$$f(y_j; \psi) = \binom{m}{y_j} \left(\frac{\psi}{1+\psi}\right)^{y_j} \left(\frac{1}{1+\psi}\right)^{(m-y_j)}$$

$\log(\psi) = \alpha$, $y_j = 0, 1, \dots m$ e $j = 1, \dots, k$.

$$f(y_j; \alpha) = \binom{m}{y_j} \left(\frac{e^{\alpha}}{1+e^{\alpha}}\right)^{y_j} \left(\frac{1}{1+e^{\alpha}}\right)^{(m-y_j)}$$

$$\Rightarrow \log\{f(y_j; \alpha)\} = \log\binom{m}{y_j}$$

$$+ y_j\alpha - y_j \log(1+e^{\alpha}) - (m-y_j)\log(1+e^{\alpha})$$

$$= y_j\alpha + m\log(1+e^{\alpha})$$

$$L(\alpha) = \sum_{j=1}^{k} \log\{f(y_j; \alpha)\}$$

$$= k\bar{y}\alpha - km\log(1+e^{\alpha})$$

$$U_{\alpha} = \frac{dL(\alpha)}{d\alpha}$$

$$U_\alpha = k\bar{y} - \frac{kme^\alpha}{1+e^\alpha}$$

$$U_{\hat{\alpha}} = 0 \Rightarrow \frac{me^{\hat{\alpha}}}{1+e^{\hat{\alpha}}} = \bar{y}$$

$$\Rightarrow \frac{\bar{y}}{m} = \frac{e^{\hat{\alpha}}}{1+e^{\hat{\alpha}}}$$

$$\Rightarrow \hat{\alpha} = \log\left\{\frac{\bar{y}/m}{1-\bar{y}/m}\right\} = \log\left(\frac{\bar{y}}{m-\bar{y}}\right)$$

$\bar{y} \neq 0$, $\overline{m} \neq 0$

$H_0: \alpha = 1$

$H_1: \alpha \neq 1$

$$\xi_{RV} = 2\left[L(\hat{\alpha}) - L(1)\right]$$

$$= 2\left\{k\bar{y}\hat{\alpha} - km\log(1+e^{\hat{\alpha}}) - k\bar{y} + km\log(1+e)\right\}$$

$$\xi_{RV} = 2k\left[\overline{y}\,(\hat{\alpha}-1) + m\log\left\{\frac{1+e}{1+e^{\hat{\alpha}}}\right\}\right]$$

$$\xi_{RV} \underset{m\to\infty}{\overset{H_0}{\sim}} \chi^2_1$$