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Economic forces and the stock market revisited $\stackrel{\text{\tiny $\stackrel{$}{$$}$}}{=}$

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Abstract

The pricing of the Chen, Roll, and Ross (CRR) macrovariables is re-examined and found to be surprisingly sensitive to reasonable alternative procedures for generating size portfolio returns and estimating their betas. These methods include the full-period post-ranking return approach used in many recent studies. Strong evidence of pricing is obtained only for their industrial production growth factor and, in another contrast, for the VW market index. In particular, the corporate-government bond return spread, an important factor in CRR, is insignificantly negative for the 1958–1983 period, corroborating the cross-sectional regression results.

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1. Introduction

An important body of research in financial economics is concerned with the forces that determine the prices of risky securities, and there are a number of competing theories of asset pricing. These include the original capital asset pricing models (CAPM) of Sharpe (1964), Lintner (1965) and Black (1972), the intertemporal models of Merton (1973), Long (1974), Rubinstein (1976), Breeden (1979), and Cox et al. (1985), and the arbitrage pricing theory (APT) of Ross (1976). In each case a relation between expected return and one or more measures of exposure to systematic risk is derived.

In a CAPM framework, a security's systematic risk is measured by its beta with respect to a diversified stock index, the latter viewed as a proxy for the value-weighted market portfolio of all assets. Black et al. (1972), Blume and Friend (1973), and Fama and MacBeth (1973) are important early examples of such work. Roll (1977) criticizes the early studies, however, emphasizing that they are really tests of the mathematical hypothesis that the stock index is mean-variance efficient, and would reflect on the CAPM only if the true market portfolio were used in the tests.¹

Sobered by Roll's conclusions, many researchers have turned to other approaches to risk-return analysis. Motivated by the APT, Roll and Ross (1980) employ factor-analytic methods to estimate multiple measures of systematic risk. Restrictions on the covariance matrix of returns are used to statistically identify the factor sensitivities, as the underlying factors are not observed in this context. Brown and Weinstein (1983), Connor and Korajczyk (1988) and Lehman and Modest (1988) are other studies of this sort.

A third approach to asset pricing empirical work, pursued in this paper, is advanced by Chan et al. (1985) and Chen et al. (1986), henceforth CCH and CRR. These studies look at pricing relative to a set of observable macroeconomic variables, or factors, selected primarily based on economic intuition.² Although the authors appeal to the APT in motivating their work, the strong intuition underlying their choice of factors is derived, in large part, from the intertemporal models cited above. Thus, one can also view the factors more formally as a "multivariate proxy" for the unobservable equilibrium benchmark (see Shanken, 1987). In the spirit of Roll's critique, this view makes explicit the joint hypothesis concerning the factors that is inherent in such analyses.

This paper makes several contributions to research in the area. First, we examine some issues that arise when inferring whether a particular macroeconomic variable is a priced factor. Both CCH and CRR employ versions of the two-pass cross-sectional regression (CSR) methodology developed by Fama and MacBeth (1973). The reported "*t*-statistics" are impressive, suggesting that several of the factors are priced; i.e., the betas on these factors help explain the cross-sectional variation in mean asset returns. However, as shown in Shanken (1992a), the usual Fama–MacBeth standard errors on the estimated prices of risk are generally biased downward (asymptotically) due to the well-known errors-in-variables (EIV) problem in the second pass CSRs.³ We examine the extent to which inferences are affected by taking into account this source of measurement error.

Our main finding concerns the lack of robustness of the CRR/CCH results. Twenty size-based portfolios serve as the assets in their studies. Alternative experiments in which securities are grouped on the basis of market beta, standard deviation of return, or price level were conducted

¹ See Shanken (1987) and Kandel and Stambaugh (1987) for less restrictive conditions under which the CAPM is testable.

² Earlier work by Schipper and Thompson (1981) looks at the relation between stock returns and several macrovariables but does not test the associated pricing relations.

³ Unless noted otherwise, "asymptotic" refers to limiting results as the time series length approaches infinity and is applicable, strictly speaking, only to the "contemporaneous beta" scenario discussed below.

as well (see Footnote 8 of CRR), but were not successful. Although we too employ size-based groupings, the results are surprisingly sensitive to the specific way in which the portfolio returns are generated and β s are estimated. In particular, only the market index is significantly priced in our cross-sectional regressions on full-period post-ranking betas estimated from the size portfolio returns.

While we use the same factors as CRR, Ferson and Harvey (1991) explore several variations on the definition of the bond risk premium factor and report that its pricing is "highly sensitive to the definition of the variable and the subperiod". In other related work, Warga (1989) finds that the CCH pricing estimates can be quite sensitive to employment of a jacknife procedure in which data is deleted 1 month at a time in estimating the betas. He does not assess the impact on inferences about the risk premia, however.

Several methodological innovations in this paper deal with the use of factors that are derived from portfolio returns. These methods are relevant, in particular, whenever some factors in the model are defined as the spread between two portfolio returns, as is now common in the literature, e.g., Fama and French (1993). Additional economic restrictions implied in this case are incorporated in both estimation and testing of the linear risk-return relation. Shanken's (1985a) CSR T^2 test of linear factor pricing is adapted to this context and a new test, focusing directly on the factor portfolio constraints, is presented. Our application of these methods to the factors used by CRR/CCH allows us to combine evidence from the corporate and government bond markets with that from the common stock portfolios. We find that the risk premia estimates and the magnitude of the EIV adjustments to the standard errors are often sensitive to the inclusion of bond returns.

