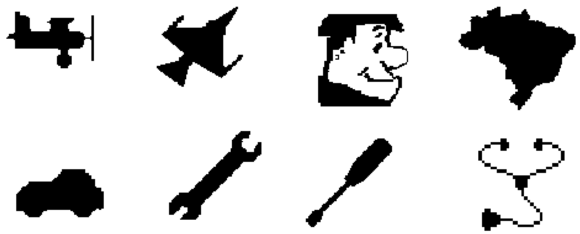
# Shape Analysis and Classification

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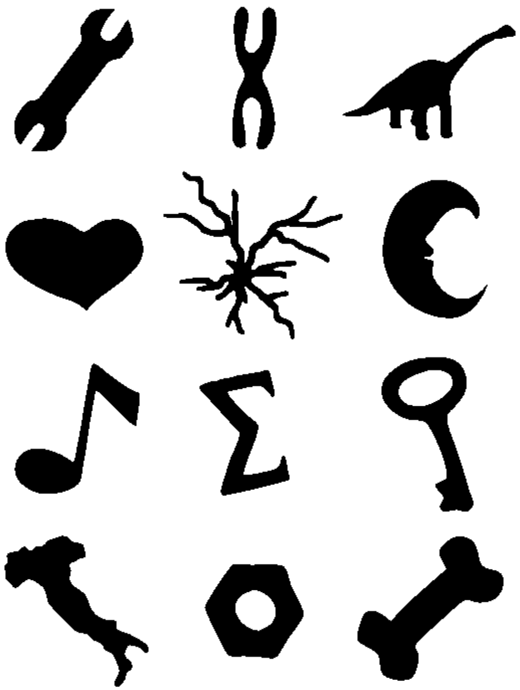
http://www.vision.ime.usp.br/~cesar/shape/

# Shape Analysis and Classification



## SHAPE CHARACTERIZATION

# Shape measures

- Basic idea: extracting measures from shapes
- Useful standard measures:
  - mean (and other statistical moments, such as standard deviation, etc)
  - median, min, max
  - ratios such as max/min, etc.
- Acquisition information may play an important role (e.g. pixel dimensions)

# Shape measures

- Perimeter:
  - number of boundary pixels (approx.)
  - for 8-connected chain-coded boundaries ($N_e$ and $N_o$ denote, respectively, the number of even and odd codes in the chain-coded contour representation):

$$P = N_e + N_o \sqrt{2}$$

# Shape measures

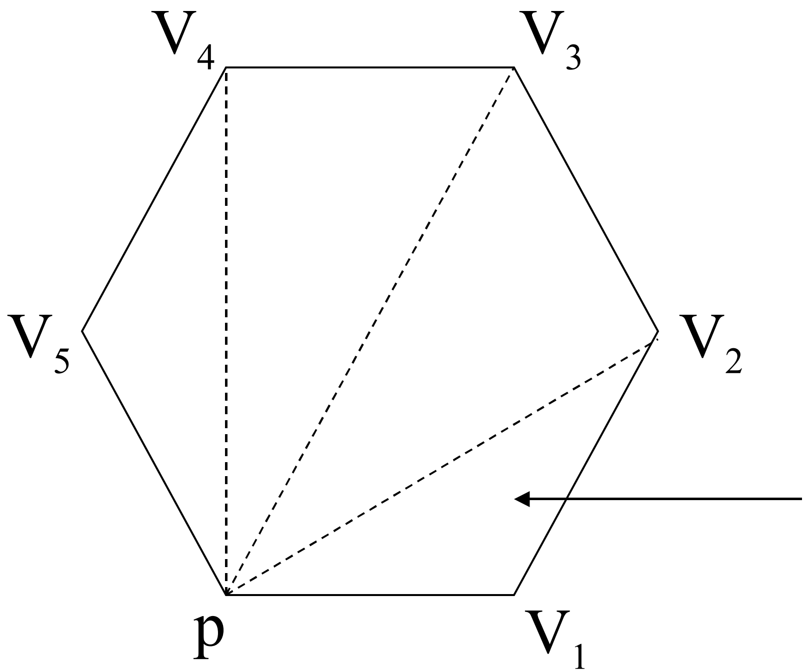
- Perimeter:
  - If the contour is represented as a complex-valued signal $u(n) = x(n) + j\, y(n)$:

$$P = \sum_{n=0}^{N-1} \left| u(n) - u(n-1) \right|$$

# Shape measures

- Area:
  - May be approximated by the number of shape pixels, in case of region-based representations (e.g. by binary images)
  - The area of polygons may be calculated using triangle decomposition and vectorial products

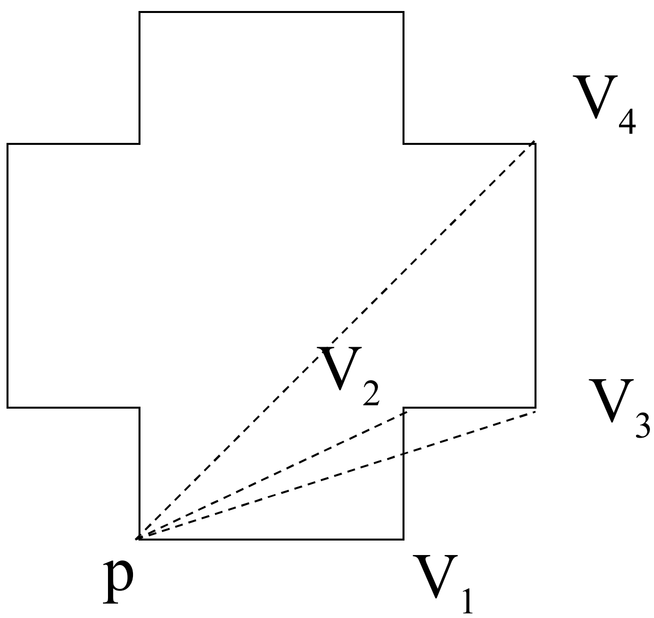
# Shape measures




$$A(V_0 V_1 V_2) = \frac{1}{2} \left| (V_1 - V_0) \times (V_2 - V_0) \right|$$

¿ $(x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0)$

# Shape measures




Signed area compensates for concavities

# Shape measures

**Algorithm:** ***Area-Based Object Sorting***

```
1- Label each connected component in the image;
2- Calculate the histogram of the labeled image;
3- Sort the connected components as a function of the
respective histogram heights, ignoring the background pixels;
```

# Shape measures

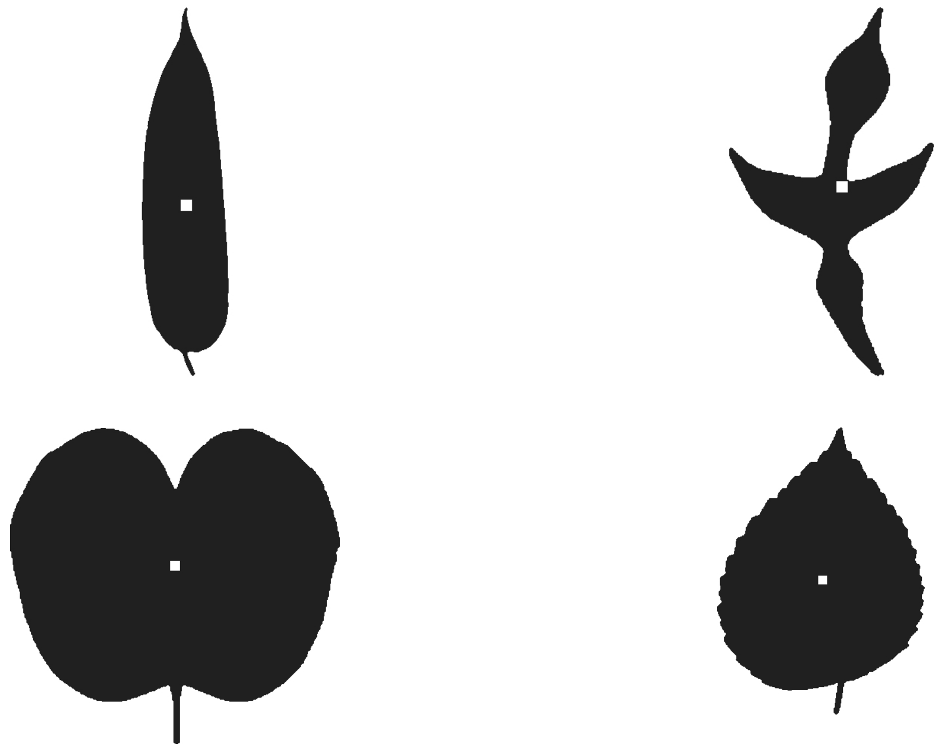
- Centroid:
  - If the contour is represented as a complex-valued signal $u(n) = x(n) + j\,y(n)$:

$$M = \frac{\sum_{n=0}^{N-1} u(n)}{N}$$

  - Region based representations: average value of all pixel coordinates

# Shape measures

Centroid



# Shape measures

- Centroid useful measures:
  - Maximum distance $D_{max}$ between the centroid and the boundary points
  - Minimum distance $D_{min}$ between the centroid and the boundary points
  - Mean distance $D_{mean}$ between the centroid and the boundary points
  - Histogram of the distances between the centroid and the boundary points
  - Ratios: $\frac{D_{max}}{D_{min}}$ $\frac{D_{max}}{D_{mean}}$ $\frac{D_{min}}{D_{mean}}$

