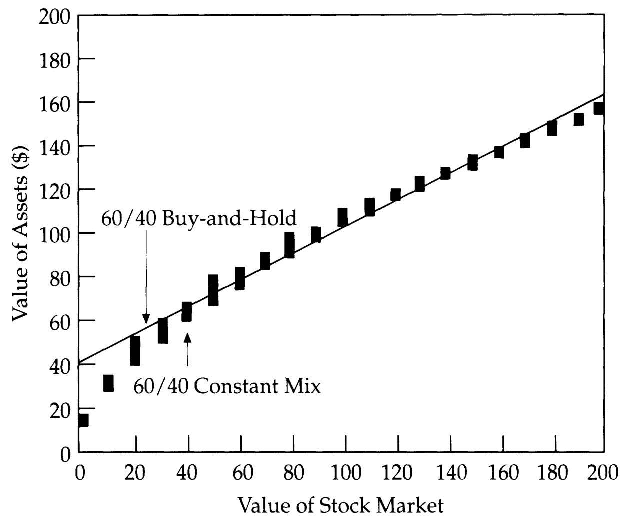
Conversely, if the market moves from 100 to 90 and then to 80, both types of investors will lose, but the buy-and-hold investor will lose less. In general, a constant-mix approach will underperform a comparable buy-and-hold strategy when there are no reversals. This will also be the case in strong bull or bear markets, when reversals are small and relatively infrequent, because most of the marginal purchase and sell decisions will turn out to have been poorly timed.

The value of a constant-mix investor's assets after several rebalancings will depend on both the final level of the stock market and on the manner in which stocks move from period to period before reaching that final level. The relation depicted in the payoff diagram will thus be somewhat fuzzy. Cases in which the market ends up near its starting point are likely to favor constant-mix strategies, while those in which the market ends up far from its starting point are likely to favor buy-and-hold strategies.

Figure 8 provides an example. The horizontal axis plots the level of the stock market after a number of decisions (rebalancings) have been undertaken. The vertical axis shows the final value of the investor's assets. The straight line shows the value of a buy-and-hold investor's portfolio. Each of the squares represents one of 2,000 possible outcomes for a constant-mix investor who rebalances after any 10-point move in the stock market.[7]

Here, neither strategy dominates the other. A constant-mix policy tends to be superior if markets are characterized more by reversals than by trends. A buy-and-hold policy tends to be superior if there is a major move in one direction.[8]

Figure 8. Payoff Diagram for Constant-Mix and Buy-and-Hold Strategies



CONSTANT-PROPORTION STRATEGIES

Constant-proportion strategies take the following form:

$$\text{Dollars in Stocks} = m\,(\text{Assets} - \text{Floor}),$$

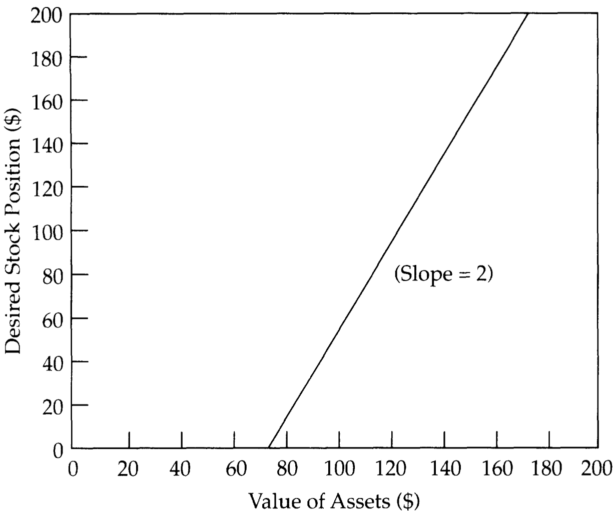
where m is a fixed multiplier. Constant-proportion portfolio insurance (CPPI) strategies are constant-proportion strategies with multipliers greater than one.[9]

To implement a CPPI strategy, the investor selects the multiplier and a floor below which he does not want the portfolio value to fall. This floor grows at the rate of return on bills and must initially be less than total assets. If we think of the difference between assets and the floor as a "cushion," then the CPPI decision rule is simply to keep the exposure to equities a constant multiple of the cushion.

