EDUCATION AND PRODUCTION

Use of Nonlinear Programming to Optimize Performance Response to Energy Density in Broiler Feed Formulation

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ABSTRACT A nonlinear programming optimization model was developed to maximize margin over feed cost in broiler feed formulation and is described in this paper. The model identifies the optimal feed mix that maximizes profit margin. Optimum metabolizable energy level and performance were found by using Excel Solver nonlinear programming.

Data from an energy density study with broilers were fitted to quadratic equations to express weight gain, feed consumption, and the objective function income over feed cost in terms of energy density. Nutrient:energy ratio constraints were transformed into equivalent linear constraints. National Research Council nutrient requirements and feeding program were used for examining changes in variables.

The nonlinear programming feed formulation method was used to illustrate the effects of changes in different variables on the optimum energy density, performance, and profitability and was compared with conventional linear programming. To demonstrate the capabilities of the model, I determined the impact of variation in prices. Prices for broiler, corn, fish meal, and soybean meal were increased and decreased by 25%. Formulations were identical in all other respects. Energy density, margin, and diet cost changed compared with conventional linear programming formulation. This study suggests that nonlinear programming can be more useful than conventional linear programming to optimize performance response to energy density in broiler feed formulation because an energy level does not need to be set.

(*Key words*: feed formulation, least cost, maximum profitability, nonlinear programming, optimization)

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INTRODUCTION

Linear programming has been an effective method of determining least-cost feed mixes. However, broiler production response to dietary energy density (constant nutrient weight per megacalorie) diminishes with increasing nutrient density and is curvilinear (Parks, 1982; Gous, 1986; Pesti et al., 1986; Mack et al., 2000). Static methods of diet formulation ignore the importance of economics and are not adequate to optimize the feeding of commercial broilers. Reducing feed costs may make the cost side of the equation look attractive but the resulting loss in performance may have negative effects on profitability.

Because the response of birds to energy density is a diminishing returns phenomenon, it should be evaluated economically to estimate an economic optimum level rather than a biological maximum. Fisher et al. (1973) have shown that requirements of animals are variable and depend on marginal cost of nutrients and marginal revenue of the product. What is required is the ability to derive production functions to enable the prediction of broiler performance over a range of nutrient input levels and to handle nonlinear constraints.

Therefore, since it is widely accepted in poultry nutrition that nutrient requirements should be expressed as grams per megacalorie to take into acount the effect of energy on feed intake (Scott et al., 1982; Waldroup et al., 1990; Leeson et al., 1996), a broiler response function can be derived in terms of dietary energy density from either experimental or industry data to analyze profitability.

At present, there is no reliable dynamic computer method of diet formulation to determine how changing prices of broiler and feed ingredients affect performance and dietary energy density that maximizes margin over feed cost. The effect of these changes on broiler performance and profitability can be evaluated through the use of nonlinear programming that can handle curvilinear response functions. Previous work has attempted to maximize liveweight or minimize cost per pound of meat and optimize concentrations of protein and energy by using quadratic programming (Pesti et al., 1986; Talpaz et al., 1986). However, both techniques are a partial solution to the profit equation.

Also, the proper use of such a computer model depends on an accurate quantitative description of how animals respond to incremental changes of energy density. An

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attempt to relate quantitatively several performance variables to dietary energy density was made by Waldroup et al. (1976). Knowledge of these relationships plus relevant price information could be used to formulate diets that optimize profit (Gous, 1998).

The objective of this study was to demonstrate the merits of a nonlinear programming optimization model to determine the impact of variation in ingredient and broiler meat price on the optimal feed mix and margin over feed cost.

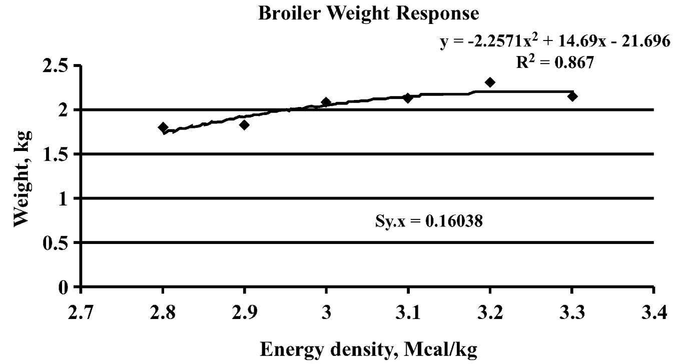

FIGURE 1. Broiler weight response to energy density at 42 d of age.