# Shape measures

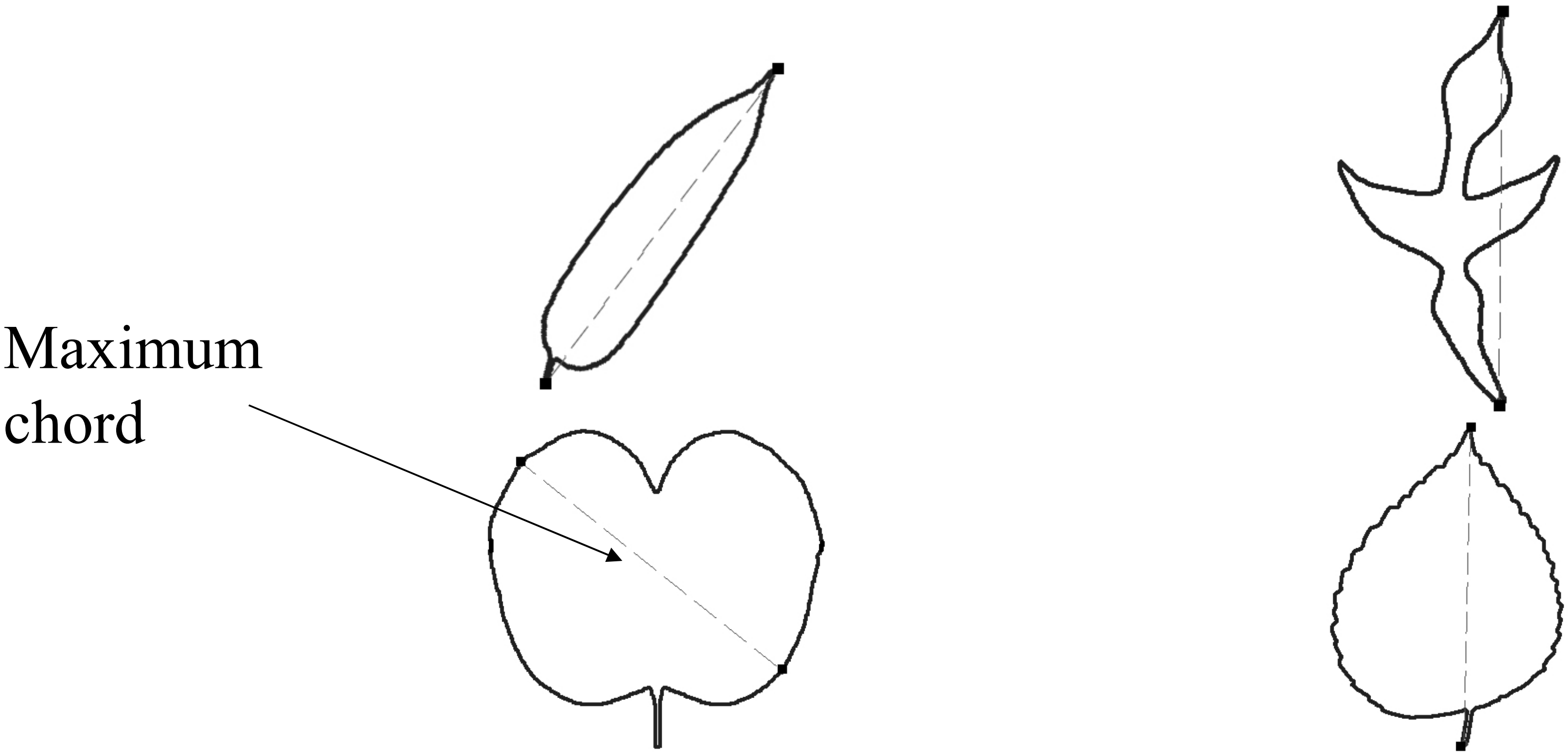
- Mean distance to the boundary:

$$\beta = \frac{1}{N} \sum d\left(r, boundary\left(g\right)\right)$$

- May be calculated from the distance transform
- Derived complexity measure: $$f = \frac{A}{\beta^2}$$

# Shape measures

- Diameter



# Shape measures

- ✓Norm features
  - ✓`2*n* Euclidean norm
  - ✓RMS size
  - ✓Mean size
  - ✓Centroid size
  - ✓Normalized centroid size
  - ✓Baseline distance
  - ✓Landmark-based shape diameter

# Shape measures

- The Karhunen-Loève Transform
- Let $X$ be a random vector with covariance matrix $K$.
- Let $v_i$, ($i = 1, 2, \ldots, N$) be the eigenvectors of $K$, represented in terms of the following matrix:

$$OMEGA = \begin{bmatrix} \leftarrow \vec{v}_1 \rightarrow \\ \leftarrow \vec{v}_2 \rightarrow \\ \vdots \\ \leftarrow \vec{v}_N \rightarrow \end{bmatrix}$$

