$Z \sim ZABI(m, \mu, \tilde{\pi})$

$$f_Z(z; \mu, \tilde{\pi}) = \begin{cases} \tilde{\pi} & \text{σε } z = 0 \\ \dfrac{(1-\tilde{\pi})\, f_Y(z; \mu)}{1 - P(Y=0)} & \text{σε } z = 1, 2, \ldots, m \end{cases}$$

$Y \sim B(m, \mu)$

$$E(Z) = 0 \times \tilde{\pi} + \frac{(1-\tilde{\pi})}{1-(1-\mu)^m} \sum_{z=1}^{m} z\, f_Y(z; \mu)$$

$$= \frac{(1-\tilde{\pi})}{1-(1-\mu)^m} m\mu = \boxed{\frac{m\mu(1-\tilde{\pi})}{1-(1-\mu)^m}}$$

$$E(Z^2) = 0^2 \times \tilde{\pi} + \frac{(1-\tilde{\pi})}{1-(1-\mu)^m} \sum_{z=1}^{m} z^2 f_Y(z; \mu)$$

$$= \frac{(1-\tilde{\pi})}{1-(1-\mu)^m} E(Y^2)$$

$$= \frac{(1-\tilde{\pi})}{1-(1-\mu)^m} \left\{ m\mu(1-\mu) + m^2\mu^2 \right\}$$

$$\operatorname{Var}(Z) = \frac{(1-\pi)}{1-(1-\mu)^m}\left\{m\mu(1-\mu) + m^2\mu^2\right\}$$

$$+ \frac{m^2\mu^2(1-\pi)^2}{\left\{1-(1-\mu)^m\right\}^2}$$

$$= \boxed{\frac{m\mu(1-\pi)}{1-(1-\mu)^m}\left[(1-\mu) + m\mu + \frac{m\mu(1-\pi)}{1-(1-\mu)^m}\right]}$$

Supon $Z_i \overset{iid}{\sim} ZABI(m, \mu, \pi)$

$(i=1,\dots,k)$ $\quad R \sim B(k, \pi)$

$$L(\mu, \pi) = r\log(\pi) + (k-r)\log(1-\pi)$$

$$+ (k-r)\log\left\{1-(1-\mu)^m\right\} + \sum_{i=r+1}^{k}\left[\log\binom{m}{z_i}\right.$$

$$\left. + z_i\log(\mu) + (m-z_i)\log(1-\mu)\right]$$

$$U_\pi = \frac{\partial L}{\partial \pi} = \frac{r}{\pi} - \frac{(k-r)}{1-\pi}$$

$$U_{\hat{\pi}} = 0 \quad \Rightarrow \quad \boxed{\hat{\pi} = \frac{r}{k}}$$

$$U'_{\pi} = -\frac{r}{\pi^2} + \frac{(k-r)}{(1-\pi)^2}$$

$$E\{-U'_{\pi}\} = \frac{k}{\pi} - \frac{k}{1-\pi} = \frac{k}{\pi(1-\pi)} = K_{\pi\pi}$$

$$\boxed{Var(\hat{\pi}) = \frac{\pi(1-\pi)}{k}}$$

$$K_{\pi\mu} = 0$$

$$U_{\mu} = \frac{\partial L}{\partial \mu} = \frac{(k-r)\, m\, (1-\mu)^{(m-1)}}{1-(1-\mu)^m} + \frac{k\bar{z}}{\mu}$$

$$- \frac{k(m-\bar{z})}{1-\mu}$$

$U_{\hat{\mu}} = 0$ processo iterativo

$$U_\mu = \frac{m(k-\pi)(1-\mu)^{(m-1)}}{1-(1-\mu)^m} + \frac{k\bar{z}}{\mu} - \frac{k(m-\bar{z})}{1-\mu}$$

$$U'_\mu = \frac{m(k-\pi)}{\{1-(1-\mu)^m\}^2}\Big[-(m-1)(1-\mu)^{(m-2)}\{1-(1-\mu)^m\}$$

$$-m(1-\mu)^{(m-1)}(1-\mu)^{(m-1)}\Big] - \frac{k\bar{z}}{\mu^2} + \frac{k(m-\bar{z})}{(1-\mu)^2}$$

$$= -m(k-\pi)\frac{(1-\mu)^{(m-2)}}{\{1-(1-\mu)^m\}^2}\{(m-1)+(1-\mu)^m\}$$

$$-\frac{k\bar{z}}{\mu^2} + \frac{k(m-\bar{z})}{(1-\mu)^2}$$

$$K_{\mu\mu} = E\{-U'_\mu\} = mk(1-\pi)\frac{(1-\mu)^{(m-2)}}{\{1-(1-\mu)^m\}^2}\{(m-1)+(1-\mu)^m\}$$

$$+\frac{kE(\bar{z})}{\mu^2} - \frac{k(m-E(\bar{z}))}{(1-\mu)^2}$$

$$E(\bar{z}) = E(z) = \frac{m\mu(1-\pi)}{1-(1-\mu)^m} \qquad Var(\hat{\mu}) = \frac{1}{K_{\mu\mu}}$$