Figure 9 shows the exposure diagram for a CPPI strategy with a floor of $75 and a multiplier of two. As with buy-and-hold strategies, investors who like CPPI strategies have zero tolerance for risk (hence no exposure to stocks) below a specified floor. However, with CPPI, tolerance for risk increases more quickly above the floor than with buy-and-hold strategies.

Exposure diagrams for CPPI strategies are similar to those for buy-and-hold strategies. This is not surprising: Buy-and-hold strategies are constant-proportion strategies with a multiplier of one and a floor equal to the value invested in bills. Constant-mix strategies also represent special

Figure 9. Exposure Diagram for CPPI Strategy



Note: Floor equals $75; multiplier equals two.

cases of the constant-proportion formula. They have floors of zero and multipliers with values between zero and one. For a constant-mix strategy, the multiplier corresponds to the percentage invested in stocks.

For the payoff diagram for a CPPI strategy, we assume $100 of wealth, a floor of $75 and a multiplier ($m$) of two. Because the initial cushion is $25, the initial investment in stocks must be twice this, or $50. The initial mix is thus 50/50 stocks/bills.

Now imagine that the stock market falls from 100 to 90. The investor's stocks will fall 10 per cent, from $50 to $45. Total assets will then be $95, and the cushion will equal $20 ($95 − $75). According to the CPPI rule, the appropriate stock position is $40 (2 × $20). This requires the sale of $5 of stocks and investment of the proceeds in bills. If stocks fall further, more should be sold. If they increase in value, stocks should be bought. And so on.

From this analysis, we see that a CPPI strategy *sells stocks as they fall and buys stocks as they rise.*

Under a CPPI strategy, the portfolio will do at least as well as the floor, even in a severe bear market. Such a strategy puts more and more into bills as stocks decline, reducing the exposure to stocks to zero as the assets approach the floor. The only scenario in which the portfolio might do worse than the floor is if the market drops precipitously before one has had the chance to rebalance. Just how precipitous the decline must be depends on the multiplier. With a multiplier of two, the market can fall by as much as 50 per cent with no rebalancing before the floor is endangered. More generally, the market can fall by as much as $1/m$ with no rebalancing before the floor is endangered.

In a bull market, the CPPI strategy will do very well. It calls for buying stocks as they rise, with each marginal purchase paying off handsomely. In a flat market, a CPPI strategy will do relatively poorly, owing to the same phenomenon that makes constant-mix strategies perform so well—reversals. Reversals hurt CPPI strategies because they sell on weakness only to see the market rebound, and buy on strength only to see the market weaken.

Figure 10 illustrates these principles. The horizontal axis plots the level of the stock market after a number of decisions (rebalancings) have been undertaken. The vertical axis shows the final value of the investor's assets. Each of the squares represents one of 2,000 possible outcomes for a CPPI investor (with a floor of $75 and a multiplier of two) who rebalances after any 10-point move in the stock market.[10] (The appendix contains an exact formula for the payoff when rebalancing is continuous and costless.)

Figure 10. Payoff Diagram for CPPI and Two Buy-and-Hold Strategies

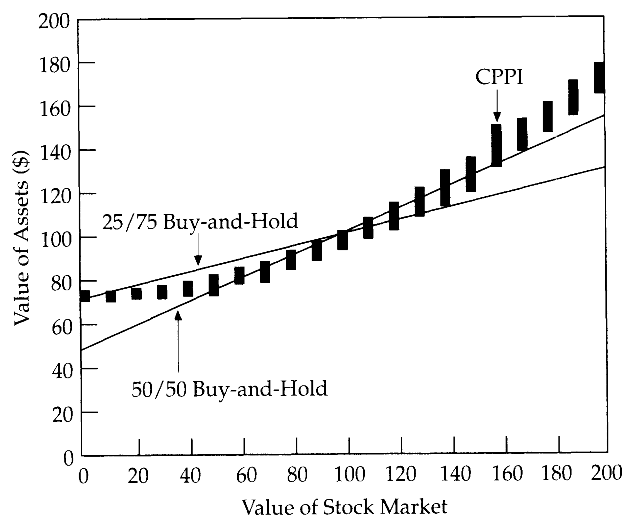


Figure 10 also gives the payoffs from two buy-and-hold strategies. The steeper of the two lines corresponds to a policy with the same initial asset mix (50/50) as used in the CPPI strategy. The second involves investment of $75 in bills and $25 in stocks to insure that the portfolio's value will never fall below the floor of $75.