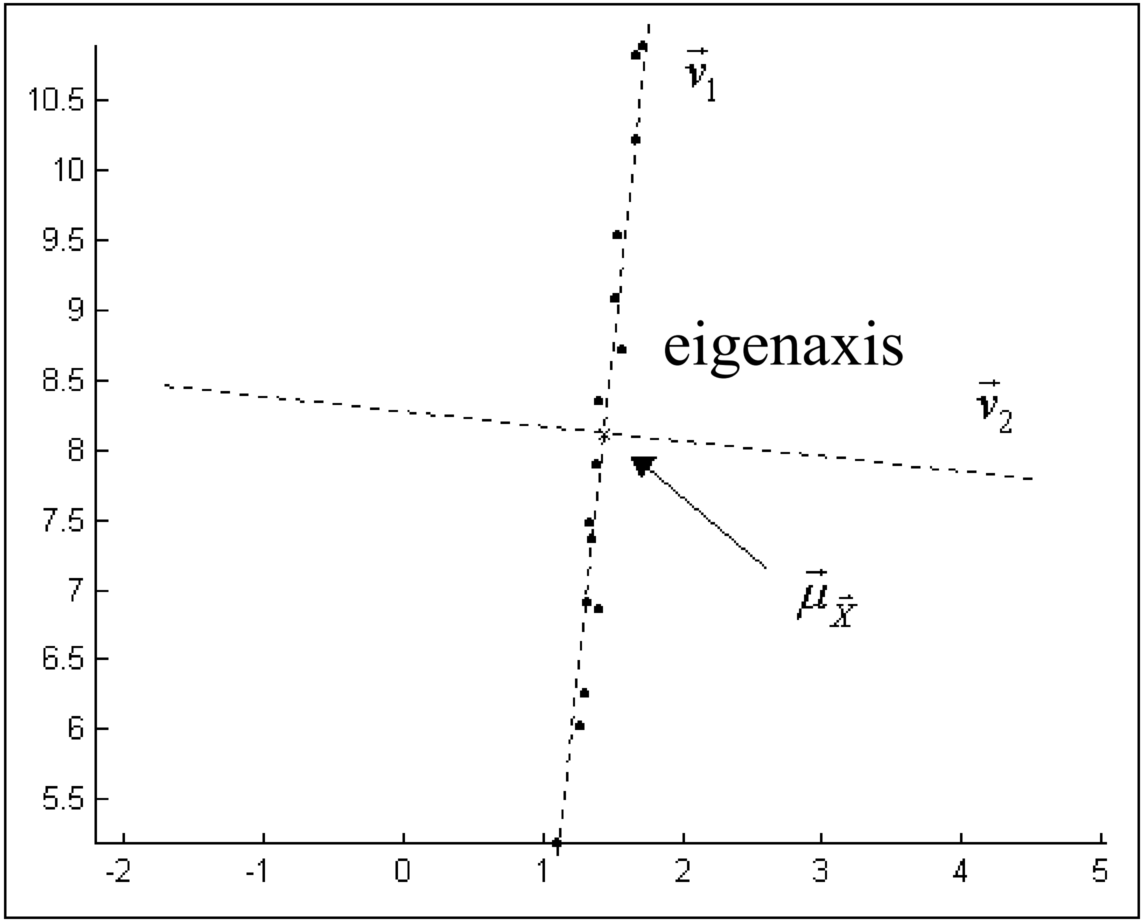
# Shape measures

- The Karhunen-Loève Transform is defined as

$$\vec{\hat{X}} = OMEGA\,\vec{X}$$

- Useful for dimensionality reduction

# Shape measures



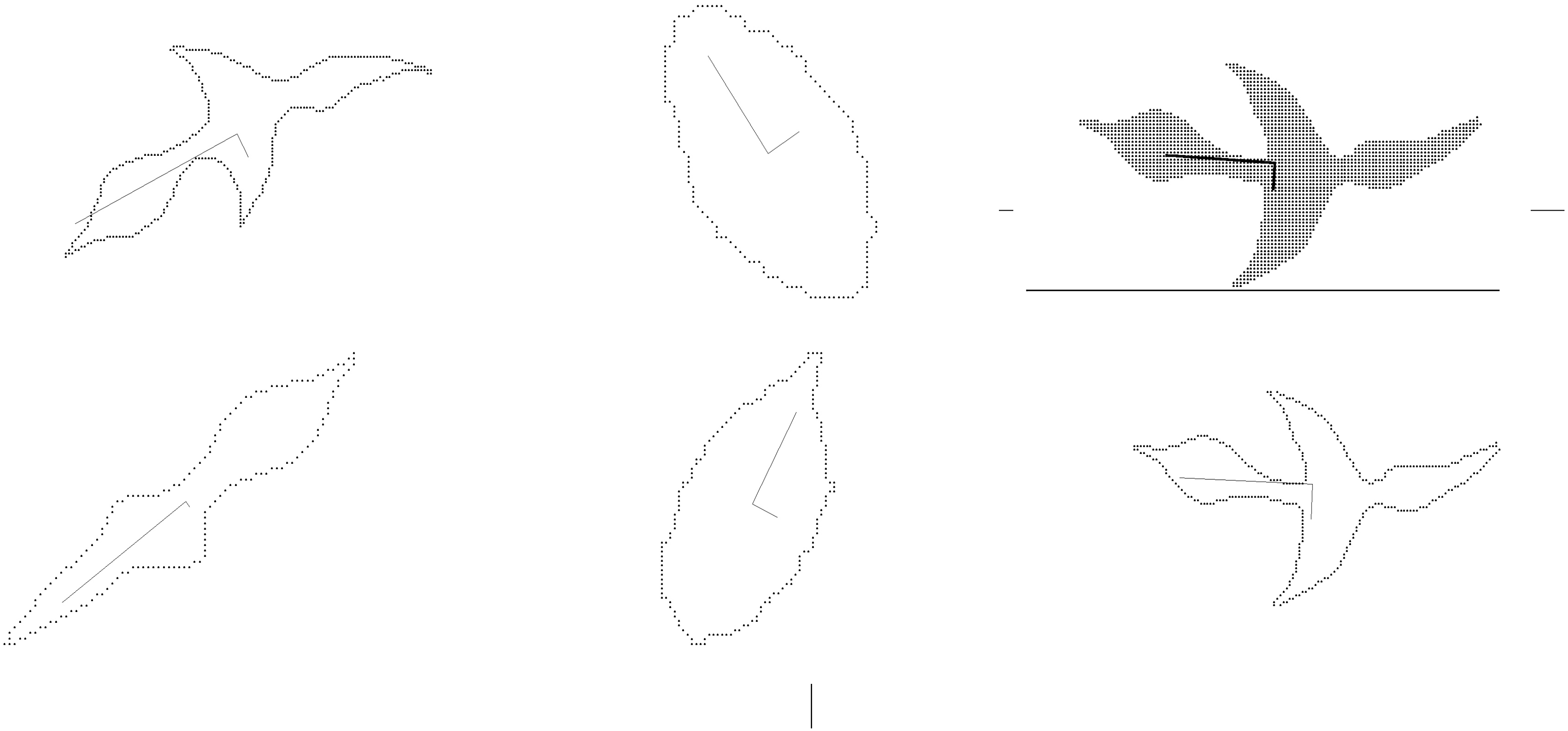
# Shape measures

- Major and minor axis of a shape

$$\left\{ \left(x_1, y_1\right), \left(x_2, y_2\right), \left(x_3, y_3\right), \ldots, \left(x_n, y_n\right) \right\}$$

- Regions, contours, etc.

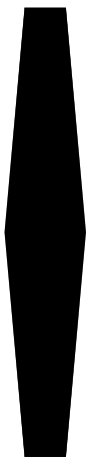
# Shape measures



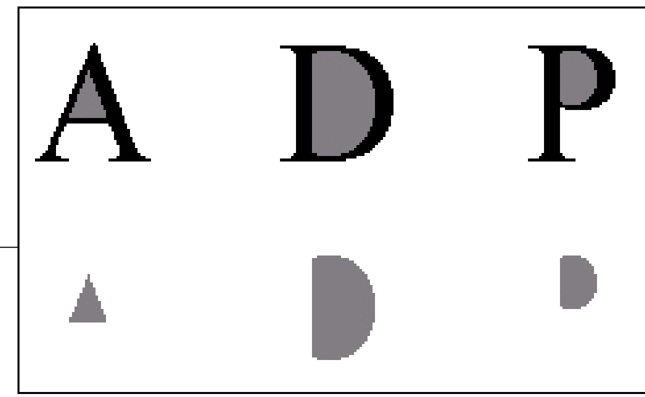
# Shape measures

- ✓ Shape measures from major/minor axes
  - ✓ The lengths of the principal axes, which can be defined as the associated eigenvalues
  - ✓ The *aspect ratio*, also known as *elongation*, defined by ratio the between the major and the minor axes' sizes
  - ✓ Rectangularity, defined as $$\frac{area(shape)}{area(MER)}$$
  - ✓ $$\frac{length(major\ axis)}{perimeter(shape)}$$

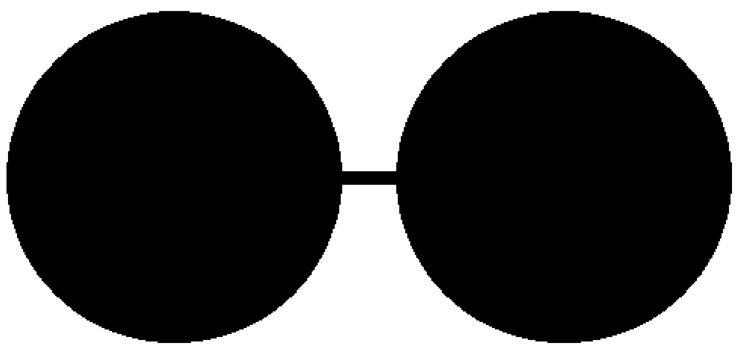
# Shape measures



Thickness




Hole features



Breaking the shape

# Statistical moments

- As in the PCA case, the shape points are taken as samples from a given shape distribution:

$$\left\{\left(x_1, y_1\right), \left(x_2, y_2\right), \left(x_3, y_3\right), \ldots, \left(x_n, y_n\right)\right\}$$

- Let $g(p,q)$ be a binary image representing the shape.

# Statistical moments

- The statistical moments of $g$ are defined as:

$$m_{r,s} = \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} p^r q^s g(p,q)$$

# Statistical moments

- Central moments are used for translation invariance

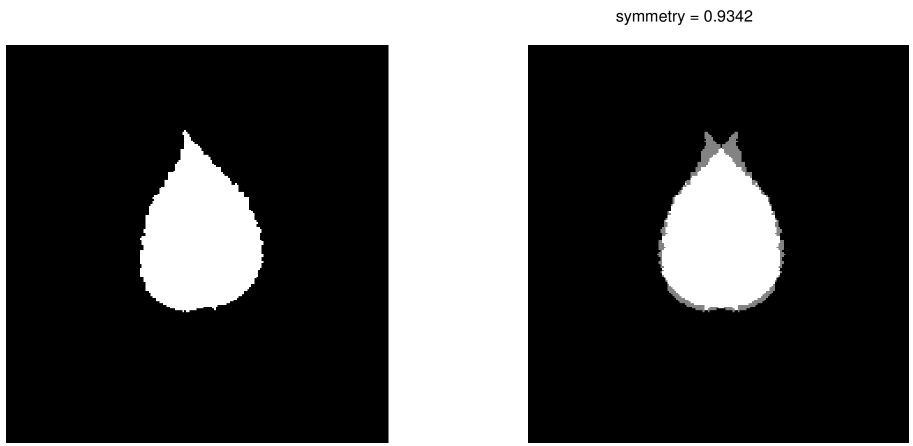
$$\mu_{r,s} = \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} (p-\bar{p})^r (q-\bar{q})^s g(p,q)$$

$$\bar{p} = \frac{m_{1,0}}{m_{0,0}} \qquad \bar{q} = \frac{m_{0,1}}{m_{0,0}}$$

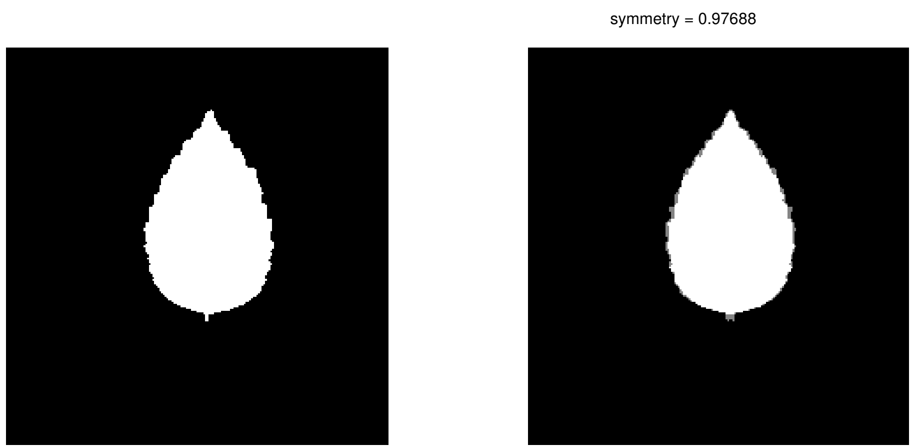
# Bilateral symmetry

- Simple method:
  - Reflection around major axis (passing through the centroid)
  - Sum
  - N: number of foreground pixels
  - N2: number of foreground pixels with gray-level=2
  - Symmetry measure: N2/N

