$$Y_{ij} \overset{ind}{\sim} \mathcal{P}(\mu_i)$$

$$\log(\mu_1) = \alpha - \Delta$$
$$\log(\mu_2) = \alpha + \Delta$$

$(i=1,2)$ $(j=1,\dots,m)$

$$X = \begin{bmatrix} 1 & -1 \\ \vdots & \vdots \\ 1 & -1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix} \quad W = \mathrm{diag}\{\omega_1,\dots,\omega_1,\omega_2,\dots,\omega_2\}$$

$$X^T W X = \begin{bmatrix} 1 & \cdots & 1 & 1 & \cdots & 1 \\ -1 & \cdots & -1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \omega_1 & & & & \\ & \ddots & & & \\ & & \omega_1 & & \\ & & & \omega_2 & \\ & & & & \ddots \\ & & & & & \omega_2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ \vdots & \vdots \\ 1 & -1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} m(\omega_1+\omega_2) & m(\omega_2-\omega_1) \\ m(\omega_2-\omega_1) & m(\omega_1+\omega_2) \end{bmatrix}$$

$$(X^T W X)^{-1} = \frac{1}{m^2(\omega_1+\omega_2)^2 - m^2(\omega_1-\omega_2)^2} \begin{bmatrix} m(\omega_1+\omega_2) & m(\omega_1-\omega_2) \\ m(\omega_1-\omega_2) & m(\omega_1+\omega_2) \end{bmatrix}$$

$$\Big\downarrow$$

$$4m^2\omega_1\omega_2$$

Então

$$Var(\hat{\alpha}) = Var(\hat{\Delta}) = \frac{\omega_1+\omega_2}{4m\omega_1\omega_2}$$

$$Cov(\hat{\alpha},\hat{\Delta}) = \frac{\omega_1-\omega_2}{4m\omega_1\omega_2}$$

$$\omega = (d\mu/d\eta)^2/V \qquad V = \mu(\mu-1)$$

$$\mu = e^{\eta} \Rightarrow \frac{d\mu}{d\eta} = \mu$$

$$\Rightarrow \omega = \frac{\mu^2}{\mu(\mu-1)} = \frac{\mu}{\mu-1} \qquad \omega_1 = \frac{\mu_1}{\mu_1-1}$$

$$\omega_2 = \frac{\mu_2}{\mu_2-1}$$

③

$$U_\Delta = Z^T W^{\frac{1}{2}} V^{-\frac{1}{2}} (y-\mu)$$

$$Z^T = (-1 \dots -1 \quad 1 \dots 1)$$

$$\sqrt{\frac{\omega}{V}} = \sqrt{\frac{\cancel{\mu}}{(\mu-1)\cancel{\mu}(\mu-1)}} = \sqrt{\frac{1}{(1-\mu)^2}} = \frac{1}{(1-\mu)}$$

$$U_\Delta = (-1 \dots -1 \quad 1 \dots 1) \begin{bmatrix} (1-\mu_1)^{-1} & & & & \\ & \ddots & & & \\ & & (1-\mu_1)^{-1} & & \\ & & & (1-\mu_2)^{-1} & \\ & & & & \ddots \quad (1-\mu_2)^{-1} \end{bmatrix}$$

$$\times (y_{11}-\mu_1, \dots, y_{1m}-\mu_1, y_{21}-\mu_2, \dots, y_{2m}-\mu_2)^T$$

$$= \sum_{j=1}^{m} \frac{(y_{2j}-\mu_2)}{\mu_2 - 1} - \sum_{j=1}^{m} \frac{(y_{1j}-\mu_1)}{\mu_1 - 1}$$

$$\hat{Var}_0(\hat{\Delta}) = \frac{2\hat{\omega}^0}{4m[\hat{\omega}^0]^2} = \frac{1}{2m\hat{\omega}^0}$$

$$= \frac{(\bar{y}-1)}{2m\bar{y}}$$

$$\hat{U}^0_\Delta = \sum_{j=1}^{m} \frac{(y_{2j}-\bar{y})}{\bar{y}-1} - \sum_{j=1}^{m} \frac{(y_{1j}-\bar{y})}{\bar{y}-1}$$

$$= \frac{1}{\bar{y}-1}\left\{ m\bar{y}_2 - \cancel{m\bar{y}} - m\bar{y}_1 + \cancel{m\bar{y}} \right\}$$

$$= \frac{m(\bar{y}_2-\bar{y}_1)}{\bar{y}-1}$$

$$\xi_{SR} = \left[\hat{U}^0_\Delta\right]^2 \hat{V}ar_0(\hat{\Delta})$$

$$= \frac{m^{\cancel{2}}(\bar{y}_2-\bar{y}_1)^2}{[\bar{y}-1]^{\cancel{2}}} \frac{\cancel{(\bar{y}-1)}}{2\cancel{m}\bar{y}} = \boxed{\frac{m(\bar{y}_2-y_1)^2}{2\bar{y}(\bar{y}-1)}}$$

$$\xi_{SR} \underset{m\to\infty}{\overset{H_0}{\sim}} \chi^2_1$$