The rest of this paper is organized as follows. Section 2 discusses important aspects of the experimental design of this study. Pricing results and tests of multibeta risk-return linearity are presented in Section 3 for the basic 5-factor model, with size portfolios as the assets. Bonds as well as stocks are treated as assets in the 5-factor "restricted" model of Section 4, and Section 5 considers an expanded model that includes a stock market factor. Section 6 summarizes the paper and offers some conclusions.

2. Experimental design

The two-pass methodology involves (i) estimation of beta(s) for each asset in a first-pass timeseries regression of asset returns on the given factor(s), and (ii) estimation of the risk-return parameters, i.e., the zero-beta rate and price(s) of risk, by a CSR of the returns for the given assets on the betas estimated in the first pass. These CSRs are performed each month, and the results are aggregated by averaging the time series of estimates for each of the risk-return parameters.

Often in the literature, the betas used as independent variables in the CSR for a given month are estimated from prior data. We use the 5-year period ending in December of the previous calendar year and update the estimates annually. The data on the macroeconomic factors used by CRR were kindly provided by Richard Roll. Since the monthly series begins in 1953, the CSRs start in 1958 and end in 1983. Results based on contemporaneous estimation of betas and risk-return parameters are also presented. In this case, a single time-series regression for each asset and a series of CSRs are run over the same sample period.⁴ Note that contemporaneous full-

⁴ While data from 1953 to 1957 could have been included in this context, we chose the period 1958–1983 to facilitate comparison with the prior-beta results. Findings for the longer period were not very different from the results reported. CRR run CSR's through 1984 since only prior beta estimation is employed in their study.

period estimation of beta is standard in time-series tests of asset pricing models that focus on the intercepts in excess-return factor model regressions, e.g., Gibbons et al. (1989).

In order to reduce the EIV problem, CCH and CRR aggregate securities into 20 size portfolios. Given the contrast between results to be reported below and those of CCH/CRR, it is important to focus on some details of their methodology (CCH, p. 456):⁵

During each six-year interval, firms on the NYSE that exist at the beginning of the interval and have price information on December of the fifth year are chosen and ranked according to market value at the end of the fifth year. The firms are then put into one of the twenty portfolios arranged in order of increasing size... We first regress each of the 20 portfolios on the macrovariables in the first five years to estimate the variables' betas. Then we perform cross-sectional regressions of the twenty portfolios' returns on the obtained portfolios' multiple betas month-by-month in the sixth year...

Like other studies in the literature, CCH/CRR implicitly assume that the (unconditional) risk characteristics of their portfolios are (fairly) constant over each 6-year period — a 5-year beta estimation period and a subsequent year of CSRs. Given the work of Chan (1988) and Ball and Kothari (1989), however, there is reason to doubt the appropriateness of the assumption in this context.⁶

Ball and Kothari, in particular, argue that extreme return performance over a 5-year period is negatively correlated with changes in systematic risk, as measured by market beta. This could be due to changes in the risk of a firm's underlying cash flows as well as shifts in leverage. Since many of the "loser" ("winner") firms naturally find their way into the portfolios of smaller (larger) firms, as based on end-of-period rankings, there may be associated shifts in the betas of the size portfolios. If so, betas computed over the previous 5 years would give biased assessments of the true risk of the portfolio risks are overstated. Ball and Kothari do not consider the effect on inferences about the risk-return parameters, but it seems likely that risk premia estimates would be biased upward to compensate for the reduced spread in betas. Of course, the situation is more complicated when multiple measures of systematic risk are considered.⁷

An alternative approach is adopted here that avoids these selection biases, and thus provides information about the sensitivity of the CCH/CRR results to such considerations. The portfolio formation approach is similar to that used in the classic studies by Black et al. (1972), Black and Scholes (1974), and more recently in the influential paper by Fama and French (1992). Equally weighted size portfolios are formed based on securities' total market value of equity at the end of December of each year, and returns are computed for these portfolios in each month of the *following* year. The resulting portfolios can be viewed as "mutual funds" with changing compositions, but identities of their own.⁸ Betas are computed from these fund returns, either

⁵ Although the description in CRR is less complete, Nai-Fu Chen indicates, in a personal communication, that essentially the same procedures were used by CCH and CRR. He has also informed us that in an early, unpublished, version of CCH a limited investigation of some of the issues that we raise below regarding portfolio formation was conducted.

⁶ We are grateful to S.P. Kothari and Jerold Warner for helpful conversation on this point.

⁷ As Richard Roll has pointed out to us, a simple EIV analysis indicates that the bias is upward if the cross-sectional covariance between the post ranking β s and the errors (ranking β s — post-ranking β s) divided by the variance of the errors is less than -1. An estimate of this ratio based on the numbers in Table 2 of Ball and Kothari (1989) is -2.36. Of course, this computation ignores estimation errors in the β s.

⁸ This approach to size portfolio formation is also used by Chan and Chen (1988).

prior to or contemporaneously with CSRs. Since the funds' returns are not conditioned on ex post performance, the selection biases discussed earlier are avoided.

Experiments are run with 20 or 60 size portfolios in order to explore the precision gains from using more than the usual 20 assets.⁹ Like CCH, we use weighted-least-squares (WLS) regressions, although similar results are obtained using ordinary least squares (OLS), as in CRR. Our reason for using portfolios is the usual one. The second-pass estimators are biased in small samples due to the EIV problem. Using portfolios mitigates this problem since estimation error in the asset betas is reduced. While our use of EIV-corrected standard errors may provide a more realistic assessment of the precision of the estimates, it does not eliminate the small-sample bias.