# Bilateral symmetry

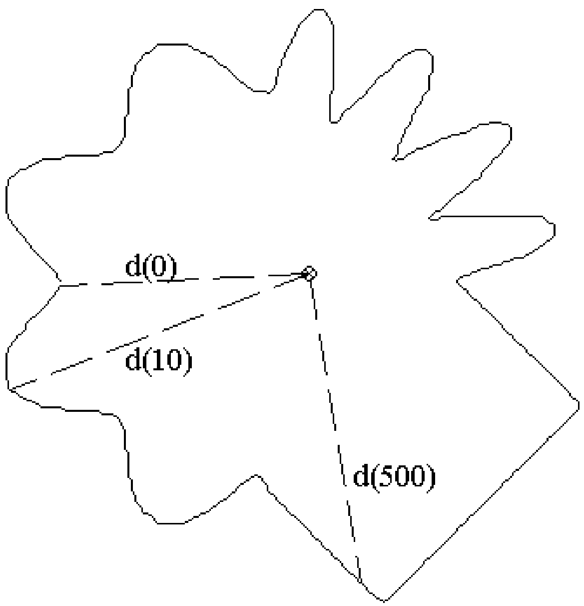


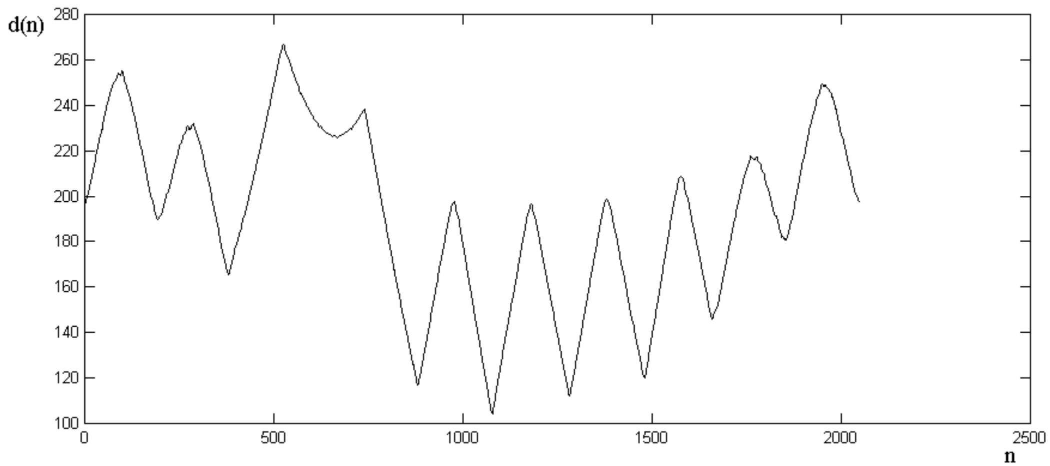

0.93




0.97

# Shape signatures




1D signals that represent the shape

# Shape signatures

- x, y, x + jy or by some function of each such complex values, such as magnitude or phase.
- Chain-code and shape number
- Curvature
- Distance to the centroid.
- Number of intersections: this signature is possible only for the above described angle-based parameterization, being defined by the number of times that the current line intersects the shape boundary.
- Angle with an axis: the angle-based parameterization can be inverted in order to produce an interesting signature.
- Affine signatures: affine curvature and affine parameterization
- Sholl diagrams

# Topological descriptors

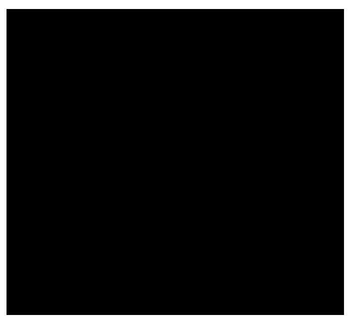
- The number of holes NH
- The number of connected components NC. It is important to note that this feature applies to composed shapes (e.g. Arabic character recognition)
- The Euler number, which is defined as E = NC – NH

B i n a r y ← NH=3, NC=7, E=4

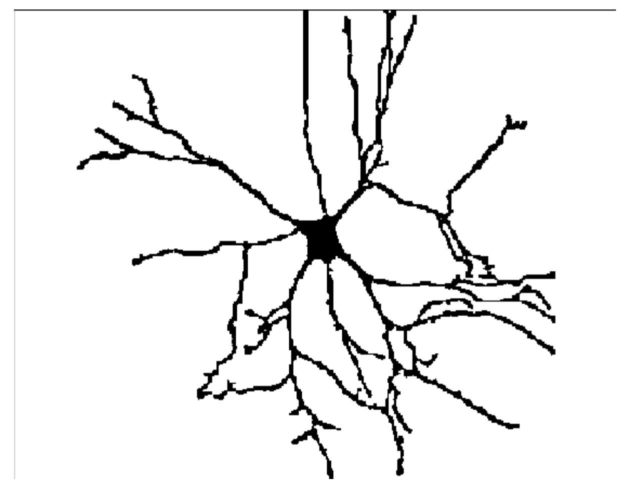
# Polygonal descriptors

- Number of corners or vertices
- Angle  and sides statistics, such as mean, median, variance and moments, to name but a few
- Major and minor sides lengths
- Major and minor sides ratio
- Major and minor angles ratio
- Ratio between the major angle and the sum of all angles
- Ratio between the standard deviations of sides and angles
- Mean absolute difference of adjacent angles

# Complexity



Simple



Complex

# Complexity

- Circularity $$\frac{P^2}{A}$$
- Thinness ratio $$4\pi\left(\frac{A}{P^2}\right)$$
- Area to perimeter ratio $$\frac{A}{P}$$
- Rectangularity $$\frac{A}{area(MER)}$$

# Complexity

- Fractal dimension: box-counting approach
- The *topological dimension:* number of degrees of freedom (points = 0, curves=1, planes=2, etc.)
- *Hausdorff-Besicovitch* dimension
- Benoit Mandelbrot: fractal set - $d \geq d_T$

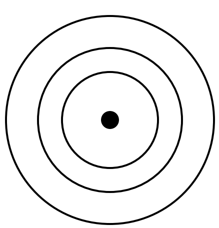
# Complexity

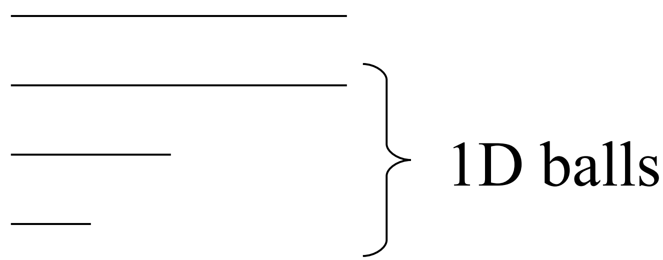
- Let $S$ be a set of $R^2$, and $M(\varepsilon)$ the number of open balls of radius $\varepsilon$ that are necessary to cover $S$.

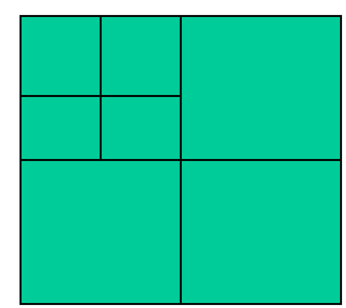
- An open ball of radius $\varepsilon$ and centered at $(x_0,y_0)$, in $R^2$, can be defined as the set $\{(x,y) \in R^2 \mid ((x - x_0)^2 + (y - y_0)^2)^{1/2} < \varepsilon\}$.