MATERIALS AND METHODS

Determination of the Broiler Response Function

Day-old male Cobb chicks were obtained from a commercial hatchery. The birds were raised in battery brooders with fluorescent lighting providing continuous daylight. At the age of 3 wk, the birds were transferred to cages without heating.

At the beginning of the experiment the birds were divided into the experimental groups, equalizing both average BW and variance. Dietary treatments included the following energy densities for both the starter and the grower phase: 2.8, 2.9, 3.0, 3.1, 3.2, and 3.3 Mcal of metabolizable energy per kilogram. The 3.2 energy level was computer-formulated to meet the requirements of the National Research Council (1994). Metabolizable energy was estimated from proximate analysis to check if deviation from formulated value was less than 5%. Nutrient levels were kept in a constant ratio to energy level. All birds consumed the diets ad libitum. Each treatment included 3 replicate groups of 10 chickens each. At the end of the study at 42 d of age, BW and feed intake were individually measured. The temperature schedule consisted of an initial temperature of 32°C and incremental weekly decreases until 22°C was reached on d 21 of the study.

Data were fitted to quadratic equations by using Excel polynomial regression to express weight gain and feed consumption in terms of energy density. For example

$$W = a + bE - cE^2$$
$$F = d - eE + fE^2$$

where W = weight (kg), E = energy density (Mcal/kg), F = feed intake (kg), and a, b, c, d, e, f are constants.

The Programming Model

Software Environment. The optimization model was formulated in Excel.[2] The nonlinear optimization model and conventional linear programming was solved using the Solver, which is the default solver of Excel (Frontline Systems, Inc., 1999). It uses the generalized reduced gradient method to solve nonlinear problems, such as the one presented in my model. The options, which are specified by the user, were set as follows: iterations = 1,000, precision = 0.00001, convergence = 0.001, estimates = tangent, derivatives = forward, and search = Newton.

Description of the Model. The model identifies the combination of feed ingredients that maximize the margin over feed cost (Table 1). In mathematic terms, the objective of the model is to identify the set of variables (vector X) that maximizes the value of the objective function (Z), which is the margin over feed cost. The objective function has the form $Z = c \times X$, where c is the vector of the objective function coefficients (it includes the price of product and the costs of ingredients). Excel cells for feed cost and energy level were used to express weight gain and feed consumption in terms of energy density (objective function) and maximize profitability. The linear programming matrix of Pesti et al. (1986) was adapted to take energy density expressed as a ratio and broiler response into account. Nutrient concentrations were kept in a constant ratio with energy level, and each nutrient constraint expressed as a ratio was transformed into equivalent linear constraints before using nonlinear programming. Energy density was entered as an extra ingredient in the left-hand side of the model. Since nutrient:energy ratio is equal to the requirement divided by energy level, then the requirement in the right-hand side of the model is equivalent to ratio times energy level

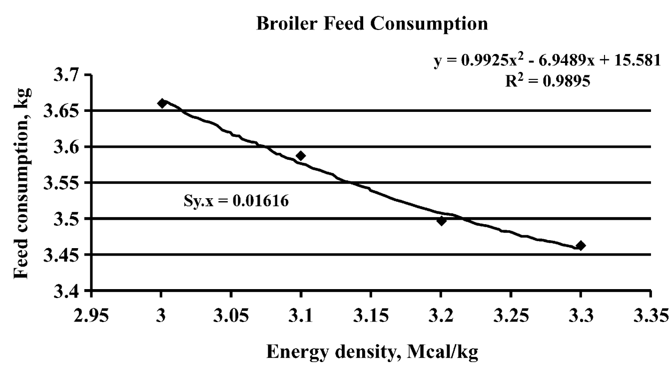

FIGURE 2. Broiler feed consumption response to energy density at 42 d of age.

[2]Microsoft, Seattle, WA.

TABLE 1. Nonlinear model[1]

	Ingredients			Energy (E)		Ranges	
Activity	×1	×3	×3	×4	RHS	Minimum	Maximum
Cost	c1	c2	c3		= Px		
Weight	w1	w2	w3		= 1,000		
Energy	e1	e2	e3	–1	= 0		
Protein (P)	p1	p2	p3	–(P/E)	≥0		
Amino acid (A)	a1	a2	a3	–(A/E)	≥0		
				1	= E	E1	E2
Objective function:	Maximize		$Py\ (a + bE - cE^2) - Px\ (d - eE + fE^2)$				

[1]Py = broiler price; Px = feed cost; E = energy level; RHS = right-hand side.

and could pass to the left-hand side of the model as a linear function.