Originally, we had intended to use a modified version of the two-pass technique, given in Shanken (1992a), and similar to that employed by Litzenberger and Ramaswamy (1979). This involves subtracting an appropriate expression from the cross-product matrix of the estimated beta vectors in order to neutralize the impact of measurement error. Under certain conditions, the modified estimator is unbiased in the limit as the number of securities tends to infinity. We find, however, that in practice the adjustment often yields a modified cross-product matrix that is not positive definite, as it should be in the limit. Clearly, more work on the properties of this methodology is needed.

We also experimented with a version of the maximum-likelihood estimator, computed under the assumption that the residual covariance matrix is diagonal. Like the modified two-pass procedure, this permits the use of a large number of securities. However, preliminary estimates obtained seemed so extreme, given the economic interpretation of the coefficients, that we decided to abandon this approach as well. Additional results are reported for a generalized least squares (GLS) version of the two-pass procedure.

3. The unrestricted 5-factor model

In this section, the pricing of the 5 CRR factors is examined and tests of the validity of the 5-factor multibeta model are presented. We begin by considering the following model for excess returns:¹⁰

$$R_{pt} = \alpha_p + \beta_{1p} MP_t + \beta_{2p} DEI_t + \beta_{3p} UI_t + \beta_{4p} UPR_t + \beta_{5p} UTS_t + \varepsilon_{pt}$$
(1)

where: R_{pt} =the excess return on size portfolio p for month t; MP=the percentage change in industrial production led by 1 month; DEI=the change in expected inflation; UI=contemporaneous unanticipated inflation; UPR=the excess return of low grade corporate bonds over longterm government bonds; UTS=the excess return of long-term government bonds over T-bills with 1 month to maturity and the ε_p values are assumed to be zero mean disturbances with constant covariance matrix conditional on the factors. Returns and factors are assumed to be independent and identically distributed over time. Using excess returns allows for a changing zero-beta rate and facilitates the imposition of some restrictions considered below, but has little effect on the results.

The assumption of independence over time is often considered an adequate approximation for stock returns. It is less clear for a macrovariable like MP, since the realized growth rate may

⁹ In an earlier version of the paper, we also reported results with N=120 size portfolios. Since the smallest size portfolio would likely consist of extremely thinly traded stocks in this case, we decided to omit these results. Conclusions drawn from these portfolios were qualitatively similar to those based on N=20 and N=60, in any event.

¹⁰ See Chen et al. (1986) for a more detailed description of the factors used here.

deviate substantially from the innovation. CCH, in particular, are explicit about having removed the seasonal in the industrial production factor. However, they also note that the pricing of the industrial production factor "becomes marginal" when the factors are pre-whitened through a simple autoregressive transformation. This should be kept in mind in what follows, since we use the original factors of CRR and do not attempt to extract innovations.

In estimating the linear expected return relation, we implicitly assume that the factors capture the relevant components of systematic return in the sense that the disturbances in (1) are uncorrelated with the appropriate equilibrium benchmark. The familiar benchmarks are the market return in the static CAPM and (the marginal utility of) consumption in some versions of the intertemporal model. In this case (see Shanken, 1987, 1992a) there exist risk-return parameters (the γ s) such that:

$$E(R_{pt}) = \gamma_0 + \gamma_1 \beta_{1p} + \dots + \gamma_5 \beta_{5p}$$
⁽²⁾

for p = 1, 2, ..., N, where N is the number of size portfolios and γ_0 is the *excess* zero-beta rate. Although approximate versions of (2) can be derived under the weaker assumption of no-asymptotic arbitrage (the APT of Ross, 1976), as argued in Shanken (1982, 1992b), such approximations do not give rise to empirically testable restrictions on a *finite* set of assets.¹¹

3.1. Pricing results

Estimates of the gammas are obtained for each month t, by regressing the size portfolio returns on estimates of the betas and a constant. Suppressing notation for estimation of the betas, we have

$$R_{pt} = \gamma_0 + \gamma_1 \beta_{1p} + \dots + \gamma_5 \beta_{5p} + \eta_{pt} \quad p = 1, 2; \dots, N$$
(3)

where η_{pt} is the (unconditionally) unexpected return. The results of estimating (3) for the full period 1958–1983 and the subperiod 1968–1977, for comparison with CRR, are presented in Tables 1 and 2, respectively. They are referred to as the "unrestricted" estimates.

Let $\hat{\Gamma} = (\hat{\gamma}_0, \hat{\gamma}_1, \dots, \hat{\gamma}_5)$ be the sample mean vector of the monthly time series of CSR estimates and let $s(\gamma_k), k=0, 1, \dots, 5$ be the usual Fama–MacBeth standard error of the mean estimate for γ_k computed from the time series of CSR estimates. These standard errors are given in parentheses below each estimate. The second, in brackets, is the square root of the EIV-adjusted asymptotic variance (see Shanken, 1992a):

$$[s^{2}(\gamma_{k}) - s_{k}^{2}](1+c) + s_{k}^{2}$$
⁽⁴⁾

where s_k^2 is the variance of the mean for factor k (zero when k=0) and

$$c = \hat{\Gamma}' S_{\rm F}^{-1} \hat{\Gamma} \tag{5}$$

with $S_{\rm F}$ the sample covariance matrix of the factors.¹²

Panel A of Table 1 uses the contemporaneously estimated betas, or what Fama and French would call the full-period post-ranking betas. We focus on the conventional unrestricted estimates for now. Surprisingly, not a single factor risk premium estimate is more than two EIV-adjusted standard errors from zero. Moreover, the *p*-values for the *F*-test of the joint hypothesis

¹¹ See Shanken (1985b) and Dybvig and Ross (1985) for an exchange on the testability of the APT.