Not surprisingly, none of the three strategies shown in Figure 10 completely dominates the others. The winner in any contest will be determined by the behavior of the market.

Concave versus Convex Strategies

From our analysis so far, it is apparent that the basic shape of the payoff diagram is not so much dependent on the specific decision rule underlying the strategy as it is on the kind of rebalancing required. We have looked at payoff curves for three kinds of rebalancing:

- do nothing;
- buy stocks as they fall, sell as they rise; and
- sell stocks as they fall, buy as they rise.

"Do nothing" strategies (buy-and-hold) give payoff diagrams that are *straight lines.*

Strategies that "buy stocks as they fall . . ." give rise to *concave* payoff curves (which increase at a decreasing rate as one moves from left to right). That is, they tend not to have much downside protection, and to do relatively poorly in up markets. They generally do very well, however, in flat (but oscillating) markets.

Strategies that "sell stocks as they fall . . ." give rise to *convex* payoff curves (which increase at an increasing rate as one moves from left to right). They tend to do very poorly in flat (but oscillating) markets. But they tend to give good downside protection and to perform well in up markets.

Constant-mix and CPPI strategies are perhaps the simplest examples of concave and convex strategies, respectively.

Strategies giving convex payoff diagrams represent the *purchase* of portfolio insurance, while those giving concave diagrams represent its *sale.*[11] Concave and convex strategies may be seen as mirror images of one another on either side of buy-and-hold strategies. Every "buyer" of a convex strategy is a "seller" of a concave strategy, and vice versa. When the portfolio of one who buys a convex strategy is combined with the portfolio of the seller of that strategy, the result is a buy-and-hold position.

There is a simple and straightforward relationship between the *shape* of a payoff diagram and the *slope* of the exposure diagram (which here corresponds to the multiplier, m).[12] Strategies with slopes less than one give rise to concave payoff diagrams, while strategies with slopes greater than one give rise to convex payoff diagrams.

There are many ways to construct strategies with concave payoff diagrams. Any procedure that "buys stocks as they fall, sells as they rise" will do. And any procedure that "sells stocks as they fall, buys as they rise" will produce a convex payoff diagram.

That convex and concave strategies are mirror images of one another tells us that the more demand there is for one of these strategies, the more costly its implementation will become, and the less healthy it may be for markets generally. If growing numbers of investors switch to convex strategies, then markets will become more volatile, for there will be insufficient buyers in down markets and insufficient sellers in up markets at previously "fair" prices. In this setting, those who follow concave strategies may be handsomely rewarded. Conversely, if growing numbers of investors switch to concave strategies, then the markets may become too stable. Prices may be too slow to adjust to fair economic value. This is the most rewarding environment for those following convex strategies. Generally, whichever strategy is "most popular" will subsidize the performance of the one that is "least popular." Over time, this will likely swell the ranks of investors following the latter and contain the growth of those following the former, driving the market toward a balance of the two.[13]

OPTION-BASED PORTFOLIO INSURANCE

Option-based portfolio insurance (OBPI) strategies begin by specifying an investment *horizon* and a desired floor value at that horizon. While not stated explicitly, OBPI strategies implicitly involve a floor value at every time prior to the horizon. For example, if the horizon is one year and the floor at year-end is $82.50, then the floor at any prior time is the present value of $82.50 discounted using the riskless rate of interest. At a 10 per cent bill rate, the initial floor is $75. The floor value grows at the riskless rate, as it does with CPPI and buy-and-hold strategies.

Once a floor is chosen and its present value calculated, the typical OBPI strategy consists of a set of rules designed to give the same payoff at the horizon as would a portfolio composed of bills and call options. Figure 11 provides an example. The bills have face value equal to the floor (e.g., $82.50). The cushion is invested in the calls. The appendix describes the method for choosing the parameters.