# Complexity

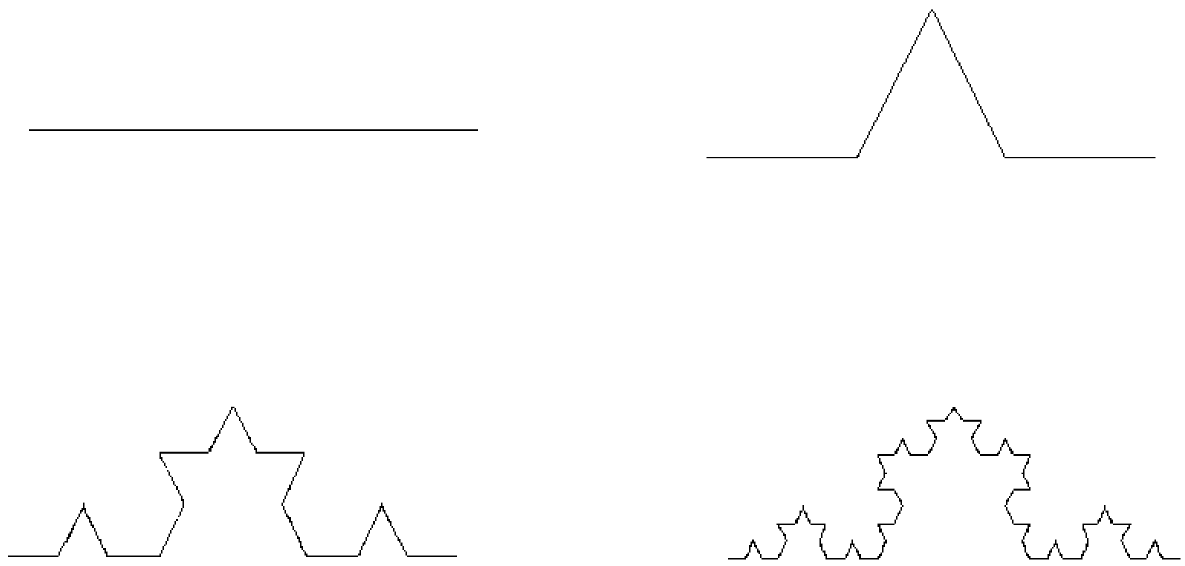
- The box-counting fractal dimension $d$ is defined as $M(\varepsilon) \sim \varepsilon^{-d}$
- Example 1: for a point, $d = 0$
- Example 2: for a straight line, $d = 1$
- Example 3: for plane, $d = 2$





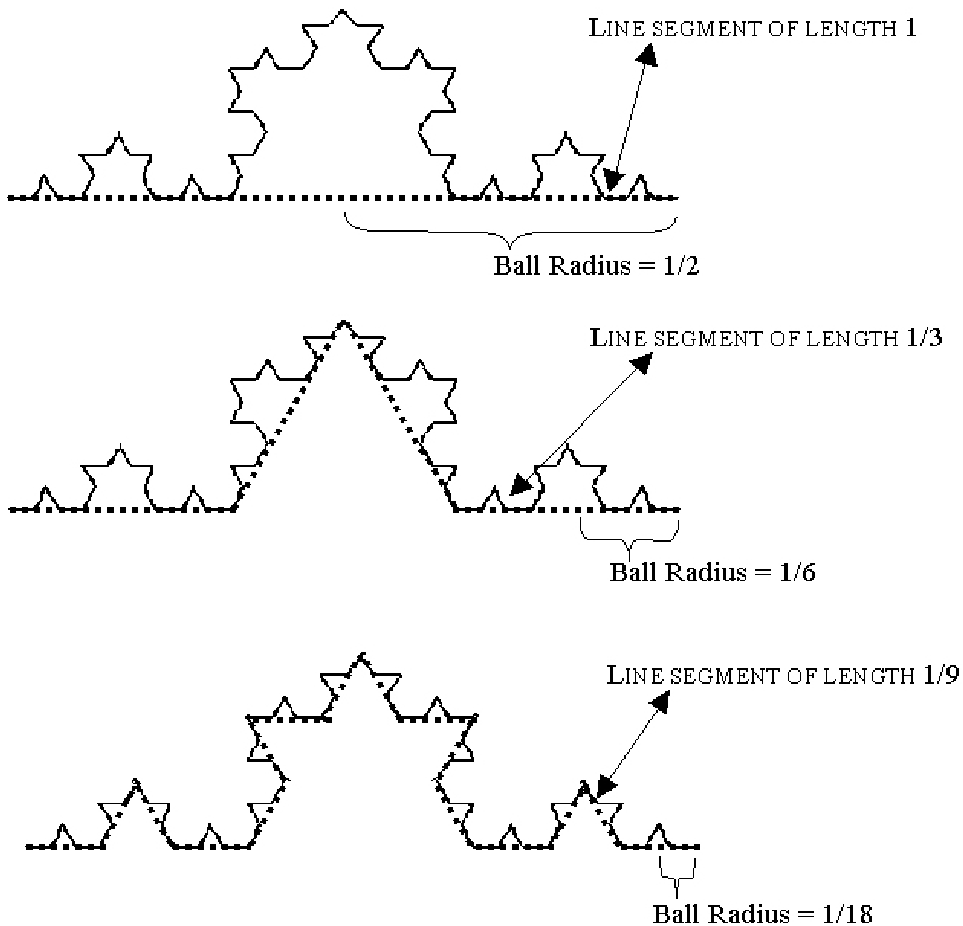



# Complexity



Koch's triadic curve

# Complexity


LINE SEGMENT OF LENGTH 1
Ball Radius = 1/2
LINE SEGMENT OF LENGTH 1/3
Ball Radius = 1/6
LINE SEGMENT OF LENGTH 1/9
Ball Radius = 1/18

# Complexity

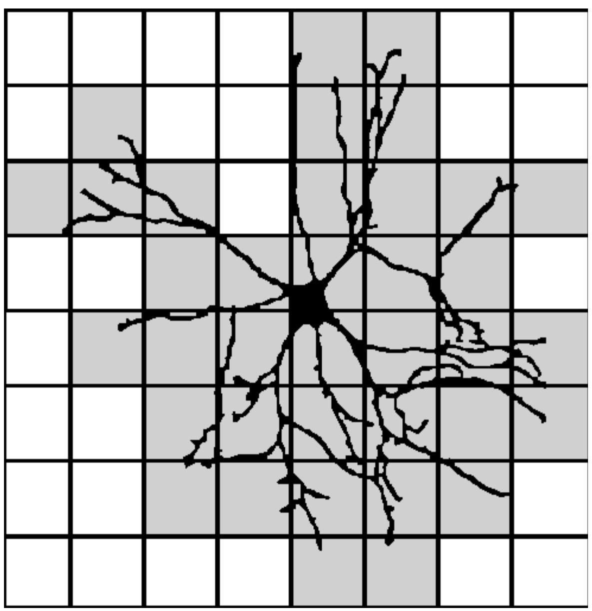
| $\varepsilon$ | $M(\varepsilon)$ | Measured Curve Length |
|---|---|---|
| $\frac{1}{2} = (1/2)\,(1) = (1/2)\,(1/3)^{0}$ | $1 = 4^{0}$ | 1 |
| $1/6 = (1/2)\,(1/3) = (1/2)\,(1/3)^{1}$ | $4 = 4^{1}$ | 1.33 |
| $1/18 = (1/2)\,(1/9) = (1/2)\,(1/3)^{2}$ | $16 = 4^{2}$ | 1.78 |
| … | … | … |