¹² In the prior-beta scenario, c is multiplied by l - l.6/n, where n is the number of years of CSRs. See the Appendix of Shanken (1992a).

Table 1	
Five-factor pricing resu	ults (1958–1983)

Specification	Constant	MP	DEI	UI	UPR	UTS	Pricing ^a
A. Contemporaneous be	tas						
Unrestricted (N=20)	30^{b}	.32	.00	04	.12	.36	1.39 (.23)
	(.32)	(.47)	(.01)	(.04)	(.26)	(.38)	
	[.34]	[.50]	[.01]	[.04]	[.28]	[.40]	
Unrestricted (N=60)	19	.55	00	04	.22	.16	1.45 (.21)
	(.27)	(.31)	(.01)	(.02)	(.21)	(.24)	
	[.28]	[.32]	[.01]	[.03]	[.22]	[.25]	
Restricted ^c (N=20)	-1.08*	2.92*	.0 3	11	02	.91	4.20* (.00)
	(.30)	(.78)	(.02)	(.08)	(.13)	(.34)	
	[.52]	[1.33]	[.03]	[.14]	[.13]	[.54]	
Restricted (N=60)	95*	2.70*	.01	11	02	.78	3.85* (.00)
	(.26)	(.67)	(.01)	(.06)	(.13)	(.30)	
	[.41]	[1.04]	[.02]	[.09]	[.13]	[.43]	
B. Five-year prior betas							
Unrestricted (N=20)	.00	1.19*	01	01	.23	.09	2.62* (.02)
	(.34)	(.46)	(.01)	(.03)	(.28)	(.30)	
	[.38]	[.50]	[.01]	[.03]	[.30]	[.32]	
Unrestricted (N=60)	.01	.91*	00	02	.37	10	2.19 (.06)
	(.28)	(.29)	(.00)	(.02)	(.20)	(.15)	
	[.30]	[.30]	[.00]	[.02]	[.20]	[.15]	
Restricted (N=20)	35	2.29*	01	03	02	.18	3.41* (.01)
	(.27)	(.66)	(.01)	(.05)	(.13)	(.28)	
	[.35]	[.84]	[.01]	[.06]	[.13]	[.38]	
Restricted (N=60)	38	1.80*	01	05	02	.21	3.79* (.00)
	(.21)	(.46)	(.01)	(.03)	(.13)	(.23)	
	[.25]	[.54]	[.01]	[.04]	[.13]	[.30]	

Estimates of the (excess) zero-beta rate and factor prices of risk based on the two-pass cross-sectional regression methodology with either contemporaneously estimated betas or betas estimated from 5 years of prior data. The factors are: MP=the percentage change in industrial production, DEI=the change in expected inflation, UI=unanticipated inflation, UPR=the return on low grade corporate bonds minus the return on long-term government bonds, and UTS=the excess return on long-term government bonds. N is the number of size portfolios employed as assets.

^a F statistic for testing the joint hypothesis that all prices of risk equal 0. P-value in parenthesis.

^b All estimates are multiplied by 100. Number in parenthesis is the time-series standard error of the mean monthly estimate. Number in brackets is the standard error adjusted for errors-in-variables in cross-sectional regressions.

^c Incorporates the restriction that the prices of risk for UTS and UPR are E(UTS)-constant and E(UPR), respectively.

* Significant at the 0.05 level.

that all prices of risk equal zero, given in the last column of Table 1, are greater than 0.20 for both N=20 and N=60.¹³

When betas are estimated from 5 years of data preceding each CSR, the risk premium for MP is significant, but none of the other estimates is more than 2 standard errors from zero in Panel B. Increasing the number of portfolios reduces the MP risk premium estimate somewhat, but it is about three times the substantially reduced standard error when N=60. Of course, focusing on the most extreme statistic can lead to an inflated impression of significance and so it is important to consider the joint test for factor pricing as well. This test tells a slightly different story. The *p*-value is 0.02 when N=20, but it increases to 0.06 when N=60, not quite significant at the usual 0.05 level.

¹³ No EIV-adjustment is needed in this case since the beta estimation error has no impact under the null hypothesis.

Specification	Constant	MP	DEI	UI	UPR	UTS	Pricing ^a
A. Contemporaneous be	tas						
Unrestricted (N=20)	35 ^b	.94	.01	01	17	.50	.33 (.90)
	(.65)	(.85)	(.02)	(.06)	(.46)	(.74)	
	[.71]	[.93]	[.02]	[.06]	[.50]	[.80]	
Unrestricted (N=60)	45	.39	00	06	.03	.19	.79 (.56)
	(.49)	(.58)	(.01)	(.04)	(.37)	(.39)	
	[.52]	[.61]	[.01]	[.04]	[.39]	[.40]	
Restricted ^c (N=20)	38	.71	.01	02	03	.34	.29 (.92)
	(.46)	(1.03)	(.02)	(.08)	(.21)	(.51)	
	[.49]	[1.09]	[.02]	[.09]	[.21]	[.54]	
Restricted (N=60)	22	.72	.00	04	03	.18	.63 (.67)
	(.36)	(.96)	(.01)	(.06)	(.21)	(.43)	
	[.39]	[1.02]	[.01]	[.06]	[.21]	[.45]	
B. Five-year prior betas							
Unrestricted $(N=20)$	67	1.22	01	.01	.42	10	1.27 (.28)
	(.67)	(.80)	(.01)	(.05)	(.58)	(.59)	
	[.75]	[.88]	[.01]	[.06]	[.64]	[.65]	
Unrestricted (N=60)	54	.66	01	04	.39	21	.91 (.48)
	(.52)	(.49)	(.01)	(.03)	(.40)	(.26)	
	[.54]	[.50]	[.01]	[.03]	[.41]	[.26]	
Restricted (N=20)	29	1.84	.00	.00	03	.25	.72 (.59)
	(.60)	(1.12)	(.02)	(.07)	(.21)	(.61)	
	[.70]	[1.29]	[.02]	[.08]	[.21]	[.74]	
Restricted $(N=60)$	22	1.31	.01	02	03	.18	.76 (.58)
	(.45)	(.80)	(.01)	(.06)	(.21)	(.48)	
	[.50]	[.87]	[.02]	[.06]	[.21]	[.54]	