The optimization model I developed is nonlinear because broiler performance depends on energy density which is unknown and this constraint is nonlinear.

Design of the Analysis

The nonlinear programming feed formulation method was used to illustrate the effects of changes in different variables on the optimum energy density, performance, and profitability and was compared with conventional linear programming. To demonstrate the capabilities of the model, I determined the impact of variation in prices. Prices for broiler, corn, fish meal, and soybean meal were increased and decreased by 25%. Formulations were identical in all other respects.

In summary, ratio constraints were transformed into equivalent linear constraints, and the objective function, income over feed cost, was expressed in terms of the energy density. Optimum energy level and performance were found by using Excel Solver nonlinear programming. National Research Council (1994) nutrient requirements and feeding program were used for examining changes in variables (Table 2).

RESULTS AND DISCUSSION

Broiler BW and feed consumption response functions are shown in Figures 1 and 2. As expected, growth response reached a plateau.

The objective function obtained was

$$\text{margin} = \text{broiler price} \times (-2.2571E^2 + 14.69E - 21.696) - \text{feed cost} \times (0.9925E^2 - 6.9489E + 15.581)).$$

Variation of Ingredient Prices

The optimal feed mix and profit margin varied markedly as ingredient prices varied by 25%. The profit margin was much higher when the prices of ingredients were low (Tables 3 and 4). The use of lower prices for protein ingredients led to an increase in energy density compared to conventional linear programming least cost feed formulation.

Variation of Broiler Price

When broiler price increased, the model changed the optimal feed mix and energy density in such a way as to

TABLE 2. Partial nonlinear programming matrix and constraint set used in optimization

Component	Corn	Soybean meal	Fish meal	Fish oil	Dicalcium phosphate	ME	RHS[1]
Cost	0.129	0.248	0.450	0.350	0.290		0.197
Weight	1.000	1.000	1.000	1.000	1.000		1.000
Protein, %	8.600	48.500	65.000			–6.25	0.000
ME, Mcal/kg	3.432	2.421	3.060	8.250		–1.0000	0.000
Calcium, %	0.040	0.200	3.730		32.000	–0.28125	0.000
Available phosphorus, %	0.100	0.300	2.430		18.000	–0.109375	0.000
Sodium, %	0.060	0.010	0.650		5.500	–0.046875	0.050
Lysine, %	0.280	3.160	5.070			–0.3125	0.000
Methionine, %	0.210	0.720	1.950			–0.11875	0.013
Methionine + cystine, %	0.370	1.470	2.600			–0.225	0.000
Choline, mg/kg	594.000	2,761.000	4,408.000			–312.5	0.000
Arginine, %	0.380	3.480	3.810			–0.34375	1.174
Threonine, %	0.290	1.870	2.820			–0.23125	0.038
Tryptophan, %	0.060	0.740	0.780			–0.05625	0.046
Valine, %	0.400	2.220	3.460			–0.25625	0.305
						1.0000	3.215
Minimum				0.000			
Maximum				0.030			

[1]RHS = right-hand side.

TABLE 3. Effect of changing the price of broiler, corn, soybean meal, and fish meal on optimum performance and energy density

Prices	Metabolizable energy (Mcal/kg)	Weight (kg)	Feed consumption (kg/kg)	FCR[1] (g/g)	Margin (US $/bird)
Linear programming					1.16
Nonlinear programming					
Normal	3.205	2.200	3.505	1.62	1.16
Corn					
+25%	3.206	2.201	3.504	1.62	1.08
–25%	3.191	2.197	3.513	1.63	1.24
Soybean meal					
+25%	3.209	2.201	3.502	1.62	1.12
–25%	3.208	2.201	3.503	1.62	1.21
Fish meal					
+25%	3.203	2.200	3.506	1.62	1.15
–25%	3.214	2.202	3.500	1.62	1.18
Broiler[2]					
+25%	3.214	2.202	3.500	1.61	1.60
–25%	3.190	2.197	3.514	1.63	0.72

[1]FCR = feed conversion ratio.

