$$Y_i \overset{iid}{\sim} \text{PTruncada}(\lambda) \quad (i=1,\dots,m)$$

$$f(y_i;\lambda) = \frac{e^{-\lambda}\lambda^{y_i}}{y_i!\,(1-e^{-\lambda})} \quad y_i = 1, 2, \dots$$

$$E(Y) = \frac{1}{(1-e^{-\lambda})} \underbrace{\sum_{y=1}^{\infty} y \frac{e^{-\lambda}\lambda^{y}}{y!}}_{\lambda} = \frac{\lambda}{(1-e^{-\lambda})}$$

$$= \boxed{\frac{\lambda e^{\lambda}}{e^{\lambda}-1}}$$

$$E(Y^2) = \frac{1}{(1-e^{-\lambda})} \underbrace{\sum_{y=1}^{\infty} y^2 \frac{e^{-\lambda}\lambda^{y}}{y!}}_{\lambda+\lambda^2} = \frac{\lambda(1+\lambda)}{1-e^{-\lambda}}$$

$$= \boxed{\frac{\lambda(1+\lambda)e^{\lambda}}{e^{\lambda}-1}}$$

$$\text{Var}(Y) = E(Y^2) - E^2(Y)$$

$$= \frac{\lambda(1+\lambda)e^{\lambda}}{e^{\lambda}-1} - \frac{\lambda^2 e^{2\lambda}}{(e^{\lambda}-1)^2} = \boxed{\frac{\lambda e^{\lambda}}{e^{\lambda}-1}\left(e^{\lambda}-1-\lambda\right)}$$

$$f(y_i;\lambda) = \frac{\lambda^{y_i}}{y_i!\,(e^{\lambda}-1)}$$

$$\Rightarrow L(\lambda) = \sum_{i=1}^{n} \left\{ y_i \log(\lambda) - \log(y_i!) - \log(e^{\lambda}-1) \right\}$$

$$= n\bar{y}\log(\lambda) - n\log(e^{\lambda}-1) - \sum_{i=1}^{n} \log(y_i!)$$

$$U_\lambda = \frac{dL(\lambda)}{d\lambda} = \frac{n\bar{y}}{\lambda} - \frac{n e^{\lambda}}{e^{\lambda}-1}$$

$$U_{\hat{\lambda}} = 0$$

$$\Rightarrow \frac{\cancel{n}\bar{y}}{\hat{\lambda}} = \frac{\cancel{n} e^{\hat{\lambda}}}{e^{\hat{\lambda}}-1}$$

$$\underbrace{\frac{\hat{\lambda} e^{\hat{\lambda}}}{e^{\hat{\lambda}}-1} - \bar{y} = 0}$$

deve ser resolvido por NR

$$\frac{d^2L(\lambda)}{d\lambda^2} = -\frac{m\bar{y}}{\lambda^2} - \left[\frac{me^{\lambda}(e^{\lambda}-1) - e^{\lambda}me^{\lambda}}{(e^{\lambda}-1)^2}\right]$$

$$= -\frac{m\bar{y}}{\lambda^2} + \frac{me^{\lambda}}{(e^{\lambda}-1)^2}$$

$$K_{\lambda\lambda} = E\left[-\frac{d^2L(\lambda)}{d\lambda^2}\right]$$

$$= \frac{m}{\lambda^2}E(\bar{Y}) - \frac{me^{\lambda}}{(e^{\lambda}-1)^2}$$

$$= \frac{m\lambda e^{\lambda}}{\lambda^2(e^{\lambda}-1)} - \frac{me^{\lambda}}{(e^{\lambda}-1)^2}$$

$$= \frac{me^{\lambda}(e^{\lambda}-1) - m\lambda e^{\lambda}}{\lambda(e^{\lambda}-1)^2}$$

$$\boxed{K_{\lambda\lambda} = \frac{me^{\lambda}(e^{\lambda}-1-\lambda)}{\lambda(e^{\lambda}-1)^2}}$$

$$\text{Var}(\hat{\lambda}) = \frac{1}{K_{\lambda\lambda}}$$