Table 2 Five-factor pricing results (1968–1977)

Estimates of the (excess) zero-beta rate and factor prices of risk based on the two-pass cross-sectional regression methodology with either contemporaneously estimated betas or betas estimated from 5 years of prior data. The factors are: MP=the percentage change in industrial production, DEI=the change in expected inflation, UI=unanticipated inflation, UPR=the return on low grade corporate bonds minus the return on long-term government bonds, and UTS=the excess return on long-term government bonds. *N* is the number of size portfolios employed as assets. *Significant at the 0.05 level.

^a F statistic for testing the joint hypothesis that all prices of risk equal 0. P-value in parenthesis.

^b All estimates are multiplied by 100. Number in parenthesis is the time-series standard error of the mean monthly estimate. Number in brackets is the standard error adjusted for errors-in-variables in cross-sectional regressions.

^c Incorporates the restriction that the prices of risk for UTS and UPR are E(UTS)-constant and E(UPR), respectively.

These results are very different from those obtained by CRR/CCH. For the comparable period and same factors, CRR report (Panel B of Table 4) *t*-statistics of 3.6, -2.0, and 2.6 for MP, UI, and UPR, respectively, when N=20 size porfolios. They do not perform joint tests of significance. Recall that the experimental design employed by CRR is the same as that used to generate our Panel B results except for the method of generating size portfolio returns. In both cases, betas are estimated using 5 years of returns prior to the month of the CSR. However, whereas CRR use 5 years of *pre-ranking* returns to estimate beta, we use 5 years of *post-ranking* returns.¹⁴

¹⁴ We have replicated the CRR results using their portfolio data and our programs, and have obtained similar results starting from scratch with the version of the CRSP tapes that was current when the earlier draft of this paper was written in 1990.

CRR also provide results for three subperiods, with significant evidence of factor pricing coming mainly from the 1968–1977 period. In this period, all five factors have t-statistics greater than 2.5 in magnitude and two exceed 3.2, an impressive display of significance for what is, by asset pricing standards, a fairly short period. In striking contrast, we find no evidence of factor pricing for this period. Not a single estimate in Table 2 is more than two (unadjusted) standard errors from zero. The point estimates in Panels A and B are often quite different from each other and from the CRR estimates. For example, the UPR estimate (N=20) is -0.17 with contemporaneous betas and 0.42 with prior betas, as compared to the CRR estimate of 1.27. Furthermore, none of the joint tests for factor pricing is significant, even at the 0.10 level.

The use of changing composition portfolio returns based on characteristic rankings, as in Tables 1 and 2, is well-established in the literature. To further investigate robustness, however, we ran an additional experiment (not shown) which, like the approach of CCH/CRR, uses *fixed* composition portfolios based on market value rankings just prior to the year over which CSRs are run. The difference is that our β s are estimated using returns from the 5 years *following*, rather than preceding, the CSRs. Again, no evidence of pricing is found for the 1968–1977 subperiod.

We also explored the use of GLS CSR estimation of the risk-return parameters in the contemporaneous beta scenario. The weighting is based on the sample covariance matrix, $\hat{\Sigma}$, of the residuals from the factor model in (1). As noted in Shanken (1992a), the GLS estimator is identical to a Gauss–Newton estimator and is asymptotically equivalent to the maximum likelihood methods suggested by Gibbons (1982) and used by Stambaugh (1982) and McElroy and Burmeister (1987), among others. Given the difficulty in precisely estimating a large covariance matrix and, especially, its inverse, we doubted that GLS with a large number of assets would be very efficient. Also, the asymptotic standard errors for the risk-return parameters do not reflect the noise in the covariance estimates. Therefore, they may understate the true variability of the GLS estimator when N is large.¹⁵

In light of these concerns, we decided to limit the experiment to 20 size portfolios. None of the estimates (not shown) is more than two adjusted standard errors from zero in this case. Interestingly, the GLS standard errors tend to lie between the WLS standard errors with 20 or 60 portfolios. Given our limited knowledge about the small-sample properties, strong conclusions are not possible. However, this finding, together with our earlier observations, suggests that using OLS or WLS with a relatively large set of assets may be the preferred approach in work of this sort.

To summarize, we find some evidence that MP has a positive price of risk over the period 1958–1983, consistent with the findings of CRR/CCH. However, the statistically significant evidence for MP is limited to the 5-year prior beta method and is diminished when a joint test of factor pricing is conducted. Moreover, there is no indication that the other factors are priced. The results are surprisingly weak across a range of experimental designs, all of which appear to be reasonable alternatives to that of CRR/CCH, including the full-period post-ranking beta approach used by Fama–French and others. Whether the differences are driven by the specific selection biases discussed earlier, or have some other basis, our findings raise significant doubts about the 5-factor model.