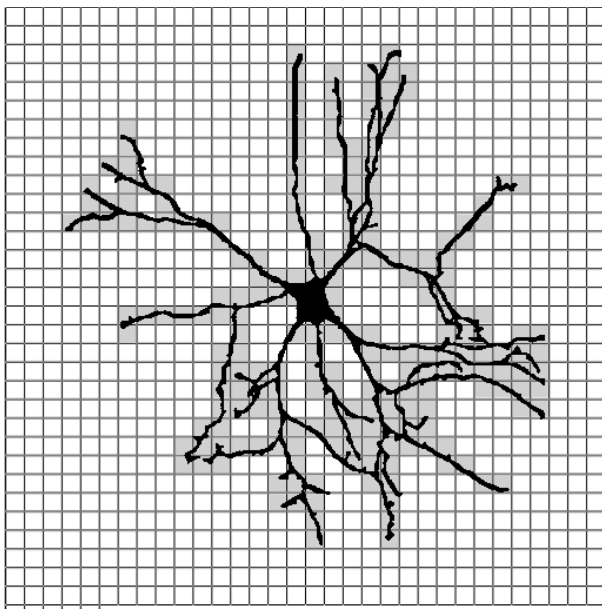
$$4 \sim (1/3)^{-d}$$

$$d = \log(4) / \log(3) \cong 1.26$$

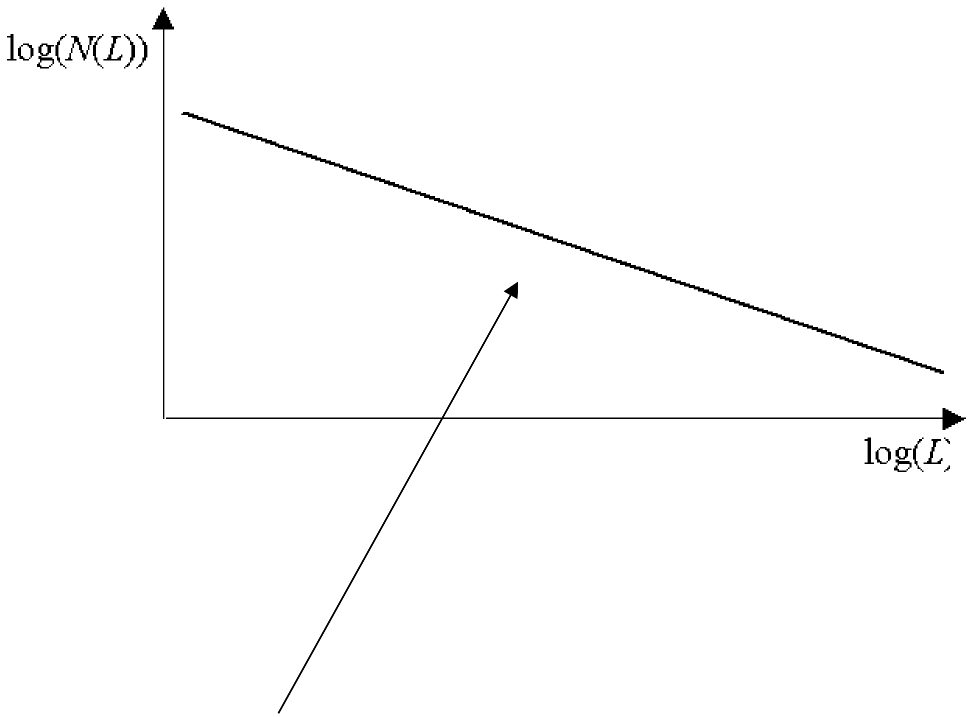
# Complexity

Estimating the box-counting dimension

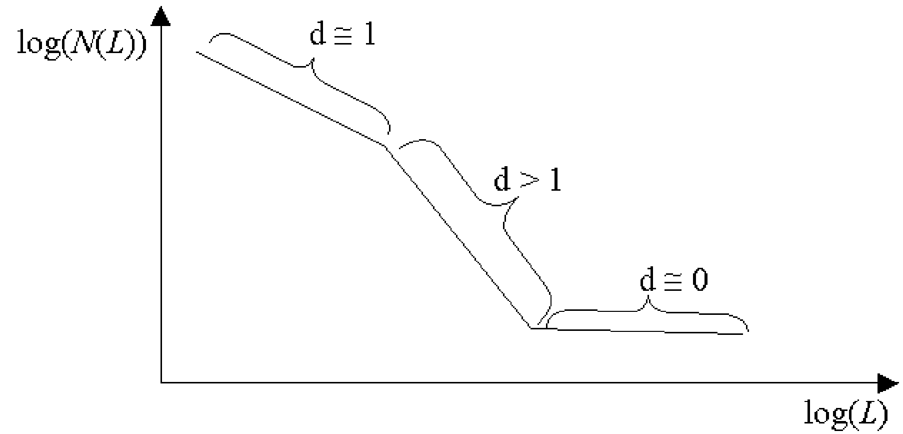




# Complexity

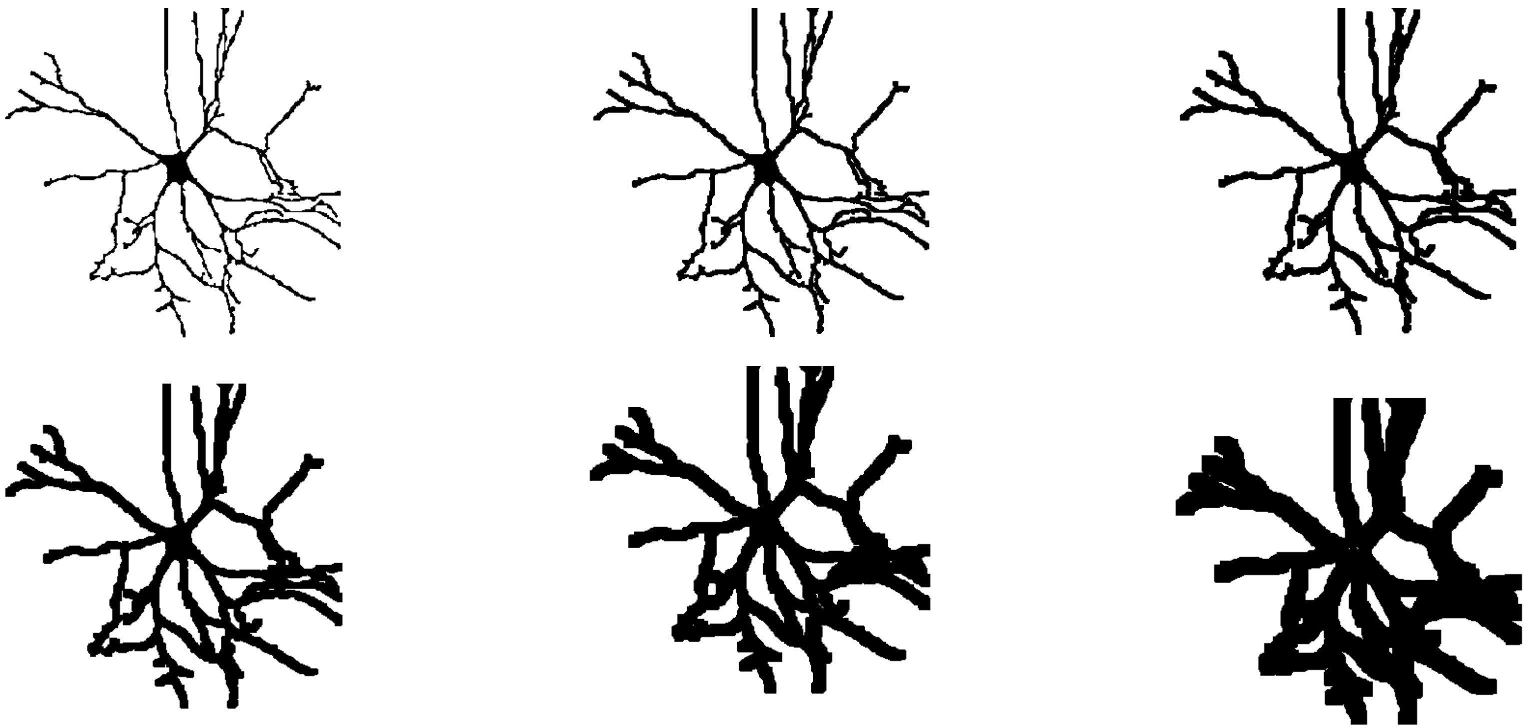



d: absolute slope of the line



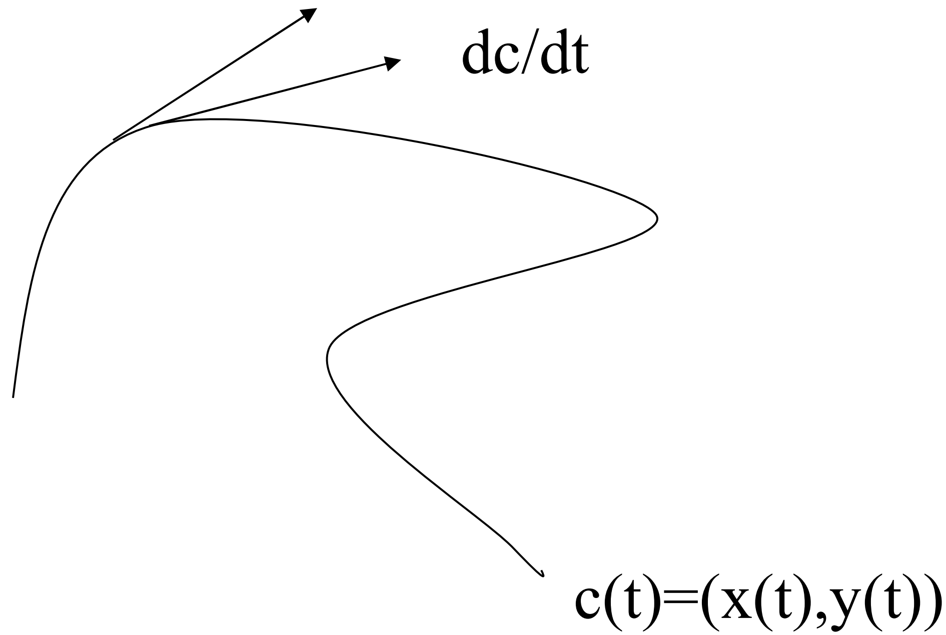
# Complexity

## The Minkowsky Sausage or Dilation Method



$$d = 2 - slope$$

# Curvature



# Curvature

**Table 6. 1:** *Summary of some important curvature properties*

| Curvature | Geometrical Aspect (assuming counterclockwise parameterization) |
|---|---|
| Curvature local absolute value maximum | Generic corner |
| Curvature local positive maximum | Convex corner |
| Curvature local negative minimum | Concave corner |
| Constant zero curvature | Straight line segment |
| Constant non-zero curvature | Circle segment |
| Zero crossing | Inflection point |
| Average high curvature in absolute or squared values | Shape complexity, related to the bending energy (Chapter 7) |