With OBPI, the exposure diagram (hence the decision rule) depends very much on the time

Figure 11. Payoff Diagram for OBPI

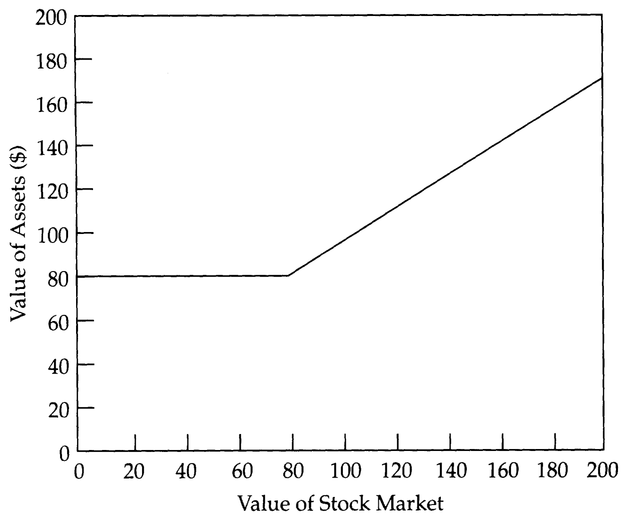
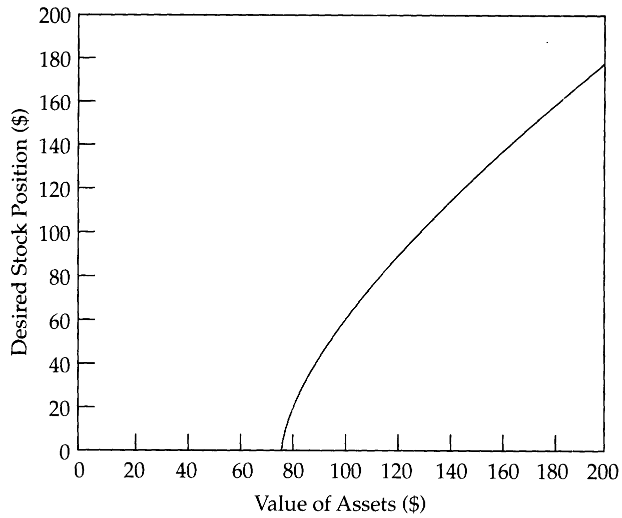


Figure 12. Exposure Diagram for OBPI



remaining before the horizon is reached. One instant prior to the horizon, OBPI involves investing entirely in bills if the assets equal the floor, and entirely in stocks if the assets exceed the floor. With more than just an instant to go before "expiration," the exposure diagram is a curve. Figure 12 shows an example in which there is one year left before the horizon. To draw this curve, one must utilize a relatively complex option pricing formula.[14] Moreover, new curves must be found as time passes.

Note in Figure 12 that the slope of the exposure curve is greater than one at all points. It begins at a value considerably greater than one and falls toward a value of one as the cushion becomes very large. This will be the case at any time prior to the horizon date. OBPI strategies are thus "sell stocks as they fall . . ." strategies. They must thus provide convex payoff diagrams. Over any period ending prior to the horizon, such payoff diagrams will plot as curves. At the horizon, as shown in Figure 11, the diagram plots as two straight lines, but with a shape that is convex overall.

With a traditional OBPI strategy, for any given (positive) cushion, the exposure to stocks increases as time passes, reaching 100 per cent of the asset value at the horizon. Such approaches are thus calendar-time dependent. This contrasts with CPPI strategies, in which the exposure depends only on the size of the cushion.

The calendar-time dependence of OBPI is particularly acute when the strategy "expires" at the horizon, because a new set of rules must then be put in place. For the long-term investor whose true horizon extends beyond the horizon specified in the OBPI strategy, this is a drawback. The asset mix just before expiration (either 0/100 or 100/0) will typically be vastly different from the mix as reset just after expiration. It is difficult to imagine circumstances in which it is sensible to effect dramatic changes in mix merely because one calendar period has ended and another has begun.

Dynamic Strategies with Resetting

When, if ever, should one "reset" the parameters of a dynamic strategy? This answer depends not only on the rationale behind the choice of strategy, but also on the type of dynamic strategy chosen. For example, as just noted, with option-based portfolio insurance, one has no choice but to reset at the horizon.

It is important to be aware that the manner in which one resets the parameters of a dynamic strategy can dramatically alter its basic characteristics. For example, we saw in Figure 12 that, with OBPI, the multiplier is different at different levels of the cushion. Thus OBPI can be considered a variation of the CPPI approach in which the multiplier is changed as the cushion changes.

As a second example, consider the following "rolling" CPPI strategy. Begin as usual with a multiplier and a floor, but then, as total assets fluctuate in value, adjust the floor so that it is always a constant fraction of assets. (Keep the multiplier constant at, say, two.) If we keep the floor at, say, 80 per cent of assets, then it would appear that we will always be assured of losing no more than 10 per cent of current assets. But, by substituting into the CPPI formula the following:

$$\text{Floor} = 0.8 \times \text{Assets},$$

we see that all we have achieved is to transform the CPPI strategy into a constant-mix strategy.[15] As shown earlier, such strategies have no downside protection at all.