3.2. Tests of the multibeta relation

Estimation results based on the Fama–MacBeth procedure implicitly take the validity of the risk-return relation in (2) as given. If the relation is violated, betas may still appear to be priced,

¹⁵ MacKinlay (1987) alludes to some simulation evidence on this point.

in that a linear function of the betas is correlated with the expected return vector. However, the correlation may be far from perfect, with substantial deviations between the linear relation and the true expected returns. In this case, it would be inappropriate to rely on the model-based estimates of expected return. Therefore, in this section we consider tests of the linearity hypothesis. Shanken (1985a) develops a multivariate CSR T^2 test of (2) against a general alternative for the contemporaneous beta scenario. The test is asymptotically equivalent to the likelihood ratio test under normality. However, as Shanken shows, the usual asymptotic approximations perform poorly when the number of assets is substantial relative to the length of the time series of returns.¹⁶ This is due to the sampling properties of Σ discussed in the previous section.

Let e be the mean vector of the time series of GLS CSR residuals for the N assets. Each component of e is an estimate of the model "mispricing" for a given asset. The CSR test statistic aggregates these estimated deviations in a quadratic form:

$$\frac{Te'\hat{\Sigma}^{-1}e}{(1+c)}\tag{6}$$

where c is the EIV-adjustment term defined in (5). Under the null hypothesis, this statistic is asymptotically distributed as χ^2 with N-K-1 degrees of freedom (K is the number of factors). Shanken argues that, in small samples, Hotelling's T^2 distribution with degrees of freedom N-K-1 and T-K-1 should provide a better approximation.¹⁷ Unusually large values of the statistic, after accounting for estimation error, suggest that the residuals systematically differ from zero, thereby violating the null hypothesis.

Since $\hat{\Sigma}$ is singular when *N* exceeds T - K - 1, the CSR test requires that the number of assets employed be limited. The test was run with 20 or 60 portfolios in the overall period and the subperiod considered earlier. None of the statistics (not shown) is significant at the 0.10 level. Thus we are unable to reject, at conventional levels, the hypothesis that expected return is a linear function of the betas in the 5-factor model.¹⁸ However, given the lack of any evidence of factor pricing using contemporaneous betas, failure to reject may be a reflection of the possibly low power of the test.

4. The restricted 5-factor model

Thus far, only size portfolios have been employed as assets in our empirical analysis. The precision of the estimators and the power of the tests may be increased by including other assets. Stambaugh (1982) emphasizes this point in testing the CAPM. Two obvious candidates for inclusion are the long-term government bond and low-grade bond portfolios from which the factors, UTS and UPR, are derived. The analysis below is a natural extension of techniques used by Black et al. (1972), to the multifactor context.

¹⁶ See related work by Jobson and Korkie (1982) and Stambaugh (1982).

¹⁷ See Amsler and Schmidt (1985) for some simulation evidence supporting the use of this approximation when K=1.

¹⁸ In Shanken and Weinstein (1990), we also explored dummy variables tests of equality of the expected return parameters when CSR estimation is conducted separately for two sets of test assets. The two sets of portfolios were based on size rankings. The test provided some indication of a size effect, but little evidence against stability of the five-factor pricing relation otherwise. In retrospect, we have doubts about the small-sample properties of the test in this application since there may not be much variation in the true factor betas within each size grouping. The stability test could prove useful in other contexts, however, and is appealing in that it does not impose restrictions on N, unlike the multivariate tests.

4.1. Econometric analysis

Assuming the bond portfolios satisfy the underlying equilibrium pricing relation, the following restrictions on the prices of risk must hold:

$$E(\text{UTS}) = \gamma_0 + \gamma_5 \tag{7}$$

and

$$E(\text{UPR}) = \gamma_4. \tag{8}$$

To establish these relations, note that the government bond portfolio has an excess-return beta of one on the UTS factor and zero on the other four factors. Similarly, the difference between the low-grade bond and government bond portfolio betas must be one for the UPR factor and zero for the remaining factors.¹⁹ Combining these facts with the multibeta expected return relation yields (7) and (8).

Substituting (7) and (8) in (2) and rearranging gives:

$$E(R_{pt}) - E(\text{UTS})\beta_{5p} - E(\text{UPR})\beta_{4p} = \gamma_0(1 - \beta_{5p}) + \gamma_1\beta_{1p} + \gamma_2\beta_{2p} + \gamma_3\beta_{3p}$$
(9)

The parameters γ_0 , γ_1 , γ_2 and γ_3 can be estimated by means of restricted CSRs with dependent variable $R_{pt} - \text{UTS}_t \beta_{5p} - \text{UPR}_t \beta_{4p}$ and independent variables $1 - \beta_{5p}$, β_{1p} , β_{2p} and β_{3p} , p = 1, 2, ..., N. Apart from these modifications, estimation and inference for γ_0 , γ_1 , γ_2 and γ_3 proceeds as in Section 3.1, with the usual OLS time-series estimates substituted for the true betas.²⁰ As suggested by (7) and (8), the other restricted price of risk estimates are $\hat{\gamma}_5 = \overline{\text{UTS}} - \hat{\gamma}_0$ and $\hat{\gamma}_4 = \overline{\text{UPR}}$, where $\overline{\text{UTS}}$ and $\overline{\text{UPR}}$ are time series sample means and $\hat{\gamma}_0$ is the (restricted) CSR estimate of γ_0 .