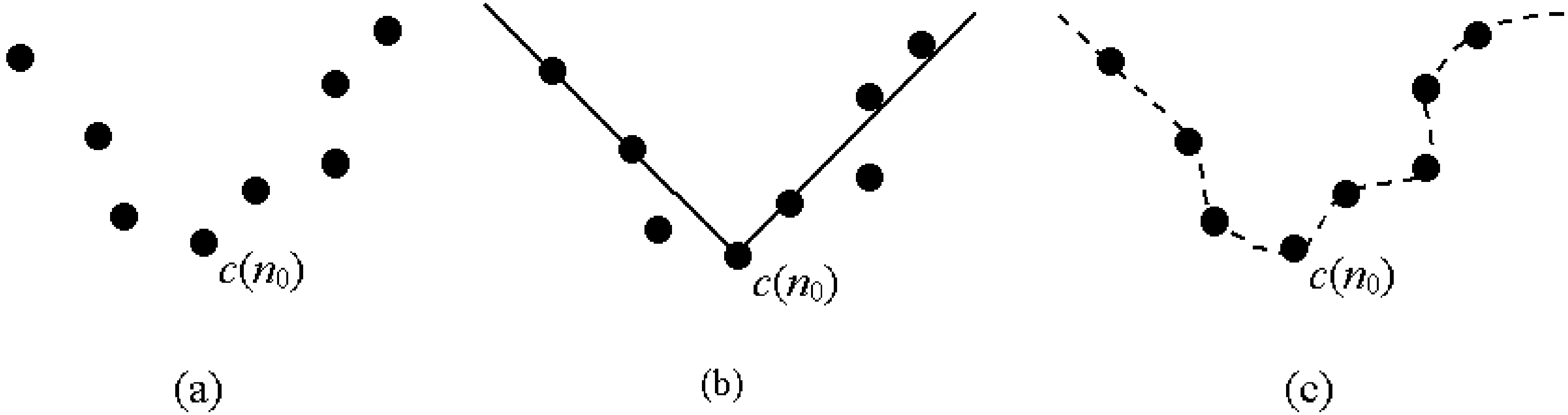
# Curvature

$$k(t)=\frac{\dot{x}(t)\,\ddot{y}(t)-\ddot{x}(t)\,\dot{y}(t)}{\left(\dot{x}(t)^2+\dot{y}(t)^2\right)^{3/2}}$$

Problem to be solved: numerical differentiation

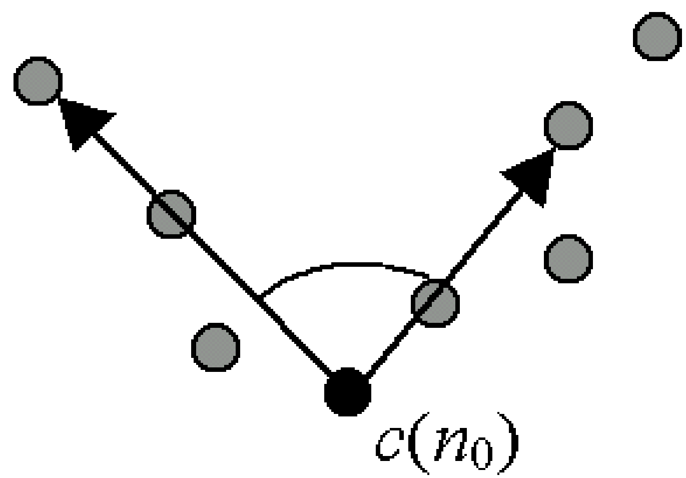
- Definition of alternative curvature measures based on angles between vectors defined in terms of the discrete contour elements
- Interpolation of  and  and differentiation of the interpolated curves