A rolling option-based portfolio insurance strategy that involves (1) rolling the horizon forward (e.g., one year) so that it always remains as far away as it was in the beginning, and (2) resetting the end-of-horizon floor so as to keep it in constant proportion to assets is also just a constant-mix strategy in disguise.[16]

One form of resetting that seems to be popular involves beginning with some portfolio insurance strategy (say, CPPI) and sticking to the rules if the market is flat or declines. If there is an appreciable up move, however, the floor is raised in order to "lock in" profits. Thus, the floor is raised if assets increase but left intact otherwise. How "ratcheting up the floor" in this manner alters the basic character of the dynamic strategy will depend very much on exactly how it is implemented. Typically, however, it can cause stocks to be sold in both up and down markets. In down markets, the selling occurs because the floor is being held fixed, thus preserving the portfolio insurance nature of the strategy. In up markets, when the cushion would ordinarily increase and thus give rise to buying, the floor is raised and the cushion thus reduced. If the floor is raised far or fast enough, the net effect will be to reduce the size of the cushion, giving rise to selling. Overall, the resulting strategy provides expected payoffs that are concave to the right and convex to the left.

Resetting can dramatically alter the character of a strategy. Resetting rules should thus be considered an integral part of the dynamic strategy, and their effects explicitly taken into account.

Option-based portfolio insurance strategies, by their nature, require resetting. However, constant-proportion strategies (CPPI, buy-and-hold and constant-mix approaches) can be implemented in perpetuity with no change in the key parameters. For investors with long time horizons, these latter strategies are attractive candidates.

SELECTING A DYNAMIC STRATEGY

Which dynamic strategy is demonstrably the best? The goal of this article is to emphasize that "best" should be measured by the degree of fit between a strategy's exposure diagram and the investor's risk tolerance (expressed as a function of an appropriate cushion).

Ultimately, the issue concerns the preferences of the various parties that will bear the risk and/or enjoy the reward from investment. There is no reason to believe that any particular type of dynamic strategy is best for everyone (and, in fact, only buy-and-hold strategies could be followed by everyone). Financial analysts can help those affected by investment results understand the implications of various strategies, but they cannot and should not choose a strategy without substantial knowledge of the investor's circumstances and desires.

APPENDIX

This appendix gives the formulas for the payoff and exposure diagrams shown in the body of the article. The following notation is used:

- A = total assets at t (payoff)
- A_0 = initial assets
- S = stock market at t (total return index)
- S_0 = initial level of stock market
- F = floor at t
- F_0 = initial floor
- E = desired stock position (exposure)
- r = bill rate

Buy-and-Hold Strategies

Let x be the initial fraction invested in stocks. The initial floor is

$$F_0 = (1 - x)A_0.$$

The floor at time t is

$$F = (1 - x)A_0e^{rt} = F_0e^{rt}.$$

The payoff at t is

$$A = F + x\ A_0S/S_0.$$

The exposure is

$$E = A - F.$$

Constant-Proportion Strategies

Let m be the multiplier. The floor at t is

$$F = F_0 e^{rt}.$$

A constant-mix strategy corresponds to the following special case:

$$F = F_0 = O.$$

The payoff at t is

$$A = F + (A_0 - F_0)\left(\frac{S}{S_0}\right)^m e^{(1-m)(r+0.5m\sigma^2)t}.$$

The exposure is

$$E = m(A - F).$$

When no leverage is allowed, the exposure is limited to total assets, A. In this case,

$$E = min(A,\ m(A - F)),$$

and there is no simple formula for the payoff in terms of the level of the stock market.

Option-Based Portfolio Insurance

Let $B(S,K,r,\sigma,t,T)$ be the Black-Scholes formula for the value at time t of a European call option (on one unit of the market) with strike price K and expiration date T:

$$B = SN(d) - Ke^{-rt}N(d - \sigma t^{0.5}),$$

where

$$d = \frac{\log\left(\frac{S}{K}\right) + (r + 0.5\sigma^2 t)}{\sigma t^{0.5}},$$

and $N \cdot (\forall)$ is the cumulative of the unit normal distribution and σ is the volatility of the market.