The asymptotic variance for $\hat{\gamma}_5$ equals the variance for $\hat{\gamma}_0$ plus the variance of UTS. Interestingly, the pricing of the UPR factor can be evaluated with a standard *t*-test on the factor mean. Since this does not involve CSRs, it is not subject to the usual EIV complications. In particular, the restricted estimator for γ_4 is unbiased and more efficient than the CSR estimator considered in Section 3.1 (see (4)). To understand why, recall that a CSR price of risk estimator can be viewed as the return on a zero-investment portfolio with betas (asymptotically) of one on the given factor and zeroes on the other factors (see Fama, 1976, Chap. 9). In the case of UPR, the factor is itself such a portfolio, but one with no residual factor model risk. Therefore, it has the smallest variance of all such portfolio estimators.

The equality between a factor risk premium and the factor mean is familiar from the Sharpe– Linter version of the CAPM and its implementation in terms of an excess return market model. In that context, however, the zero-beta rate must be assumed to equal the riskless rate or, equivalently, the riskless asset must satisfy the linear expected return relation. Here, the more general conclusion for UPR follows from the observation that it is a difference between two portfolio (excess) returns. Similar conclusions would apply to the Fama and French (1993) size and book-to-market factors in a CSR context.

¹⁹ Note that the beta of a difference of portfolios equals the difference of the portfolio betas.

 $^{^{20}}$ This entails a simple modification of the analysis in Shanken (1990) to accommodate a factor that is the difference of two portfolio returns.

The CSR T^2 test statistic for the restricted pricing relation (9) is similar to that in (6), except that *e* is now the mean vector of *restricted* CSR residuals and the restricted estimates are used to compute *c*. The degrees of freedom are $N-K_1-1$ and T-K-1, where K_1 is the number of factors (in this case 3) which do not satisfy additional constraints like (7) and (8). As usual, the former degrees of freedom is equal to the number of assets minus the number of independent variables in the second-pass CSRs, while the latter is the time-series length minus the number of independent variables in the first-pass time-series regressions.

4.2. Empirical evidence

The price of risk estimates for MP in Table 1 are significantly greater than zero for all restricted specifications over the 1958–1983 period, despite large EIV adjustments to the standard errors. The point estimates are much larger than the corresponding unrestricted estimates and the statistical significance is supported by the joint pricing tests.²¹ There is no evidence of factor pricing for the 1968–1977 subperiod.

The simple (restricted) *t*-tests for UPR, reported in Tables 1 and 2, fail to produce any evidence of pricing. This is so, even though imposing (8) reduces the standard error on the UPR premium by 50% in some cases. The estimates are (insignificantly) negative, indicating that the average return on low grade bonds is slightly less than that for long-term governments. Of course, with estimates so close to zero, we cannot rule out a positive value for the true risk premium, which would be expected a priori. The restricted estimates of the zero-beta rate over the full period are about 1% per month below the riskless rate with contemporaneous betas and are statistically significant. This anomalous result also leads to large positive restricted estimates of the UTS risk premium. The latter are less than two standard errors from zero, however, when the EIV adjustment is made.²²

4.3. A test of the bond pricing restrictions

Although the incorporation of bond pricing restrictions was motivated as a means of increasing the precision of the risk premia estimates, the restricted standard errors shown in Tables 1 and 2 are frequently larger than their unrestricted counterparts. This suggests that the restrictions may be violated. While some of the differences between restricted and unrestricted estimates look fairly large, a formal test is needed to determine whether they are statistically significant.

Once again, we use an EIV-adjusted version of Hotelling's T^2 statistic, now applied to the monthly series of differences (restricted – unrestricted) for the UPR and UTS price estimates. This provides a test of (7) and (8) conditional on (2) being a valid representation of size portfolio expected returns. Under the null hypothesis, either the restricted or unrestricted price estimates could be used to compute the EIV adjustment term 1+c. The unrestricted estimates were chosen based on our intuition that adjustments derived from the restricted estimates might not be "sensible" if the null hypothesis is, in fact, false. None of the statistics is found to be significant at the 0.10 level.

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²¹ We also computed EIV adjustments with the price for a given factor set equal to zero in the computation of c. This is consistent with asymptotic analysis under the null hypothesis that the factor's price is indeed zero. Results based on this adjustment were not very different from the results reported.

 $^{^{22}}$ The CSR T^2 test of the restricted model was conducted, as before, using 20 or 60 size portfolios. Neither test is significant at the 0.05 level.

5. An expanded factor model

The five CRR factors typically account for only about 25% or 30% of the time-series variation in our 20 size portfolio returns. The time-series R^2 rises to about 80% when the excess return on the value-weighted CRSP stock index (VW) is included as a sixth factor. Thus, it seems likely that the usual market factor captures potentially important components of systematic risk not reflected in the other macro-factors. CCH/CRR find that the addition of an equity index does not have much of an effect on the pricing of the other factors, and the index itself is not significantly priced.

Our 6-factor results for the overall period 1958–1983 are presented in Table 3. VW is treated exactly like UTS in the restricted models, with the restricted price of risk equal to $E(VW) - \gamma_0$. Surprisingly, given the CRR/CCH results, there is fairly robust evidence of positive prices of risk for VW.²³ The statistically significant evidence for MP is limited to the prior beta method in Panel B. Some indication of nonzero prices exists for other factors, but the evidence is less consistent across specifications. The joint tests of factor pricing are significant at the 0.01 level only for the restricted specifications.

The CSR T^2 tests of expected return linearity are not significant at the 0.10 level. The test for equality of the restricted and unrestricted UPR, UTS, and VW risk premia provides some limited evidence against the model in (2), with a *p*-value of 0.01 when N=60 and contemporaneous betas are used in estimation. *P*-values for the other specifications are greater than 0.10.

Overall, analysis of the expanded factor model reinforces the impression that the results of CRR/CCH are quite sensitive to reasonable alternative estimation procedures.