[2]Broiler price assumed = 0.8 US $/kg.

improve weight and feed conversion and accepted a higher energy concentration. When broiler price decreased, the opposite was true (Tables 3 and 4). Margin and diet cost changed compared with conventional linear programming formulation. Different profitabilities (margins) for similar energy densities reflected changes in broiler performance and diet composition (Table 4). The results of this study demonstrate the potential of using the model in decision-making in broiler feed formulation and indicate that the traditional concept of a static requirement can be replaced with a dynamic one.

Evaluation

As indicated above, the model identifies the optimal feed mix which maximizes margin over feed cost. However, the model may be used not only for optimization but also for evaluation of current feeding practices as well. A producer may change ingredient prices and broiler cost so that they reflect the current feeding situation. In this way, broiler performance provided by the model can be compared with the actual performance observed in the

TABLE 4. Effect of changing prices on diet formulations

Ingredient	Price ($/kg)	Normal	Corn+	Corn–	Soy+	Soy–	Fish meal+	Fish meal–	Broiler+	Broiler–	LP[1]
Corn	0.1294	72.25	62.99	73.59	63.06	65.92	66.14	72.91	71.95	72.74	72.40
Wheat middlings	0.1000	0.00	9.56	0.00	9.70	0.00	0.00	0.00	0.00	0.00	0.00
Soybean meal	0.2480	16.82	12.94	15.41	12.36	26.86	26.78	15.25	16.77	16.89	16.84
Fish meal	0.4500	4.61	6.21	5.32	6.56	0.00	0.00	5.57	4.70	4.46	4.56
Fish oil	0.3500	0.96	3.00	0.38	3.00	3.00	2.87	0.84	1.16	0.64	0.86
Limestone	0.0198	1.28	1.35	1.28	1.35	1.23	1.22	1.29	1.29	1.28	1.28
Dicalcium phosphate	0.2901	0.50	0.24	0.41	0.20	1.07	1.06	0.39	0.49	0.51	0.50
Meth MHA	2.6455	0.01	0.00	0.00	0.00	0.05	0.05	0.00	0.01	0.01	0.01
Feather meal	0.2756	3.41	3.55	3.44	3.62	1.68	1.67	3.59	3.47	3.31	3.38
Vit premix	3.1747	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Salt	0.0650	0.06	0.06	0.06	0.05	0.10	0.10	0.06	0.06	0.06	0.06
Trace minerals	0.2866	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
	Cost, $/kg	0.1724	0.1742	0.1714	0.1745	0.1736	0.1732	0.1732	0.1731	0.1712	0.1720
	Margn, $/kg	1.16	1.08	1.24	1.12	1.21	1.15	1.18	1.60	0.72	1.16
Calculated analysis											
ME, Mcal/kg		3.205	3.206	3.191	3.209	3.208	3.203	3.214	3.214	3.189	3.200
Protein, %		20.03	20.04	19.95	20.06	20.05	20.02	20.09	20.09	19.93	20.00
Lysine, %		1.00	1.00	1.00	1.00	1.05	1.05	1.00	1.00	1.00	1.00
Methionine, %		0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
Methionine-cystine, %		0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72	0.72
Calcium, %		0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90
Available phosphorus, %		0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
Sodium, %		0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15

[1]LP = linear programming.

[AUTH QUERY: Spell out D. phosphate in column 1]

farm to evaluate the efficiency of current feeding practices.

Dollar values used in this study are for establishing relationships, rather than depicting specific profit margins, because in a situation where low nutrient density feeds and by-products are used extensively, different responses and solutions can be expected. Likewise, the optimum will be different for broiler growers, whole bird integration, and portioning integration and will vary over time as costs and income change, because determining optimum energy density requires an understanding of payment structure, cost, and availability of inputs and broiler performance. As costs and income change, in different types of operation, the optimum energy density to maximize profit will change.

Decisions as to the optimum time, amino acid level, and feeding program under different situations are dependent on many variables (Gous et al., 1999). It is not possible to take into account the wide variations in biological, physical, and economic conditions that exist in different countries and companies. Although empirical models can be developed, modeling and simulation are more adequate to address more complex problems (Emmans and Fisher, 1986).

In conclusion, this study suggests that nonlinear programming can be more useful than conventional linear programming to optimize performance response to energy density in broiler feed formulation because an energy level does not need to be set. This approach replaces the traditional concept of a static requirement with a dynamic one.

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