OBPI involves investing F_0 in bills and purchasing n call options, where n and K are determined jointly by the following equations:

$$n \cdot B(S,K,r,\sigma,O,T) = A_0 - F_0$$

and

$$n \cdot K = F_T,$$

where F_T is the floor at expiration, and the initial floor is

$$F_0 = F_T e^{-rT}.$$

The first equation indicates that the value of calls purchased equals the initial cushion. The second indicates that the total exercise price equals the floor. Thus, there is always enough money to exercise the calls (the portfolio will never become levered).

At any t between 0 and T the payoff is

$$A = F + n \cdot B(S,K,r,\sigma,t,T),$$

where the floor is

$$F = F_T e^{-r(T-t)}.$$

At expiration

$$A = F_T + n \cdot \max(S - K,O).$$

The exposure is

$$E = n \cdot N(d),$$

where d can be expressed as a function of the cushion $A - F$ by solving for S using

$$B(S,K,r,\sigma,t,T) = (A - F)/n.$$

FOOTNOTES

1. It has been advocated recently in F. Black and R. Jones, "Simplifying Portfolio Insurance," *Journal of Portfolio Management* (Fall 1987), and A.F. Perold, "Constant Proportion Portfolio Insurance," Harvard Business School (August 1986). The basic procedure is rooted in R.C. Merton, "Optimum Consumption and Portfolio Rules in a Continuous Time Model, "*Journal of Economic Theory*, vol. 3 (1971). With respect to the general subject matter of this article, see also M.J. Brennan and R. Solanki, "Optimal Portfolio Insurance," *Journal of Financial and Quantitative Analysis* (September 1981), and H.E. Leland, "Who Should Buy Portfolio Insurance?" *The Journal of Finance*, vol. 35 (1980).
2. See W.F. Sharpe, "Integrated Asset Allocation," *Financial Analysts Journal* (September/October 1987):25–32.
3. For simplicity, all the payoff diagrams ignore the accrual over time of interest on any bills held. The appendix provides payoff curve formulas that include the interest earned on bills.
4. More precisely, it depicts absolute risk tolerance over the next short period of time.

5. Rebalancing is required only if the return on stocks differs from that on bills.
6. In this example, the value of assets does not fall to zero. This is because we rebalance after consecutive absolute market moves (of 10 points). If we were to rebalance after a never-ending stream of percentage declines, the value of assets would approach zero.
7. Each of the points shows the results of a simulation involving 20 periods in which the stock market could move up 10 points, down 10 points, or stay the same. In performing the simulation, each of the three possible moves was considered equally probable.

   When rebalancing occurs continuously and costlessly, an exact formula can be obtained for the payoff under a constant-mix strategy. Details are given in the appendix.
8. No consideration has been given in this analysis to transaction costs, which would lower the points associated with any dynamic strategy.
9. When m is greater than one, the formula may call for investing more than total assets in stocks. If such leverage is not permitted, the CPPI rule becomes

   Dollars in Stocks = Lesser of [m(Assets − Floor), Assets].
10. See footnote 7. In some cases, a market move of greater than 50 per cent (i.e., from 10 to 0) occurred before rebalancing. This explains the few instances in which ending asset value fell below the floor of $75.
11. This is a more general description of portfolio insurance than that given in M. Rubinstein, "Alternative Paths to Portfolio Insurance," *Financial Analysts Journal* (July/August 1985):42–45.
12. The slope of the exposure diagram equals the multiplier (the ratio of dollars invested in stocks to the cushion) only when both are constant.
13. There are, of course, many other types of investors (and issuers of securities) following different implicit and explicit trading rules. More generally, markets must be balanced across all of these.
14. See the one developed in F. Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* (May/June 1973).
15. Dollars in Stocks = m(Assets − Floor)

   = 2(Assets − 0.8 · Assets)

   = 0.4 · Assets

   = 40/60 stock/bill constant mix.
16. This is discussed in S. Benninga and M. Blume, "On the Optimality of Portfolio Insurance," *The Journal of Finance*, vol. 40 (1985):1341–52. Other forms of OBPI preserve the portfolio insurance feature (convexity) of OBPI. For example, by rolling the horizon but keeping the floor at a fixed nominal level, the exposure curve no longer varies over time. It is simply one of the curves drawn in Figure 12 (corresponding to the length of the horizon), frozen for all time.