6. Summary and conclusions

In this paper, we have examined the relation between expected returns and measures of systematic risk with respect to five macroeconomic factors studied by Chan et al. (1985) and Chen et al. (1986). Like CCH/CRR, we use a version of Fama and MacBeth's (1973) two-pass methodology with securities grouped into portfolios based on annual rankings of the market value of equity ("size"). However, whereas CCH/CRR estimate betas using backward-looking returns, relative to the ranking dates, we employ post-ranking returns throughout as, for example, in Fama and French (1992). This seemingly small change leads to strikingly different conclusions.

With an experimental design comparable to that of CCH/CRR in other respects, only the industrial production factor (MP) is significantly priced in the overall period of 1958–1983. The sample mean of the bond return premium UPR, a highly significant factor in the earlier studies, is insignificantly negative for this period. This is particularly noteworthy in that the mean is the most efficient estimator under the usual simplifying assumptions and so questions about the best way to estimate betas play no role in this conclusion. Thus, although it seems likely a priori that the true mean is positive for this factor, the conclusion cannot be established based on the data for this period.

We fail to find *any* evidence of factor pricing in the 1968–1977 subperiod, a surprising result in that CRR obtain *t*-statistics greater than 2.5 for all five factors in this period! In one

²³ Over the 1964–1986 period, Ferson and Harvey (1991) find evidence of pricing for the VW market factor considered by itself, but not in the context of a multifactor model that contains some of the factors used here.

Table 3 Six-factor pricing results (1958–1983)

Specification	Constant	MP	DEI	UI	UPR	UTS	VW	Pricing ^a
A. Contemporaneous be	etas							
Unrestricted (N=20)	-1.05^{b}	13	00	.00	.20	.32	1.48*	1.68 (.13)
	(.54)	(.49)	(.01)	(.05)	(.38)	(.38)	(.57)	
	[.58]	[.52]	[.01]	[.05]	[.40]	[.40]	[.61]	
Unrestricted (N=60)	38	.40	00	04	.32	.08	.85*	1.39 (.22)
	(.29)	(.32)	(.01)	(.02)	(.22)	(.24)	(.37)	
	[.30]	[.33]	[.01]	[.25]	[.22]	[.25]	[.38]	
Restricted ^c ($N=20$)	-1.13*	.96	02	.01	02	.96*	1.47*	3.78 (.00)
	(.22)	(.68)	(.01)	(.04)	(.13)	(.26)	(.32)	
	[.26]	[.82]	[.01]	[.05]	[.13]	[.30]	[.35]	
Restricted $(N=60)$	89*	.83	02*	05	02	.72*	1.23*	3.04 (.01)
	(.18)	(.45)	(.01)	(.03)	(.13)	(.24)	(.30)	
	[.21]	[.50]	[.01]	[.03]	[.13]	[.26]	[.31]	
B. Five-year prior beta	S							
Unrestricted $(N=20)$.30	.83	01	03	.43	04	.15	1.84 (.09)
	(.49)	(.41)	(.01)	(.03)	(.31)	(.31)	(.43)	
	[.53]	[.44]	[.01]	[.03]	[.33]	[.34]	[.46]	
Unrestricted (N=60)	.10	.85*	00	02	.42*	09	.37	1.85 (.09)
	(.32)	(.27)	(.00)	(.02)	(.19)	(.16)	(.26)	
	[.34]	[.28]	[.00]	[.02]	[.20]	[.16]	[.26]	
Restricted (N=20)	53*	1.76*	01	04	02	.36	.87*	3.10 (.01)
	(.22)	(.56)	(.01)	(.04)	(.13)	(.23)	(.29)	
	[.26]	[.66]	[.01]	[.04]	[.13]	[.30]	[.35]	
Restricted (N=60)	46*	1.46*	01	04	02	.29	.80*	3.69 (.00)
	(.16)	(.36)	(.01)	(.02)	(.13)	(.19)	(.25)	
	[.18]	[.40]	[.01]	[.02]	[.13]	[.24]	[.30]	

Estimates of the (excess) zero-beta rate and factor prices of risk based on the two-pass cross-sectional regression methodology with either contemporaneously estimated betas or betas estimated from 5 years of prior data. The factors are: MP=the percentage change in industrial production, DEI=the change in expected inflation, UI=unanticipated inflation, UPR=the return on low grade corporate bonds minus the return on long-term government bonds, UTS=the excess return on long-term government bonds, and VW=the return on the value-weighted CRSP stock index. N is the number of size portfolios employed as assets.

^a F statistic for testing the joint hypothesis that all prices of risk equal 0. P-value in parenthesis.

^b All estimates are multiplied by 100. Number in parenthesis is the time-series standard error of the mean monthly estimate. Number in brackets is the standard error adjusted for errors-in-variables in cross-sectional regressions.

^c Incorporates the restriction that the prices of risk for VW, UTS and UPR are E(VW)-constant, E(UTS)-constant and E(UPR), respectively.

more contrast with CRR, the risk premium for the value-weighted CRSP stock index return (VW) is significantly positive in most of our specifications that include the index as a sixth factor. The zero-beta rates in these specifications are implausibly low, however.

Although our most positive results are obtained for MP, one reservation about this factor should be noted. In principle, factors ought to capture innovations in the relevant state variables. However, CCH observe that the pricing of the industrial production factor "becomes marginal" when the factors are pre-whitened through a simple autoregressive transformation. Thus, lack of robustness is again a concern. A recent paper by Vassalou (2003) argues that a mimicking portfolio designed to capture news about future gross domestic product helps explain the cross-section of stock returns. Whether this result holds up to further scrutiny only time will tell.

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