# Curvature

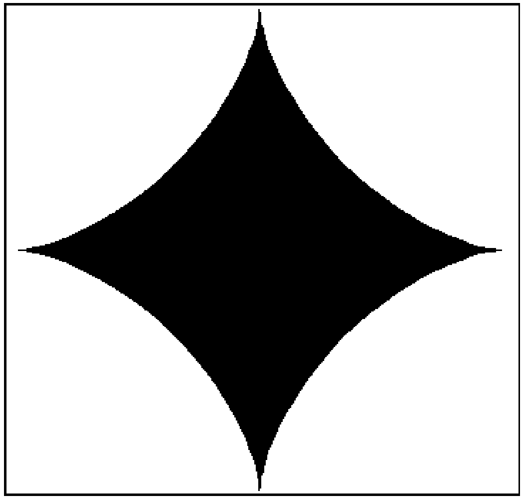


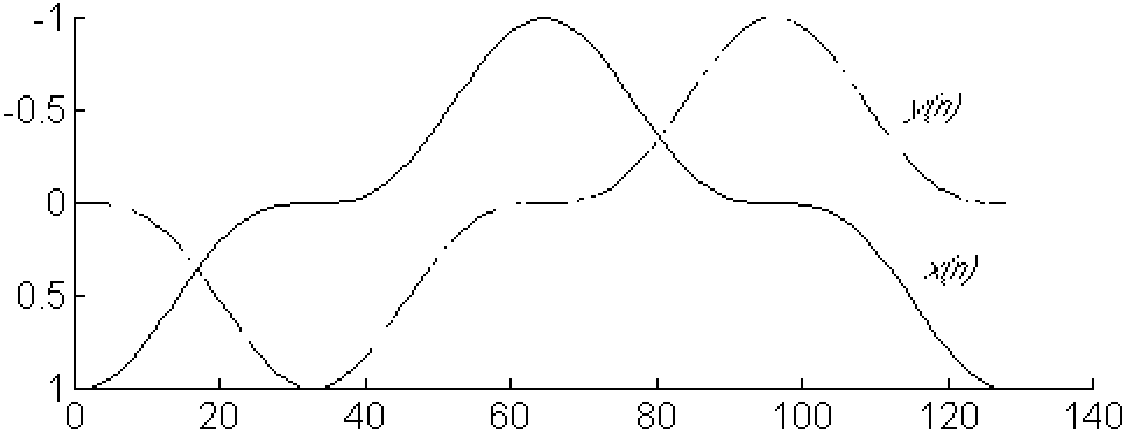
# Curvature

$$r_i(n) = \frac{v_i(n)\, w_i(n)}{\|v_i(n)\| \, \|w_i(n)\|}$$

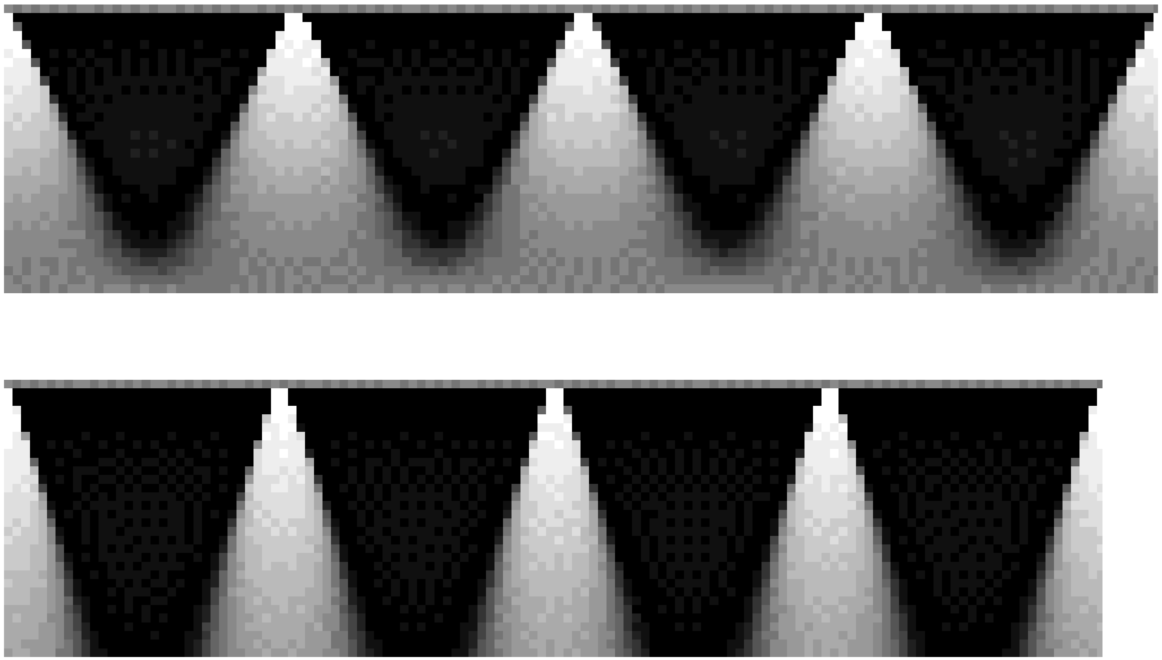


# Curvature





# Curvature



Multiscale approach obtained by varying the neighborhood size

# Curvature

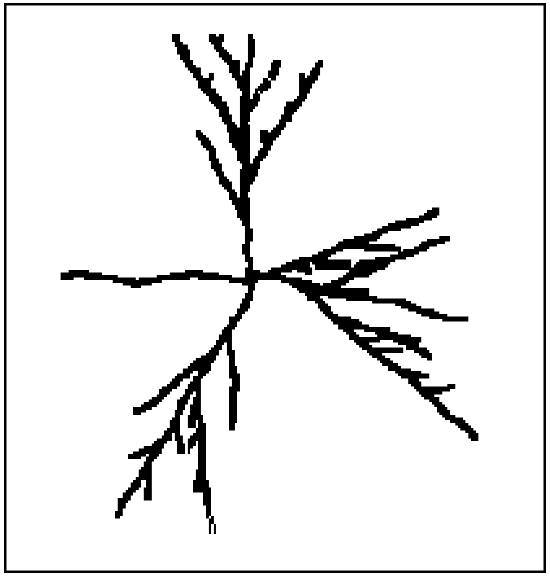
- Curvature features:
  - Sampled curvature
  - Curvature statistics
  - Maxima, minima, inflection points
  - Bending energy

$$B = \frac{1}{P} \int k(t)^2 \, dt$$

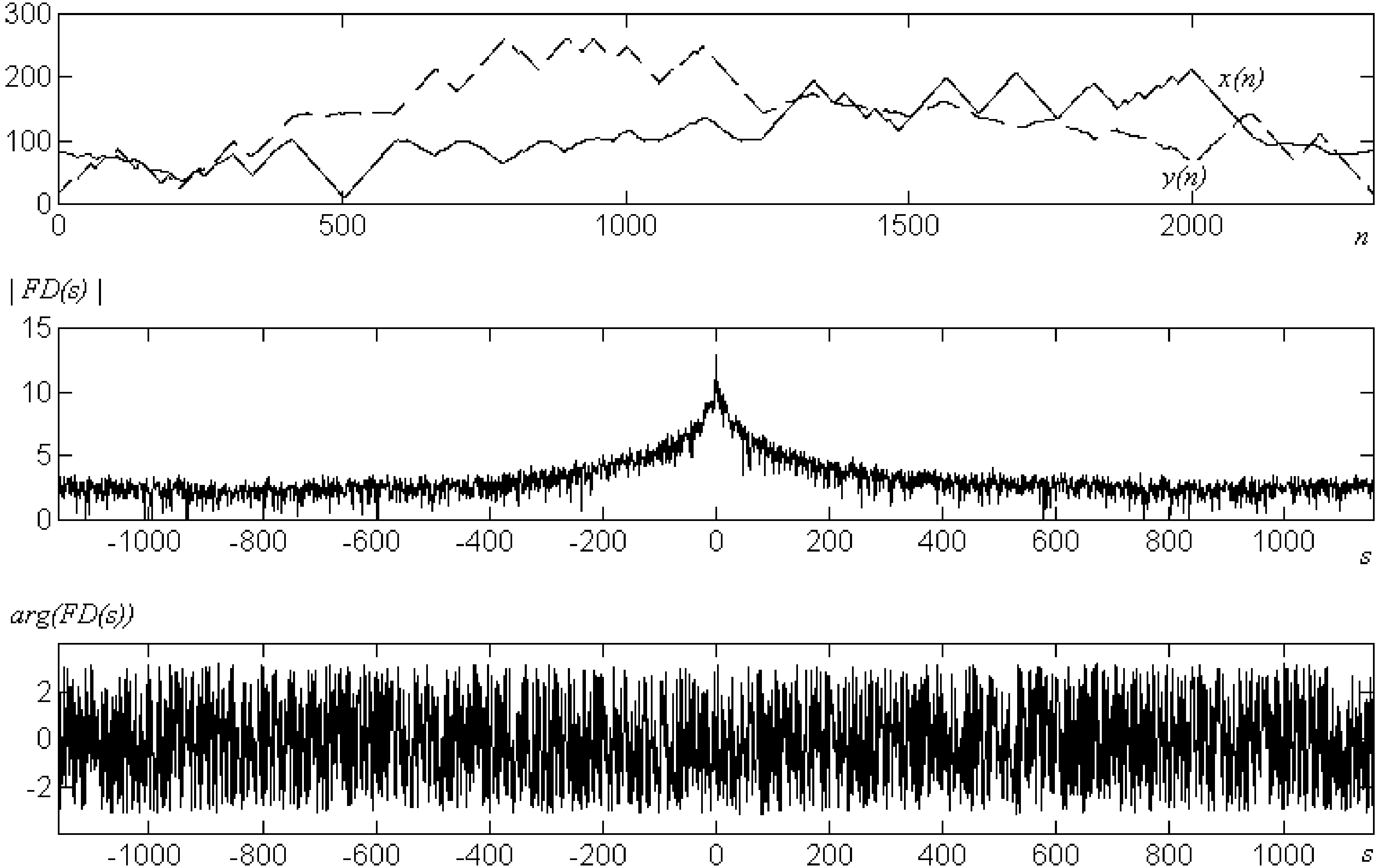
# Fourier descriptors

- Basic idea: Fourier transform of the shape
- Contours and regions
- Many variations (for invariance, etc)
- Many interesting properties have been explored in the literature

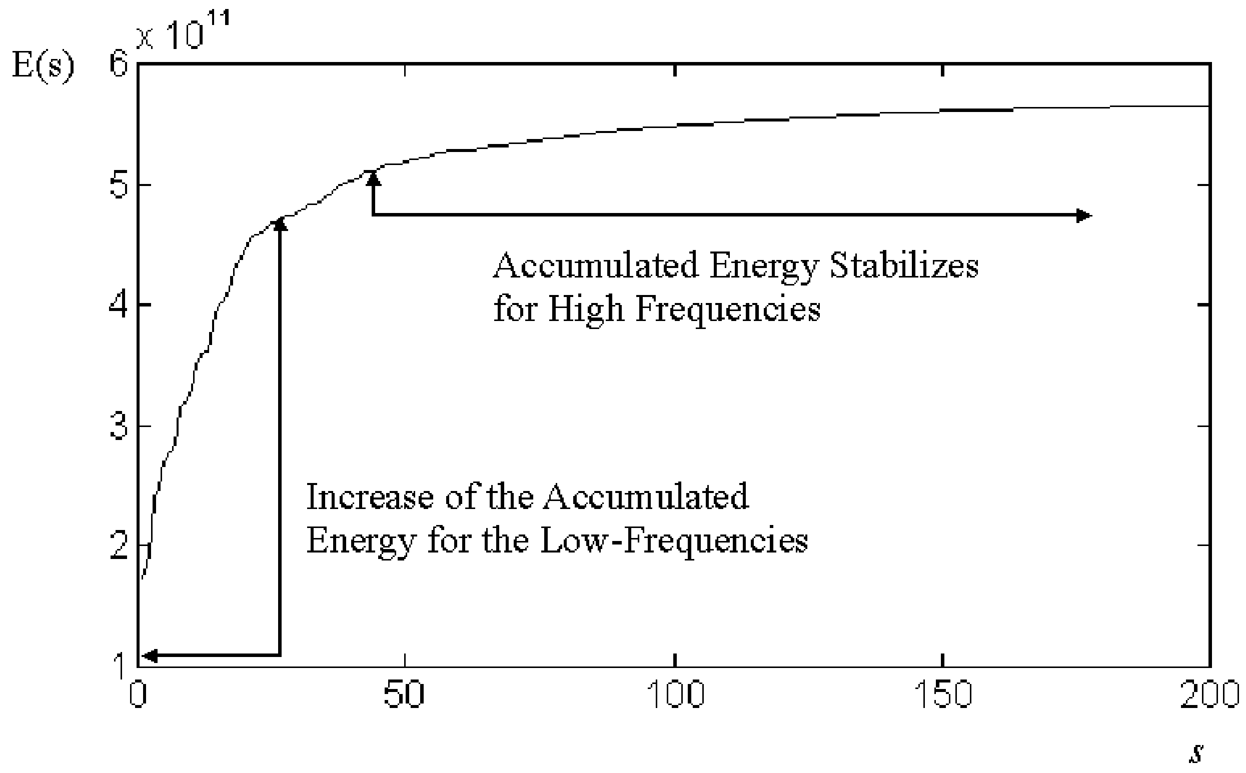
# Fourier descriptors



# Fourier descriptors



# Fourier descriptors



# Fourier descriptors